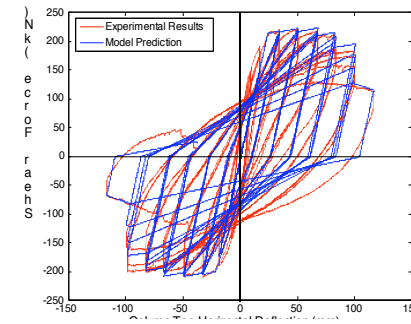
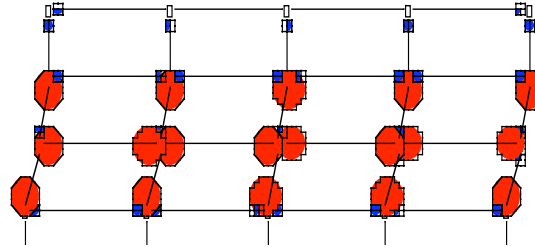
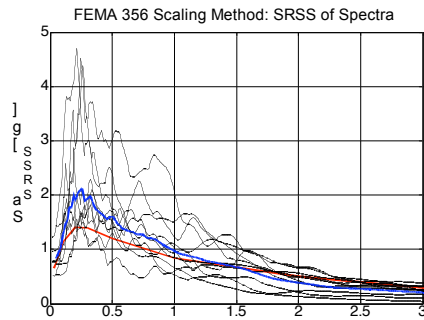


# Performance Assessment Through Nonlinear Time History Analysis



Greg Deierlein

Stanford University

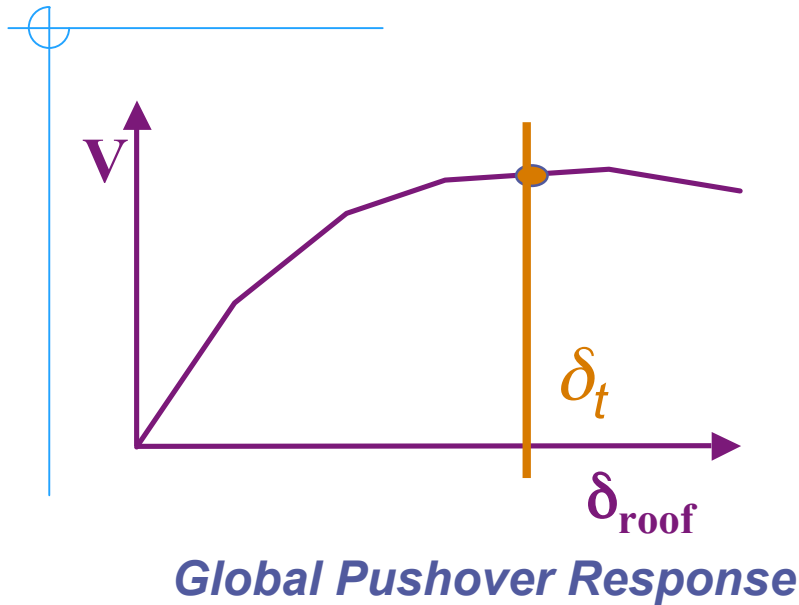
with contributions by

Curt Haselton & Abbie Liel

Stanford University



# PBEE Current "Best" Practice: FEMA 273/356



- **Nonlinear "Pushover Analysis"**

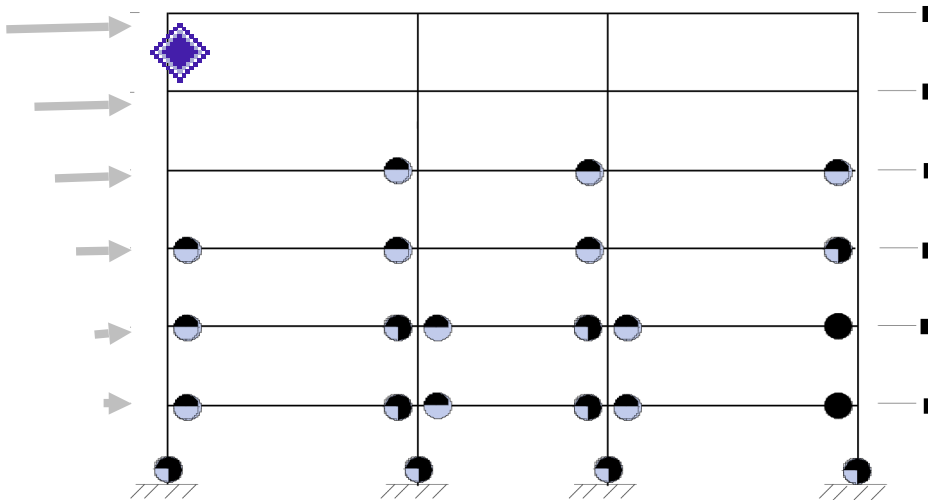
- Modeling Assumptions
- Force Distribution
- Target Displacement ( $S_a$ )

- **Component Modeling Criteria**

- "Backbone Curve"

- **Component Acceptance Criteria**

- Force Controlled Elements
- Deformation Controlled Elements



# Example: Criteria for RC Beams (FEMA 273)

**Table 6-6 Modeling Parameters and Numerical Acceptance Criteria for Nonlinear Procedures—Reinforced Concrete Beams**

Conditions	Modeling Parameters <sup>3</sup>			Acceptance Criteria <sup>3</sup>				
	Plastic Rotation Angle, radians		Residual Strength Ratio	Plastic Rotation Angle, radians				
				Component Type				
	a		b	Primary		Secondary		
				Performance Level				
IO	LS	CP	LS	CP				

**i. Beams controlled by flexure<sup>1</sup>**

$\frac{\rho - \rho'}{\rho_{bal}}$	Trans. Reinf. <sup>2</sup>	$\frac{V}{b_w d \sqrt{f'_c}}$								
≤ 0.0	C	≤ 3	0.025	0.05	0.2	0.005	0.02	0.025	0.02	0.05
≤ 0.0	C	≥ 6	0.02	0.04	0.2	0.005	0.01	0.02	0.02	0.05
≥ 0.5	C	≤ 3	0.02	0.03	0.2	0.005	0.01	0.02	0.02	0.05
≥ 0.5	C	≥ 6	0.015	0.02	0.2	0.005	0.005	0.02	0.02	0.05
≤ 0.0	NC	≤ 3	0.02	0.03	0.2	0.005	0.01	0.02	0.02	0.05
≤ 0.0	NC	≥ 6	0.01	0.015	0.2	0.0	0.005	0.02	0.02	0.05
≥ 0.5	NC	≤ 3	0.01	0.015	0.2	0.005	0.01	0.02	0.02	0.05
≥ 0.5	NC	≥ 6	0.005	0.01	0.2	0.0	0.005	0.02	0.02	0.05

**ii. Beams controlled by shear<sup>1</sup>**

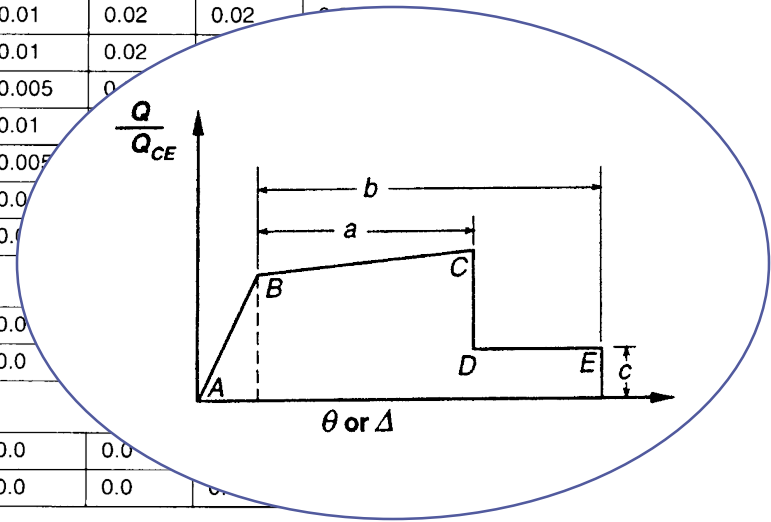
Stirrup spacing ≤ d/2	0.0	0.02	0.2	0.0	0.0	0.0	0.0	0.02	0.02	0.05
Stirrup spacing > d/2	0.0	0.01	0.2	0.0	0.0	0.0	0.0	0.02	0.02	0.05

**iii. Beams controlled by inadequate development or splicing along the span<sup>1</sup>**

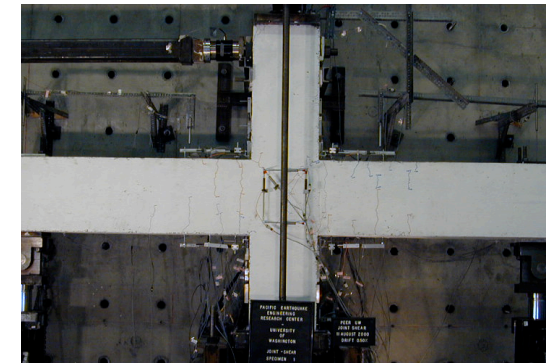
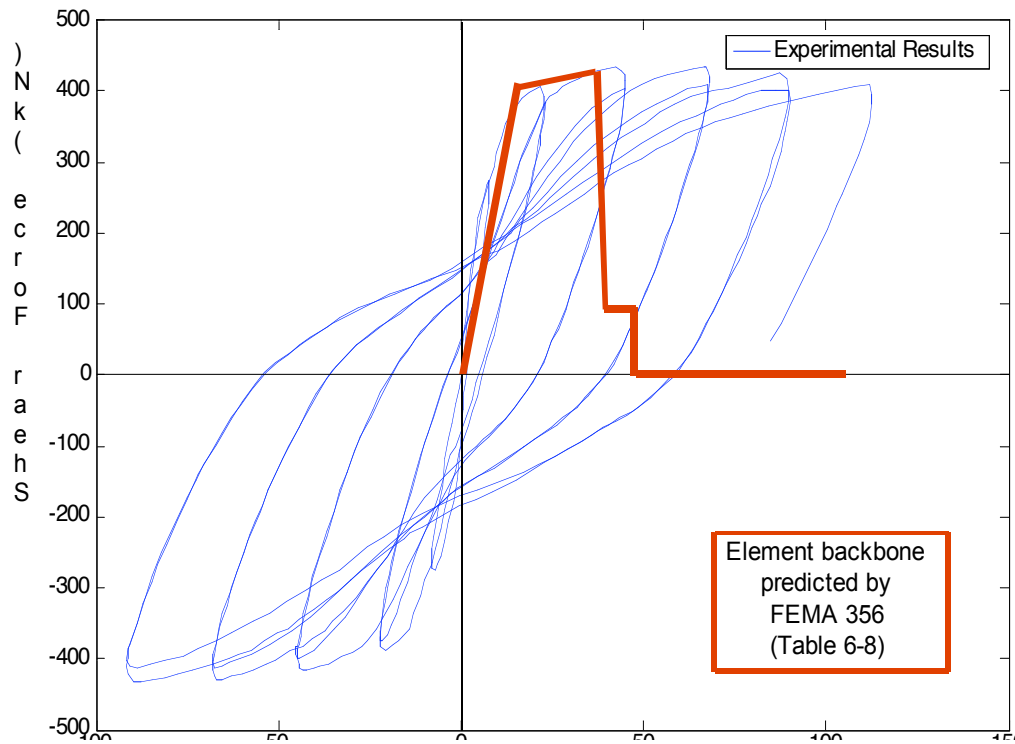
Stirrup spacing ≤ d/2	0.0	0.02	0.0	0.0	0.0	0.0	0.0	0.02	0.02	0.05
Stirrup spacing > d/2	0.0	0.01	0.0	0.0	0.0	0.0	0.0	0.02	0.02	0.05

**iv. Beams controlled by inadequate embedment into beam-column joint<sup>1</sup>**

	0.015	0.03	0.2	0.01	0.01	0.015	0.02	0.02	0.02	0.05
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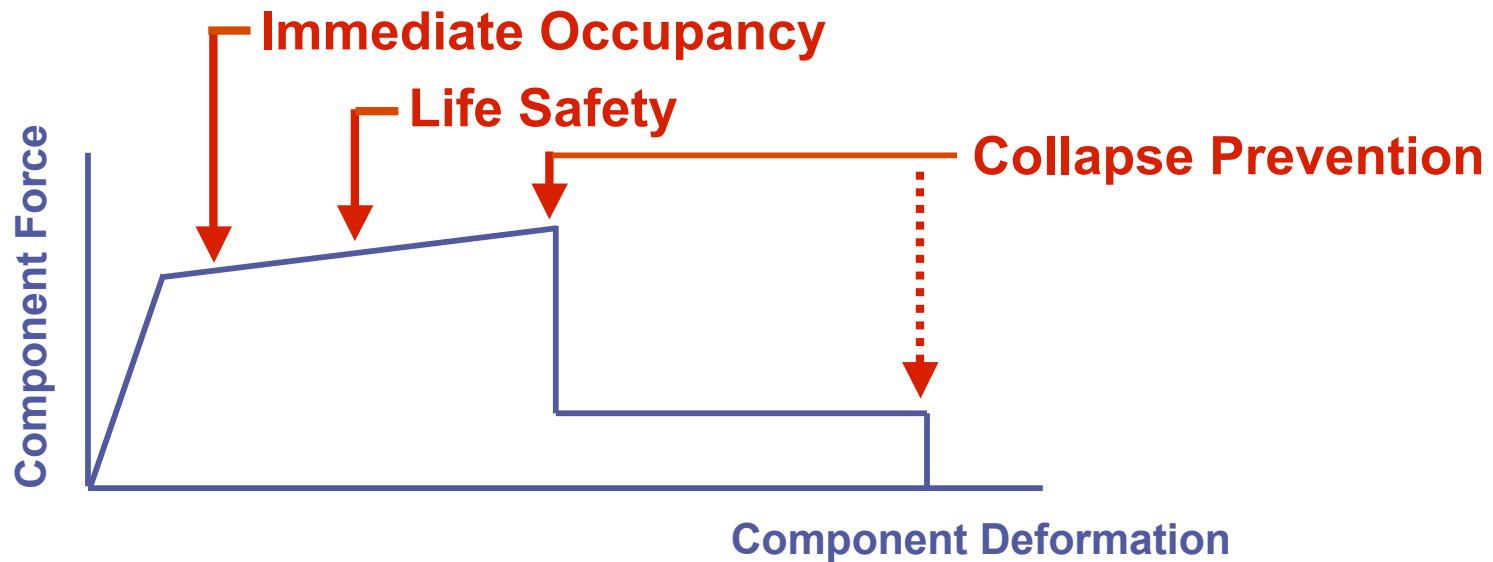
# Shortcomings of FEMA 273/356



## Component Backbone Curve:

- Overly Idealized
- Conservative
- Deterministic

# Shortcomings of FEMA 273/356



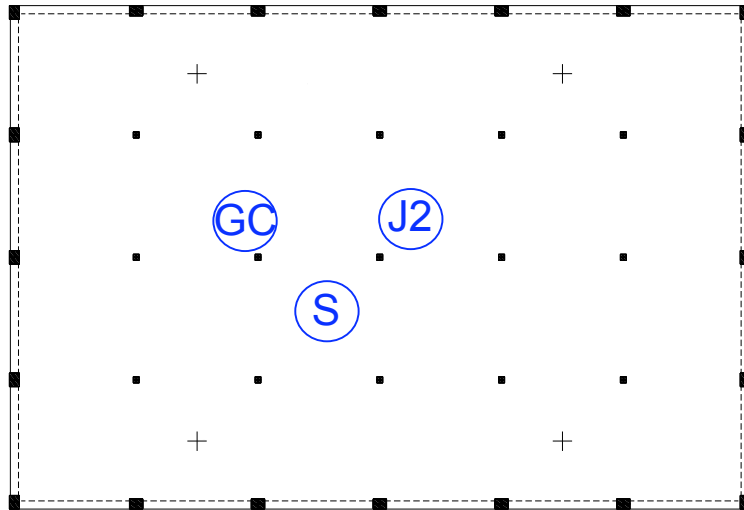
Component Backbone Curve

- Over-reliance on idealized (simplified) **local component demand** indices to predict **system response**
- Ambiguous relationships between **structural indices** and **building performance**
- Limited emphasis on static monotonic pushover approach

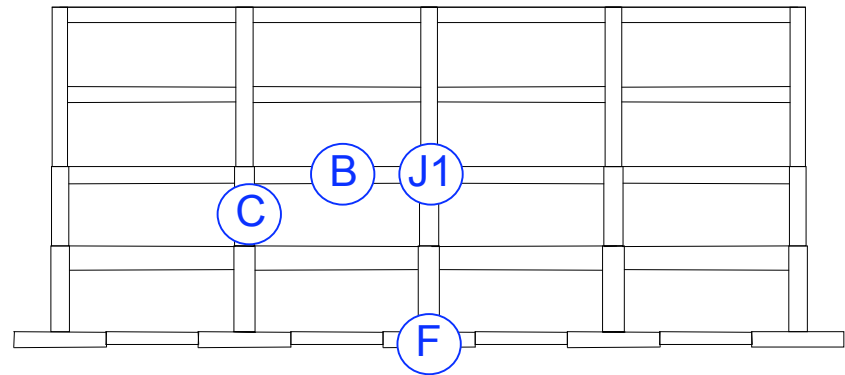
# Assessment Using Improved NLTH Analysis

- ◆ Nonlinear Component and System Modeling
- ◆ FEMA 356 Concepts with NLTH Analysis
- ◆ Preview of Comprehensive Collapse Simulation

# Structural System & Components



Floor Framing Plan



Perimeter Frame Elevation

## Moment Frames

- Beams, Columns, B-C Joints, and Foundations

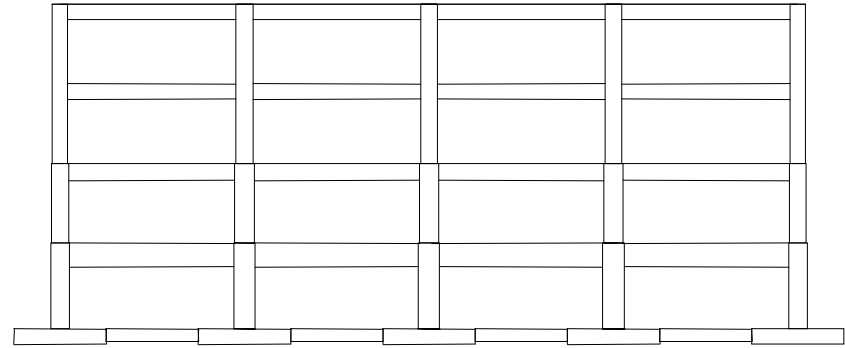
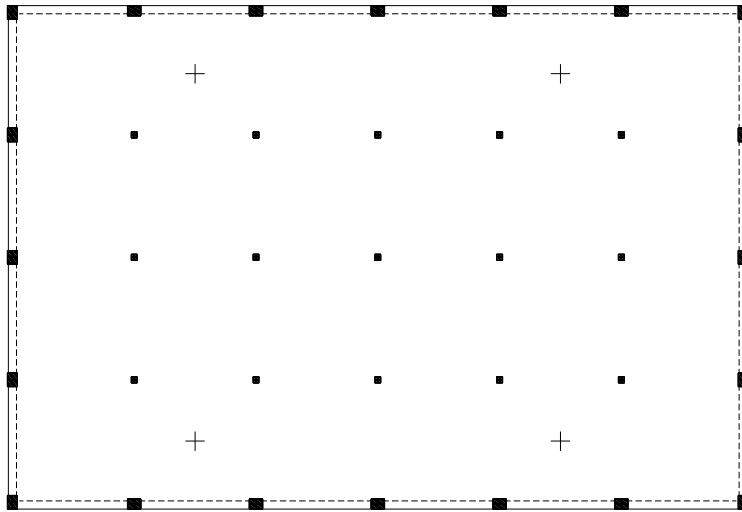
## Gravity Frames

- Slab/beams, Gravity Columns, S-C Joints, and Foundations

Shear Walls (not shown)

**Issue:** Whether or not to consider the lateral resistance of the “gravity system” in the simulation. There gravity system can provide significant enhancement in a nonlinear

# Deterioration Modes & Collapse Scenarios



## 1. Deterioration Modes of RC Elements

- Simulation vs. Fragility Models

## 2. Building System Collapse Scenarios

- Sidesway Collapse (SC)
- Loss in Vertical Load Carrying Capacity (LVCC)

## 3. Likelihood of Collapse Scenarios

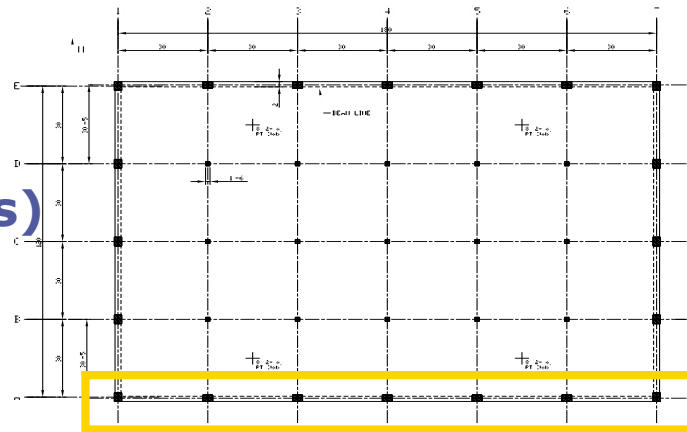
- Existing vs. New Construction
- “Ordinary” versus “Special” seismic design



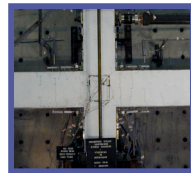
# Simulation Model



**Gravity Frame(s)**



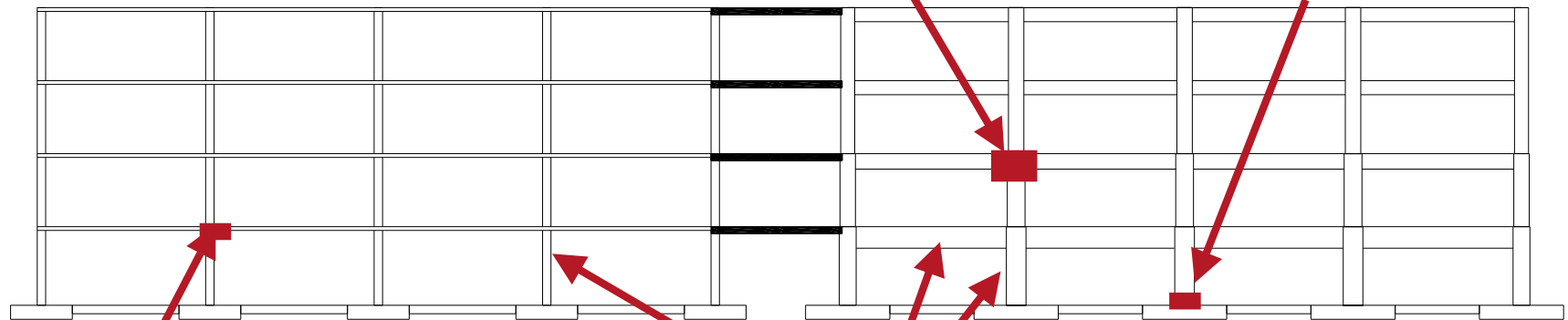
**Lateral Frame**



**Joints with both bond-slip springs and shear springs**

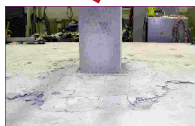


**Column bond-slip springs**

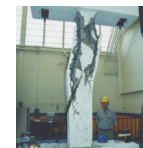


**Gravity Frame(s)**

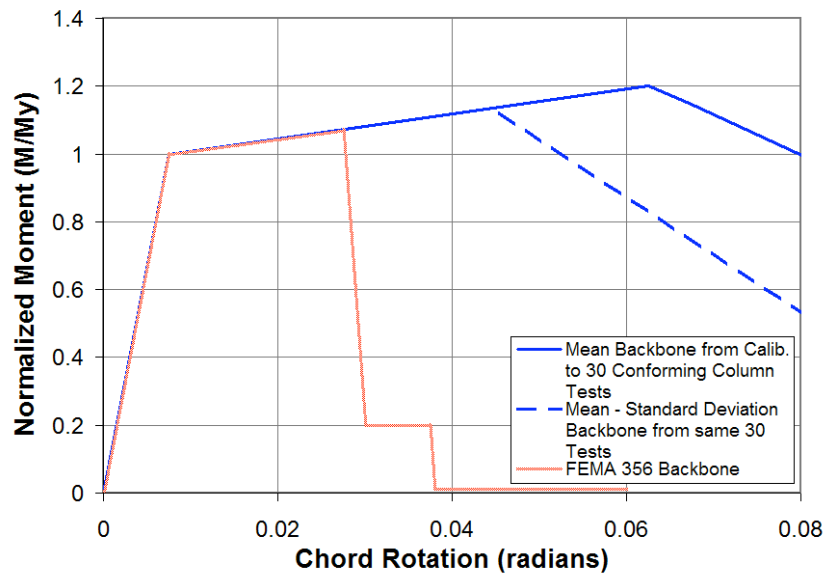
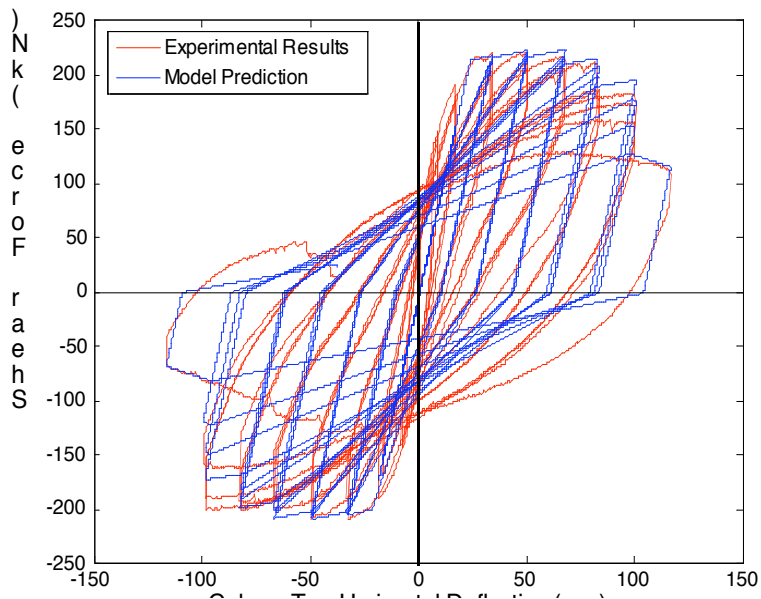
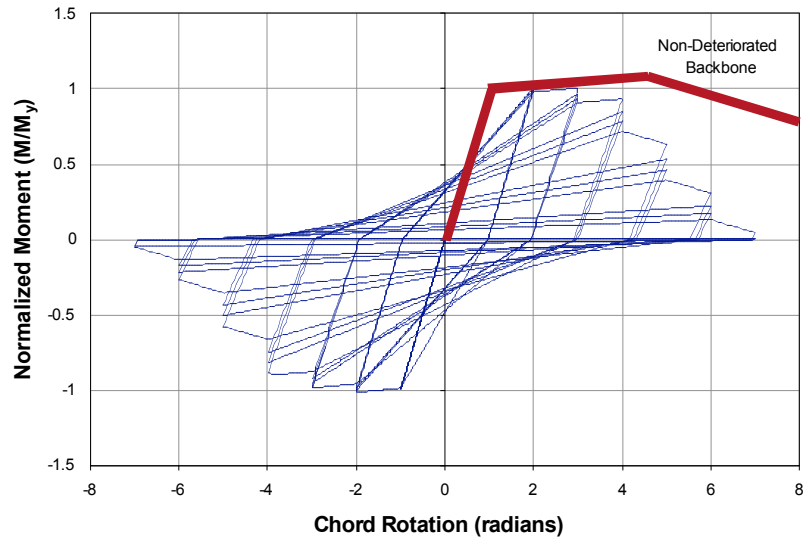
**Lateral Frame**



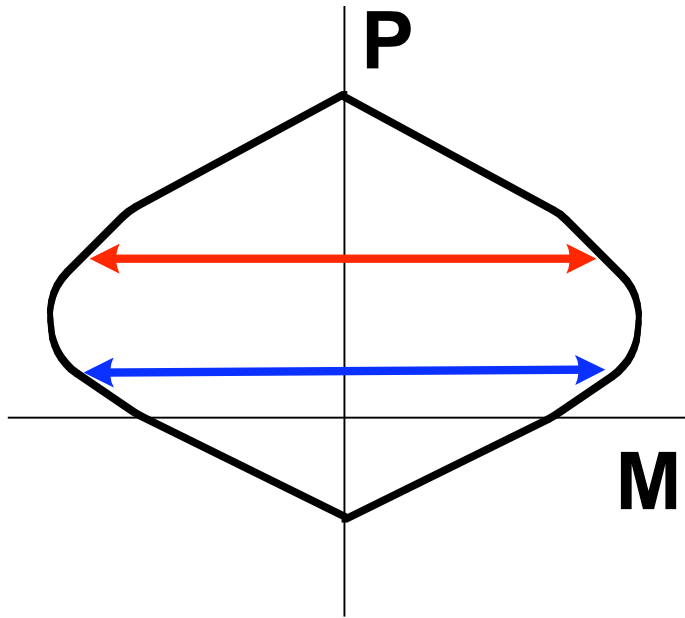
**Lumped plasticity beam-columns**



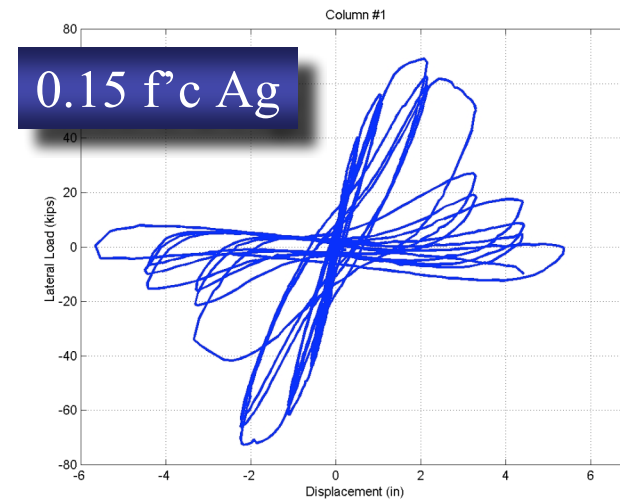
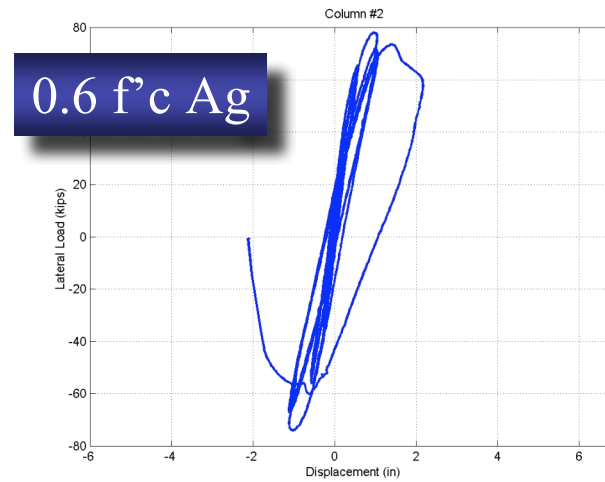
# More realistic component simulation

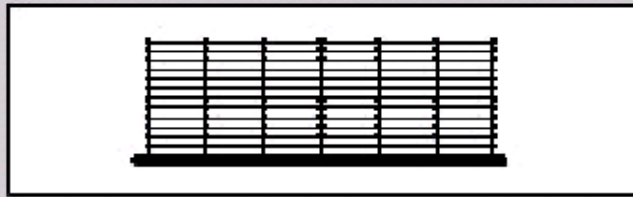


# Illustration: Axial Load & Post-Peak Response

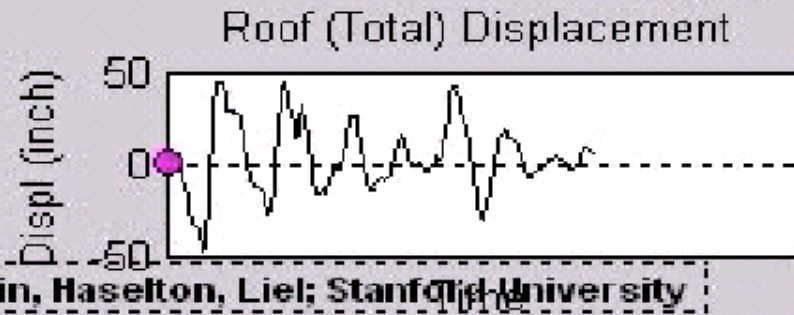
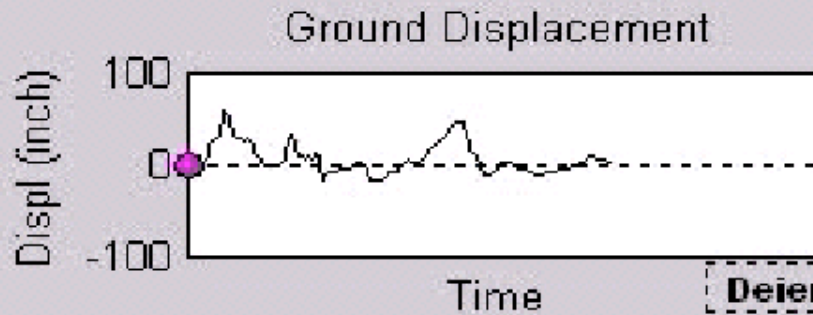
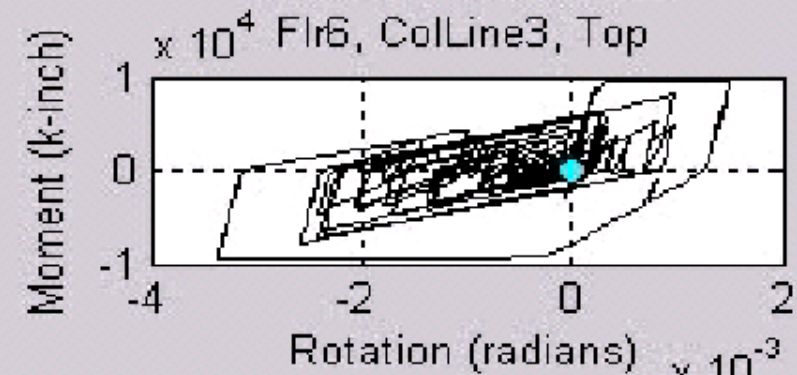
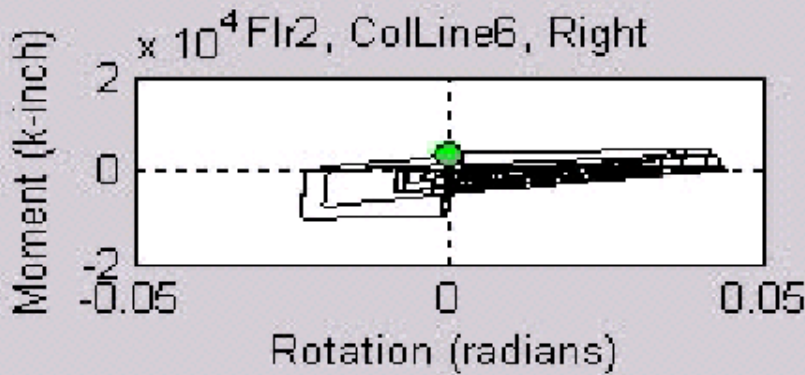
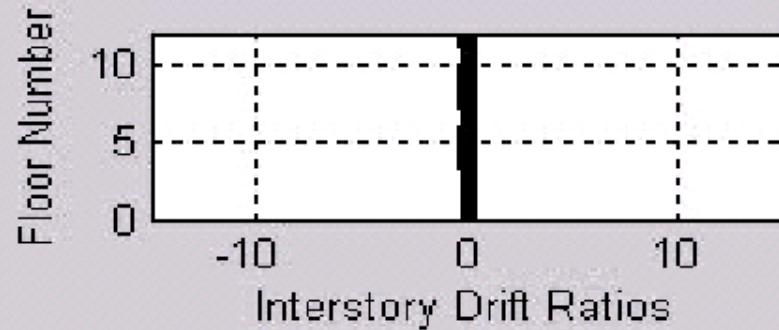


Key Parameter:  $P/P_{\text{balance}}$



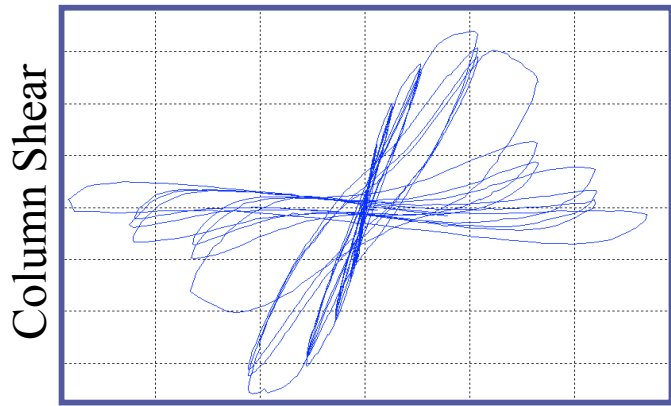


EQ: 11142,  $Sa_{code}(T=1\text{sec}): 1.79g$



Deierlein, Haselton, Liel; Stanford University

# Beam-Column Modeling Alternatives

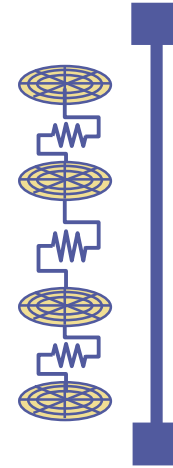


Drift

Physical  $\longrightarrow$  Phenomenological



Continuum



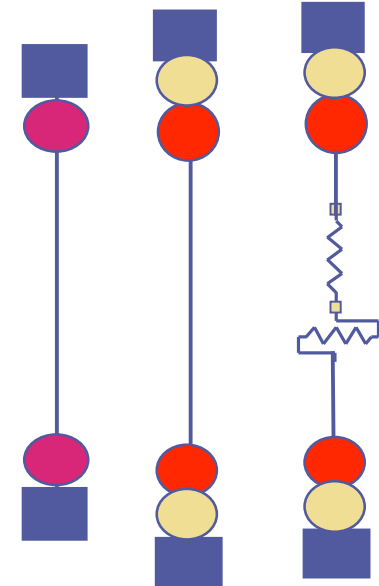
Distributed  
Inelasticity (fiber)



Concentrated  
Hinge

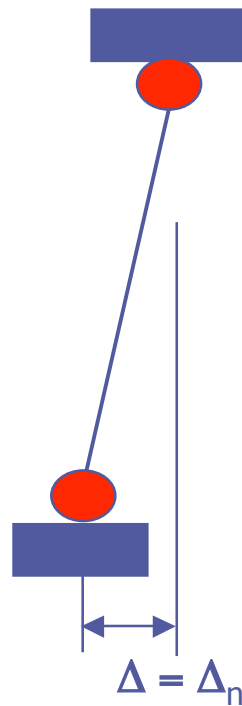
# Beam-Column Model Considerations

- Flexural Deformations
  - concrete cracking/tension stiffening
  - reinforcing bar yielding
  - concrete crushing
- Shear Deformations
  - uncracked, cracked
- Anchorage Bond Slip
  - pre and post-yield
- Critical Failure Modes and Deterioration
  - lateral tie fracture ... concrete crushing, rebar buckling
  - longitudinal bar buckling/fracture
  - PMV interaction

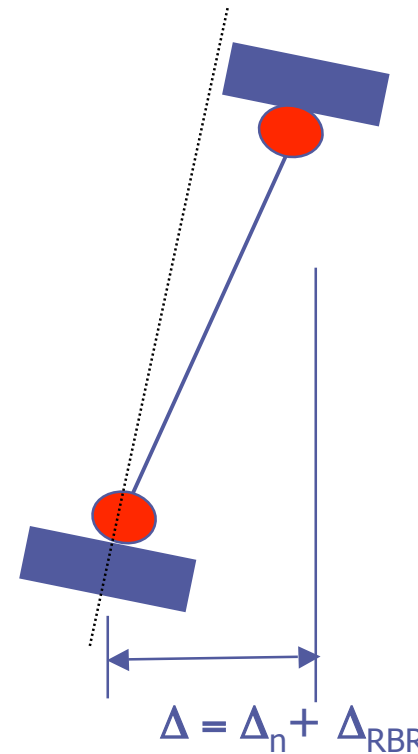


# Beam-Column Model Considerations, cont'd

- Definition of Displacements and Deformations
  - Total  $\Delta$  = Distortional (or "Natural")  $\Delta$  + Rigid Body  $\Delta$
  - Total  $\Delta$  = *Clear Story Drift*
  - Damage is typically associated with distortional deformations

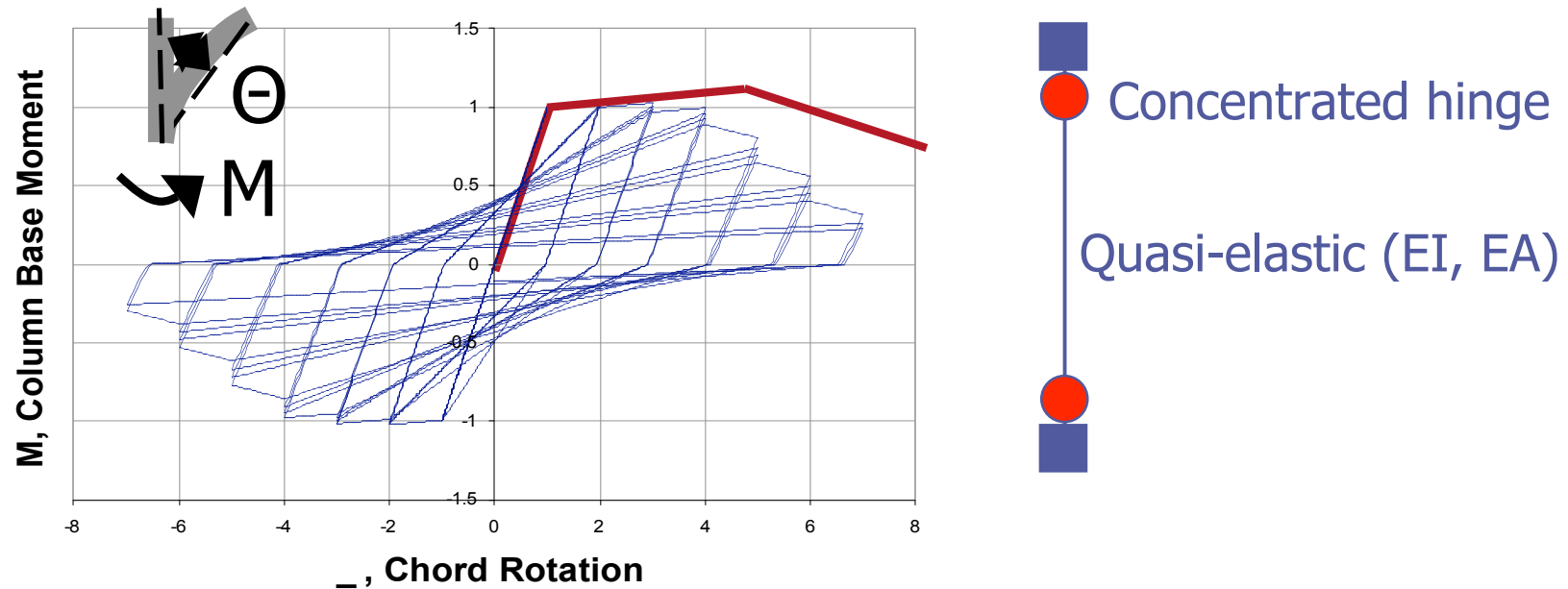


Drift from Experiment



Drift from Frame Analysis

# Standardized Inelastic Component Models

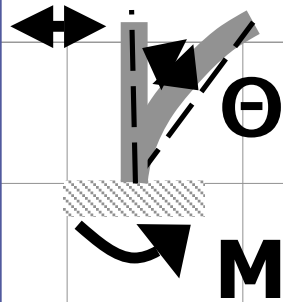
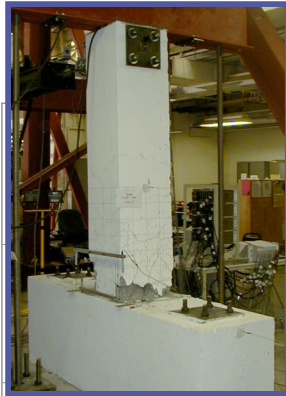


- Model Input
  - Physical Design Parameters (material, configuration, geometry, details, ...)
  - Calculated/calibrated backbone parameters (***mean and COVs*** for anchor points and hysteretic response parameters)
- Model Output: Engineering Demand Parameters (e.g.,  $\theta_{plastic}$ )

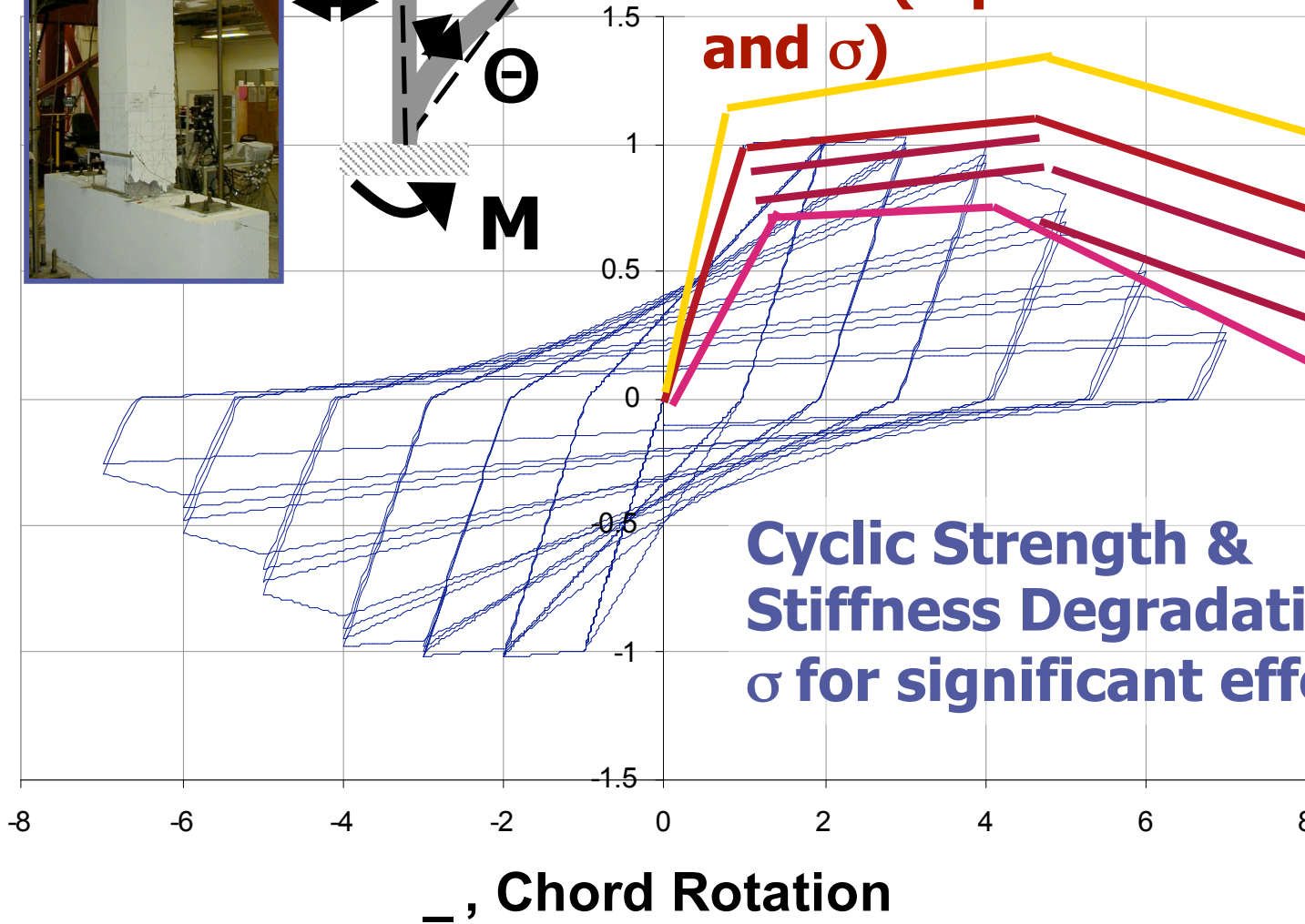


# Element Models for Collapse Analyses

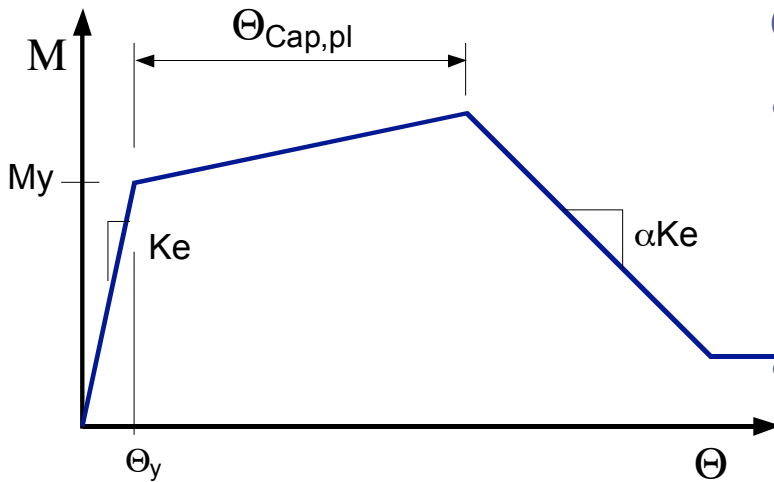
M, Column Base Moment



**Monotonic Backbone Curve (5 parameters,  $\mu$  and  $\sigma$ )**



# RC Beam-Column Simulation Model Calibration



## OVERVIEW OF CALIBRATION EFFORT

- Basic Hysteretic Model

5 parameter backbone curve

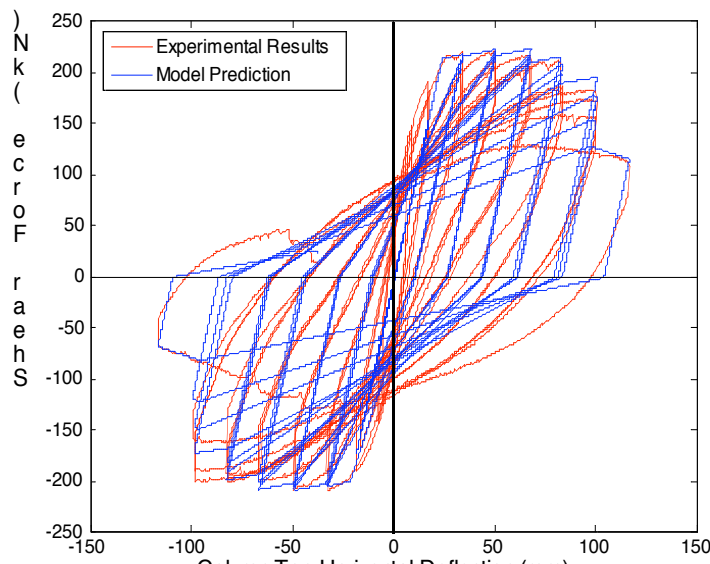
2 (x4) hysteretic parameters

- Previous RC Behavioral Studies

-Fardis et al. ( $\Theta_{cap}$ ,  $\Theta_u$ )

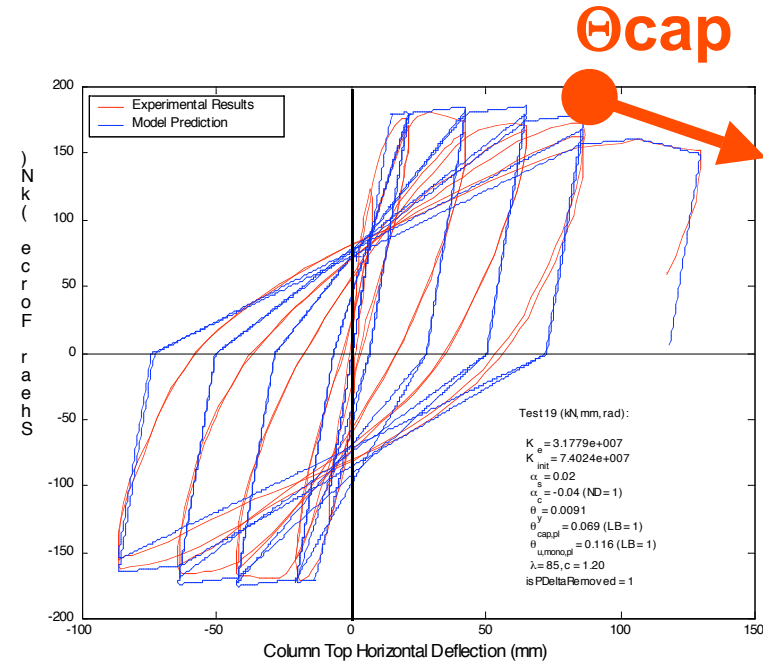
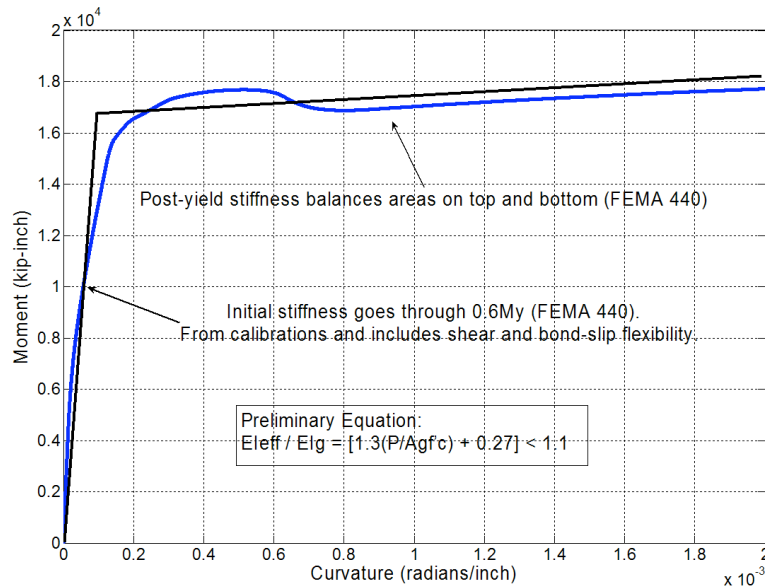
-Eberhard et al. (EDP criteria for spalling and bar buckling)

- Current effort: Systematic calibration to 226 flexurally dominated columns



- **Goal:** *Validated model to be vetted through consensus process*

# RC Beam-Column Parameters



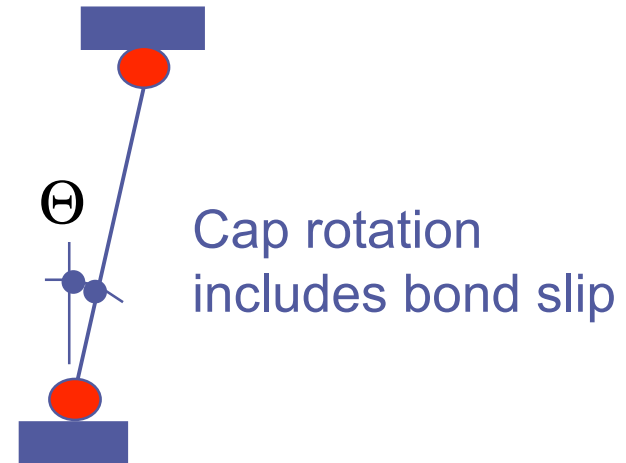
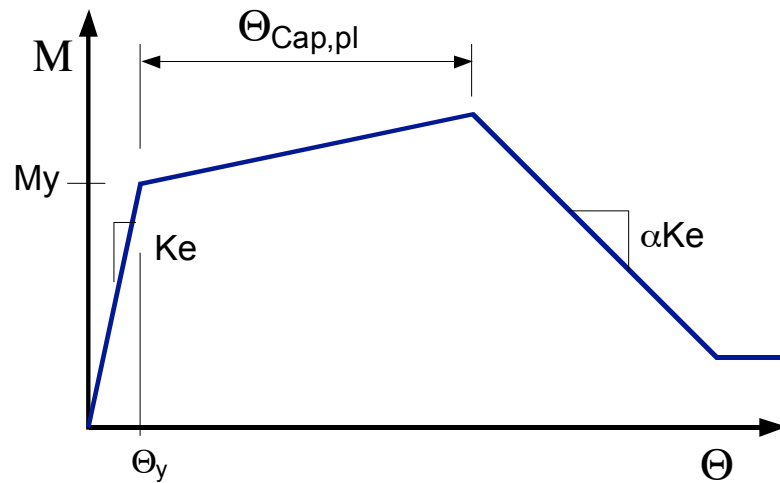
**Semi-Empirical** -- calibrated from tests, fiber analyses, and basic mechanics:

- Secant Stiffness ( $EI_{eff}$ )
- Yield Strength ( $M_y$ )
- Hardening Stiffness

**Empirical** - calibrated from tests:

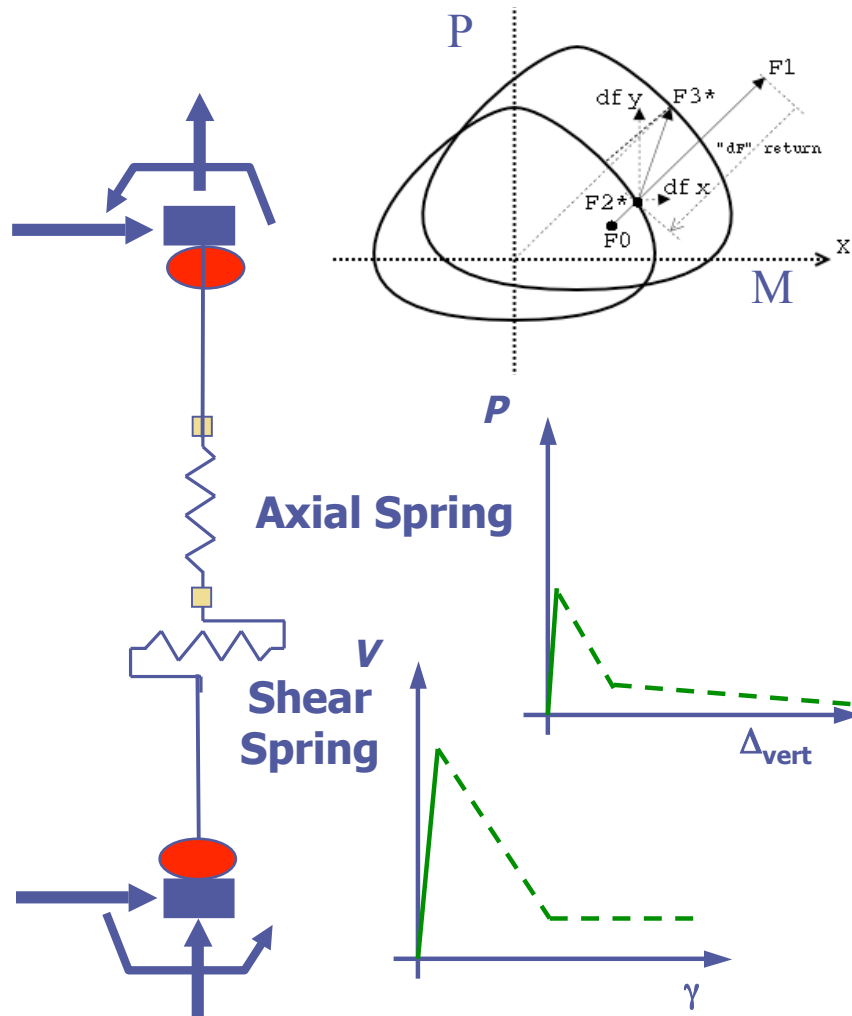
- Capping (peak) point ←
- Post-peak unloading (strain softening) stiffness ←
- Hysteretic stiffness/strength degradation

# Column Models (sample data)



COMPONENT	$\Theta_{cap,pl}$ (RAD)	COV	$\alpha$	COV
Beam - Conforming	0.07	60%	-0.05	60%
Beam - Nonconforming	0.02 to 0.05	"	-0.15	"
Column – conform, low axial	0.04 to 0.05	"	-0.05	"
Column – nonconf, low axial	0.02	"	-0.15	"
Column – nonconf, med. axial	0.01	"	-0.15	"

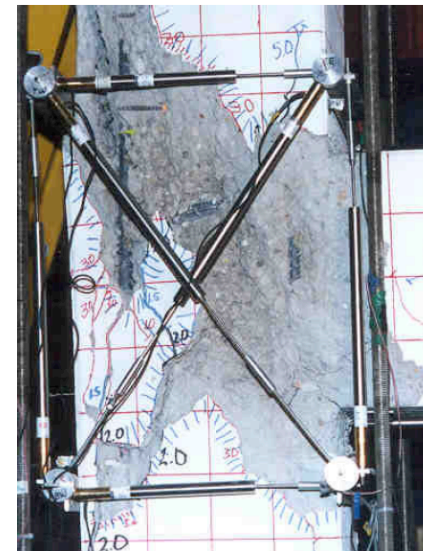
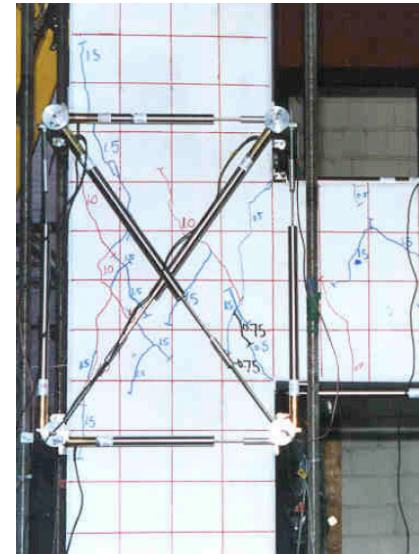
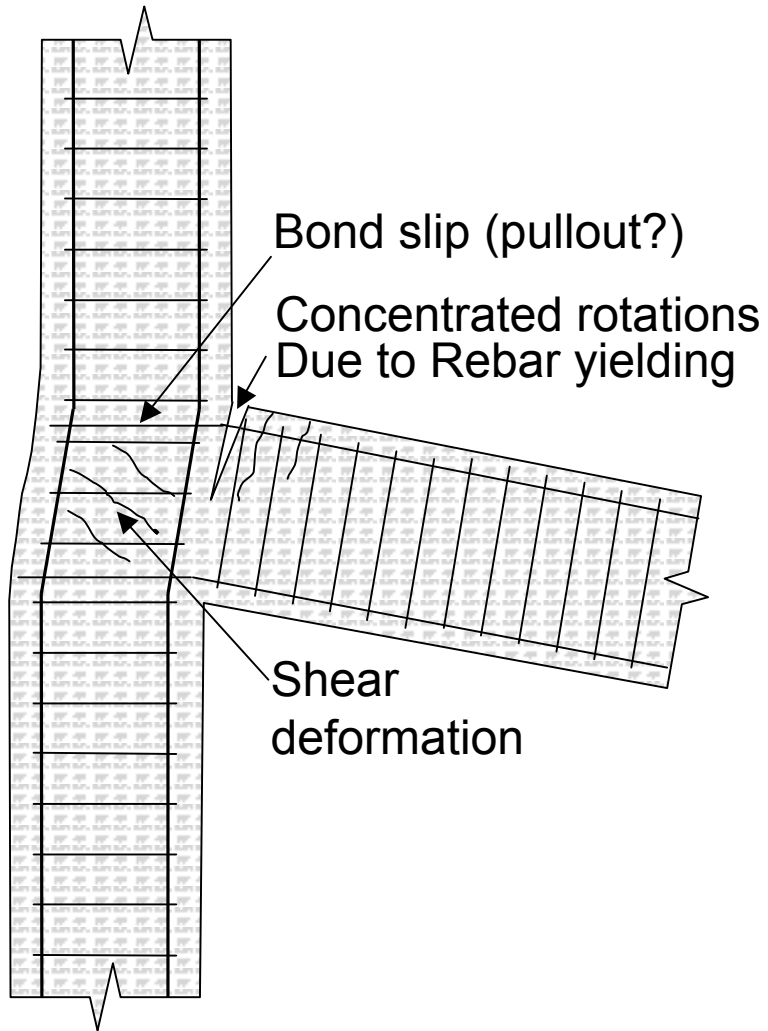
# Phenomenological P-M-V Hinge Element



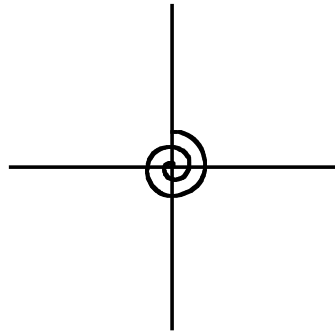
## Desired Model Features:

- direct modeling of P-M interaction through limit surface (strength, post-peak softening, hysteretic degradation)
- **DIRECT SIMULATION** (as opposed to limit state check) of column shear failure and axial failure (LVCC)
- More transparent modeling of flexibility introduced by bond slip and shear deformations.

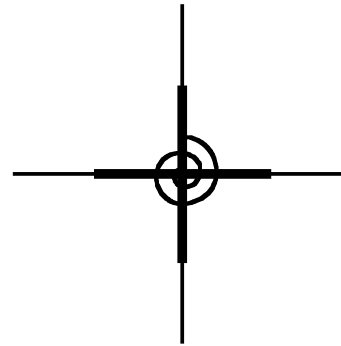
# Beam-Column Joint Models



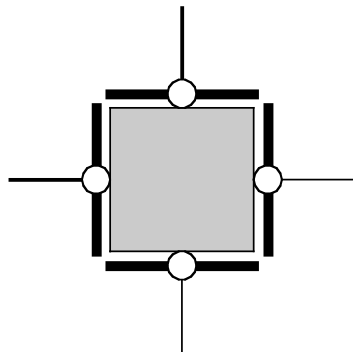
# Alternative Joint Models



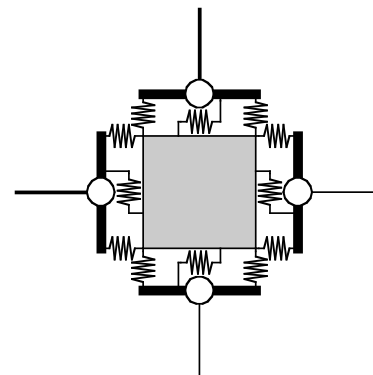
Single spring joint



Single Spring joint with rigid zones

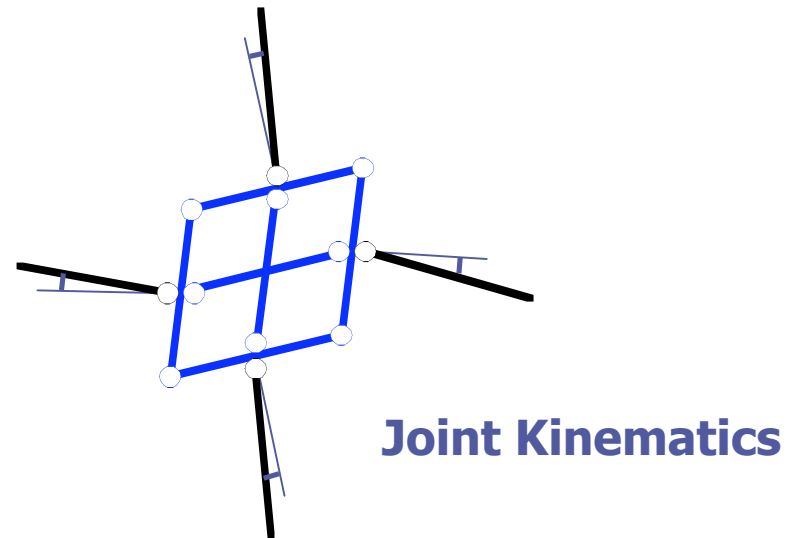
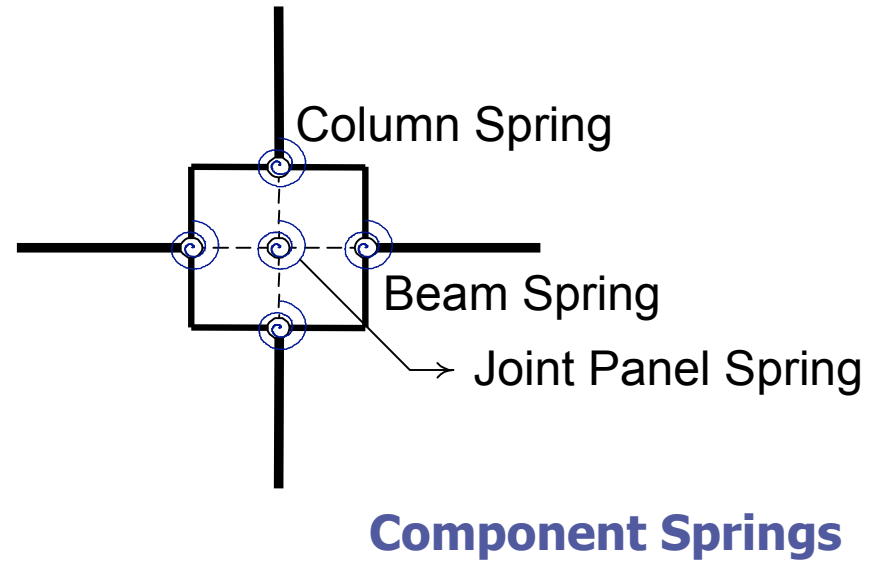
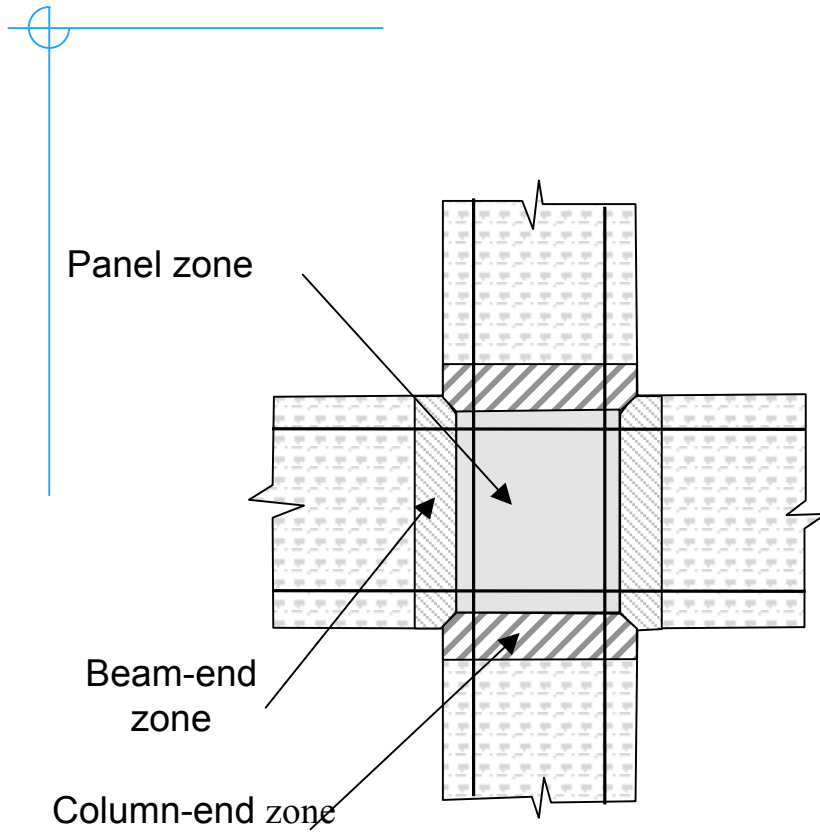


Continuum model



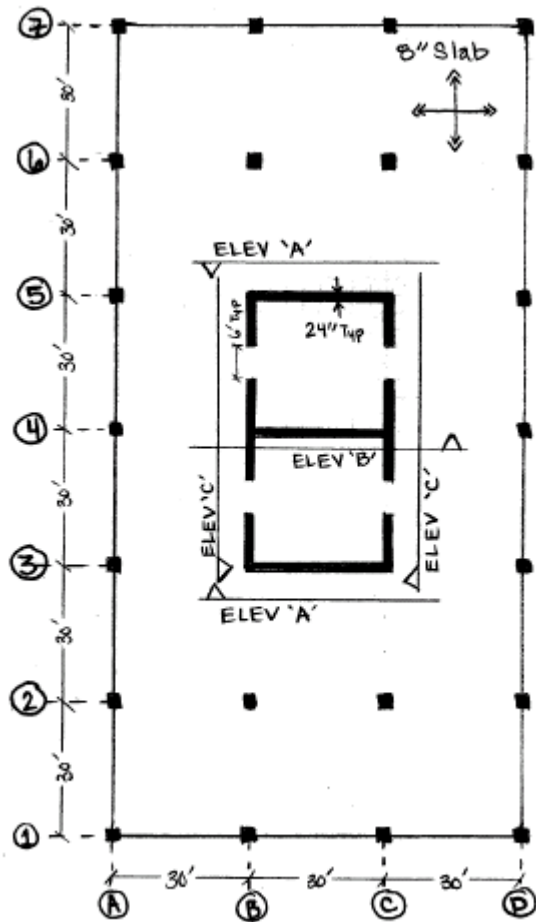
Multi spring super element

# Idealized Joint Model



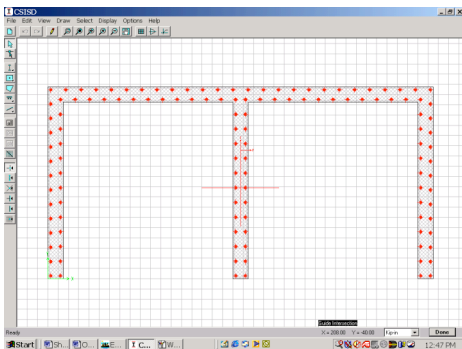
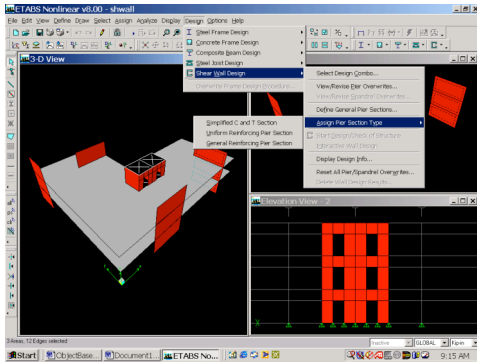


# Shear Wall Systems – Behavior Modes

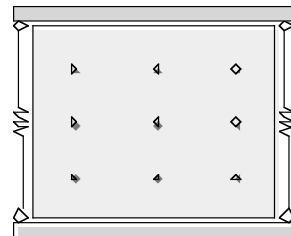
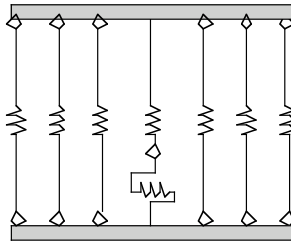
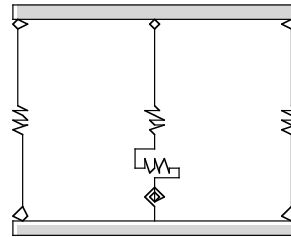


- Flexural Behavior
  - concrete cracking/tension stiffening
  - reinforcing bar yielding
  - concrete crushing
  - tie rupture – rebar buckling/fracture
- Shear Behavior
  - uncracked, cracked
  - shear failure
- Anchorage Bond Slip (base only)
  - pre and post-yield
- Coupling Beams
- Foundations
- System Compatibility
  - slab/column & slab/wall
  - column deformation

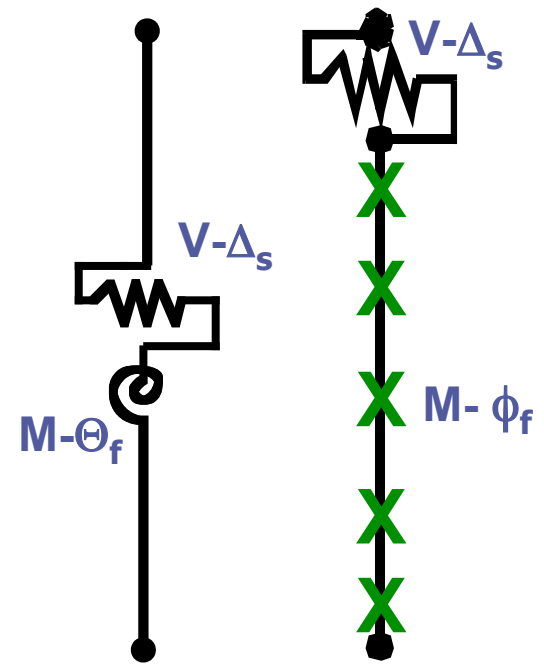
# Idealization of RC Walls



Continuum



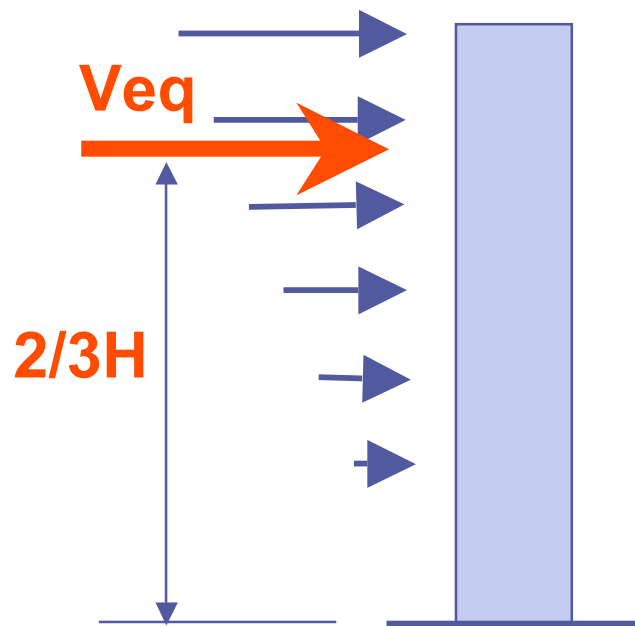
Multi-Spring



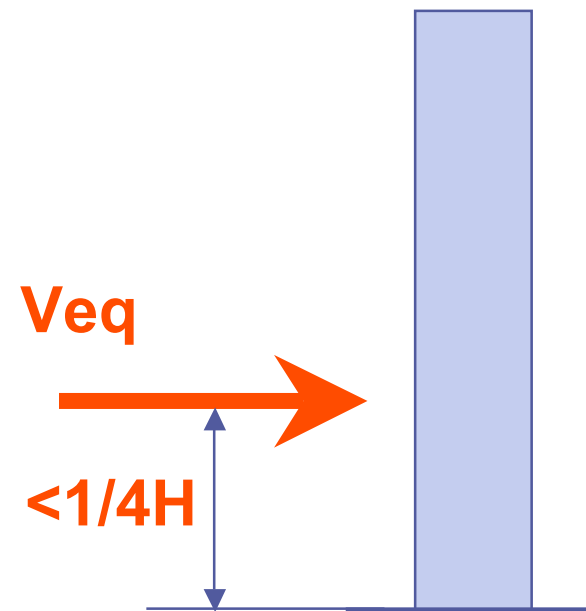
Concentrated Spring

# Shear Wall Modeling and Behavior

- Squat (short) walls versus tall flexural walls
- Inelastic Time History versus Equivalent Static Loading
  - surprising variations in flexural and shear demands
  - shears can be much higher than predicted by pushover analysis (e.g., Krawinkler & Zareian)



**Pushover**



**Time History**

# Viscous Damping with NLTH Analysis

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\{\ddot{x}_g\} + [P]$$

## Raleigh Damping:

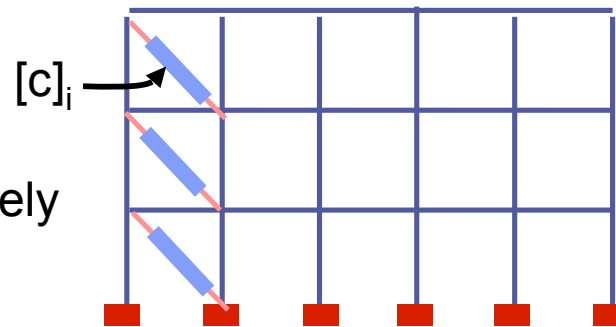
$$[C] = \alpha[M] + \beta[K]; \quad \zeta_n = \frac{\alpha}{2} \frac{1}{\omega_n} + \frac{\beta}{2} \omega_n$$

- For inelastic analysis, viscous damping should be on the order of  $\zeta_n = 5\%$ .
- Need to be careful how  $[K]$  is specified, i.e.,  $[K_e]$  versus  $[K_t]$ , since the choice will lead to variations in  $[C]$  during the analysis (see Bernal).

## Explicit Damping Elements (preferred?):

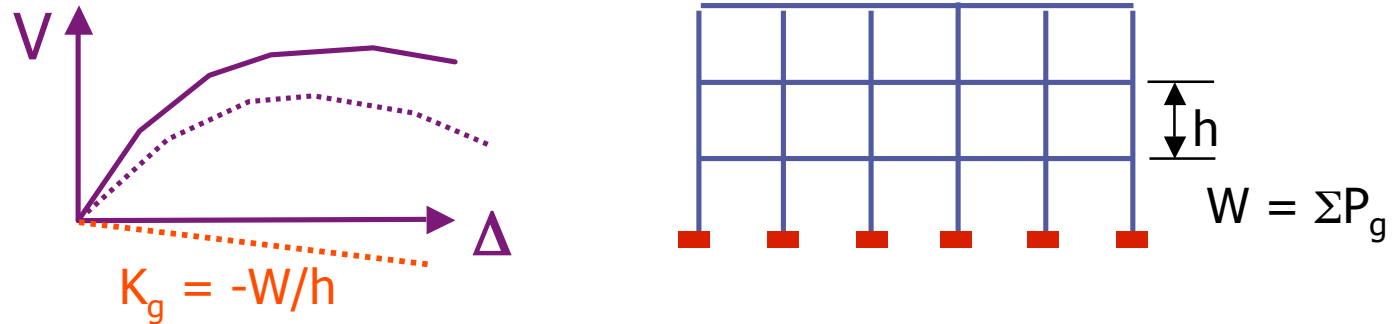
$$[C] = \sum [c]$$

$[c]_i$  configured to represent likely sources of viscous and other incidental damping.

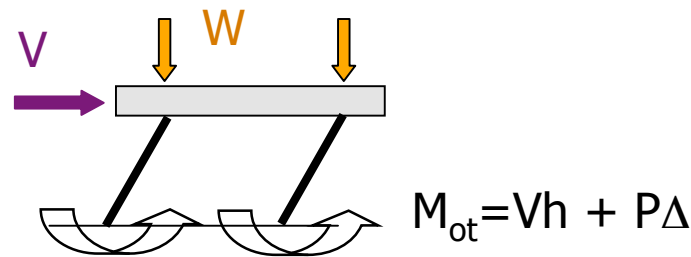


# Geometric Nonlinear (P-Δ) Effects

- ◆ Negative stiffness effect of P-Δ:



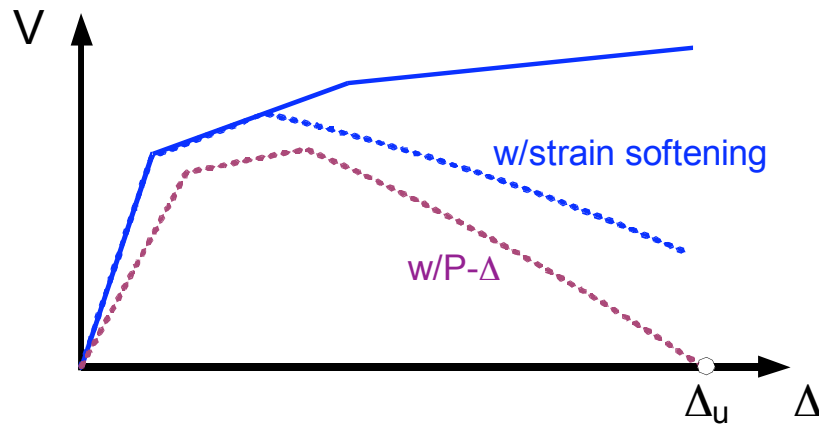
- ◆ Increases internal forces associated with overturning:



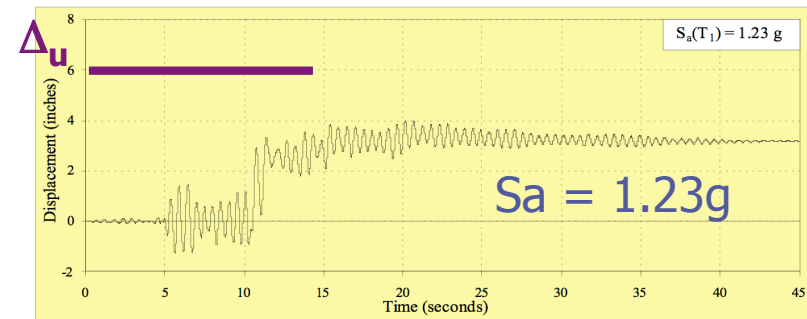
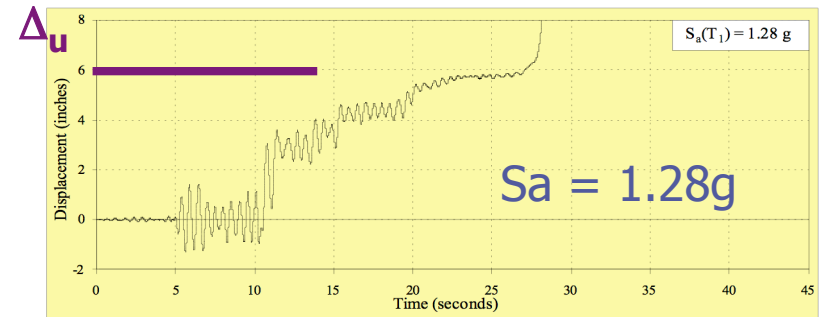
- ◆ Key Points

- "W" should represent the seismic mass that is being stabilized by the lateral system (not just the tributary gravity load)
- "Linear P-Δ" formulations accurate for drift ratios up to about 5-10%; beyond this large rotation (e.g., "co-rotational") formulations should be used to track response.

# Dynamic Sidesway Collapse



Static Pushover Analysis  
(Critical Story)



NLTH Analysis Results

- For structural systems governed by *sidesway collapse*, evidence suggests that the dynamic drift capacity is about 2/3 of the static pushover limit ( $\Delta_u$ ) --- provided that the static analysis represents strength degradation due to strain (displacement) softening and P- $\Delta$  effects.
- Collapse point can be very sensitive to ground motion intensity level and other effects.

# Assessment Using Improved NLTH Analysis

- ◆ Nonlinear Component and System Modeling
- ◆ FEMA 356 Concepts with NLTH Analysis
- ◆ Preview of Comprehensive Collapse Simulation

# FEMA 356 with NL Time History Analysis

## ◆ NL Analysis Model

- modeling assumptions (role of component backbone curves?)

## ◆ Selection of GM

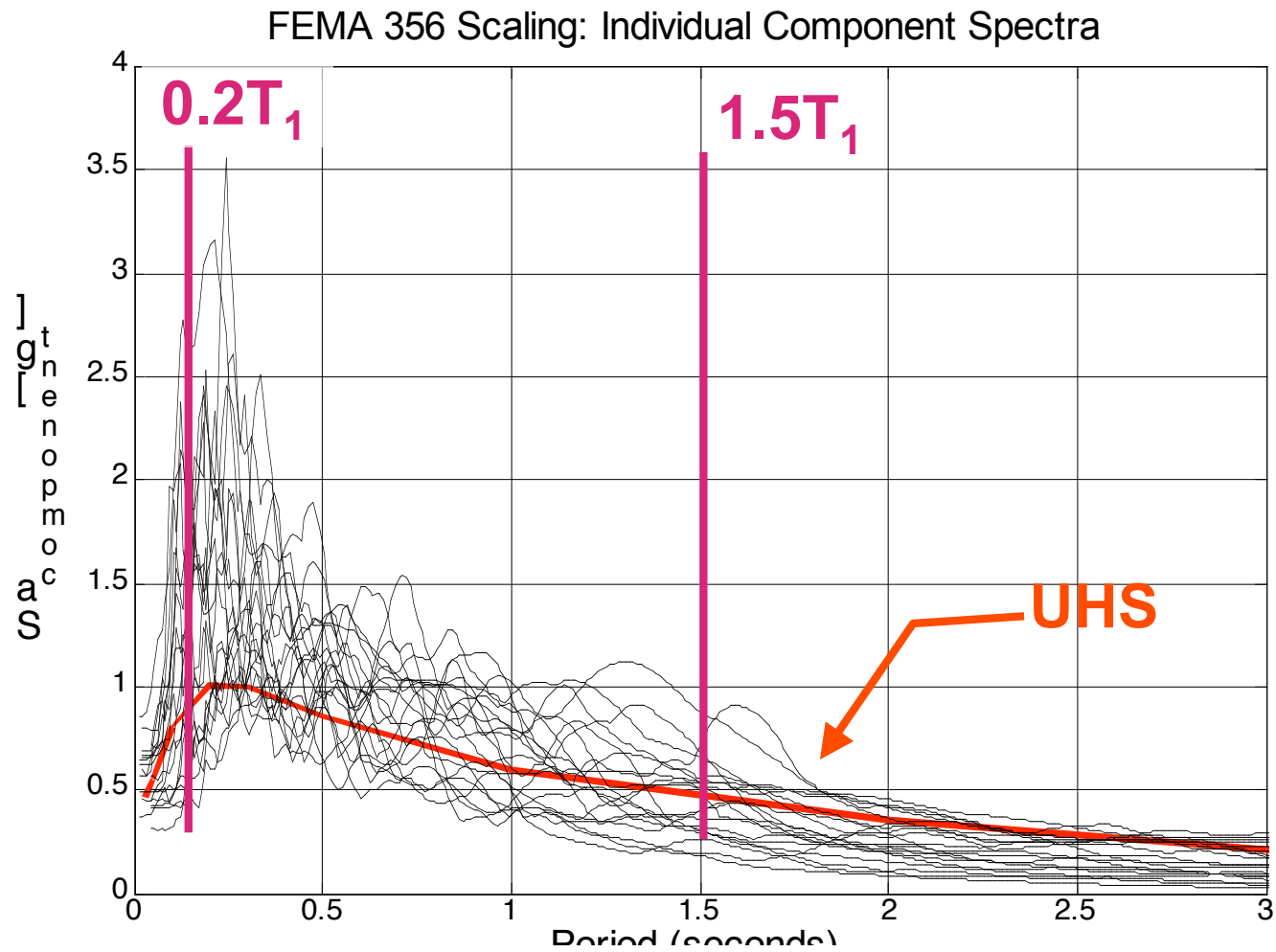
- match M, R and fault type/mechanism
- records from at least 3 events
- synthetics OK if necessary

## ◆ Scaling of GM to UHS to “design EQ”

- 5% damped spectra from SRSS of two orthogonal components
- Scale such that SRSS spectra  $> 1.4$  UHS for periods between  $0.2T_1$  and  $1.5T_1$



# Ex. – Ground Motion Scaling to 10/50 Hazard



**20 components for 10 pairs of EQ Records**

# FEMA 356 with NL Time History Analysis

## ◆ Acceptance Criteria

- Demand Parameters < Component Criteria
  - ◆ e.g.,  $\Theta_p < \Theta_{p,limit}$
- Evaluation based on either:
  - ◆ Maximum demand from results of 3 records
  - ◆ Average demand from results of >7 records

### **Concerns:**

- *statistical rationale for acceptance criteria ?*
- *implementation (which 3 records) ?*

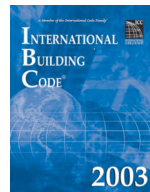
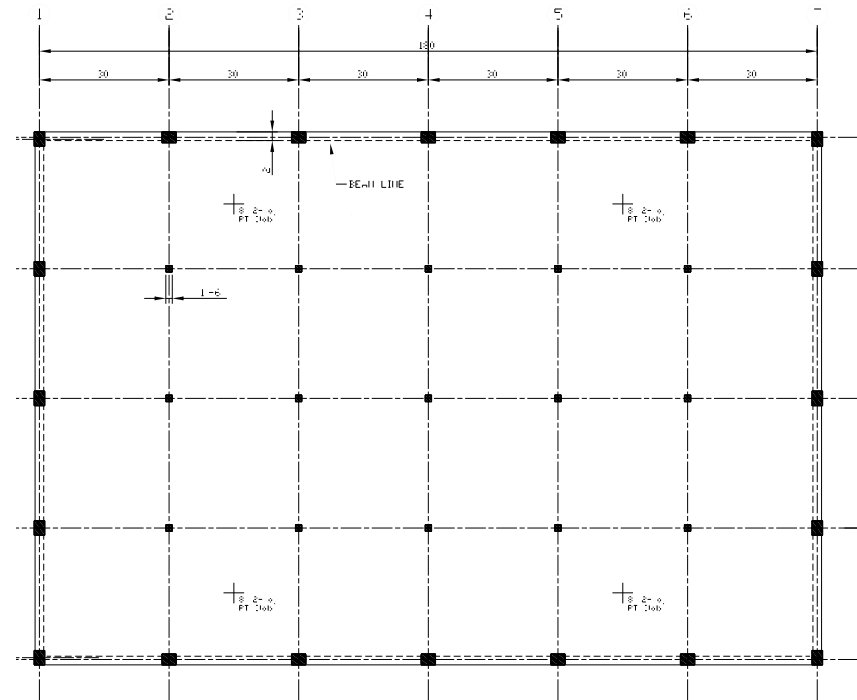
# “Enhanced FEMA 356”

- ◆ Realistic Inelastic Model
- ◆ Nonlinear Time History Analysis
- ◆ 20 ground motions (10 pairs) with their geometric mean scaled to hazard at  $S_a(T1)$
- ◆ Statistical evaluation of deformation demands to input ground motions
- ◆ Probabilistic assessment of component acceptance criteria to test data

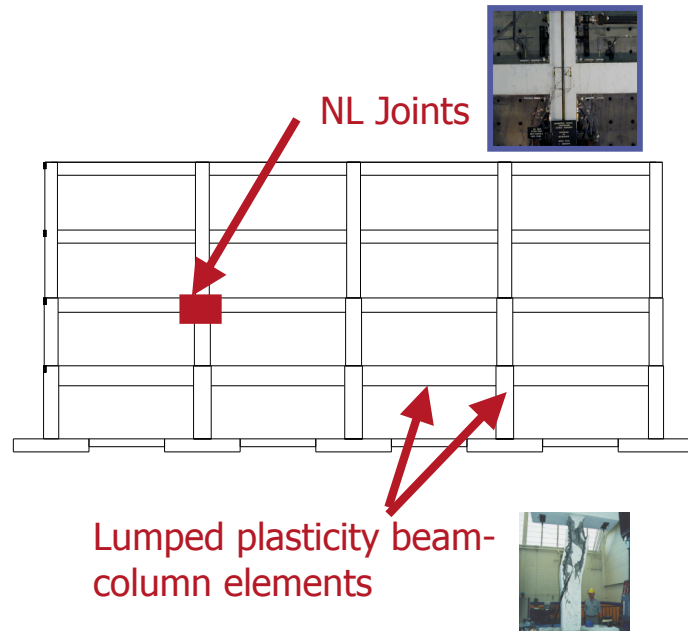
$$Probability[\Theta_p > \Theta_{p,limit-state}] = X$$

# Illustration – 4 Story SMF Building

- ◆ Office occupancy
- ◆ Los Angeles Basin
- ◆ Design Code: 2003 IBC / 2002 ACI / ASCE7-02
- ◆ Perimeter Frame System
- ◆ Maximum considered EQ demands:
  - $S_s = 1.5g$ ;  $S_1 = 0.9g$
  - $S_{a(2\% \text{ in } 50 \text{ yr})} = 0.82g$
- ◆ Design V/W of 0.094g
- ◆ Maximum inelastic design drift of 1.9% (2% limit)

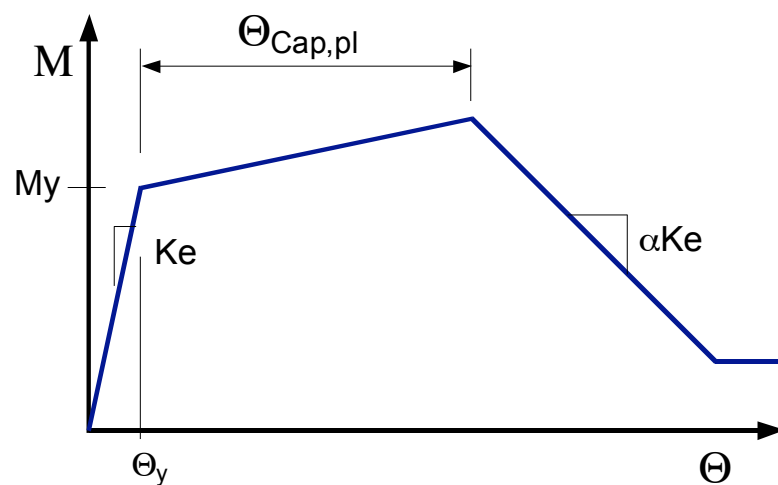


# RC Beam & Column Component Models



## OpenSees Model

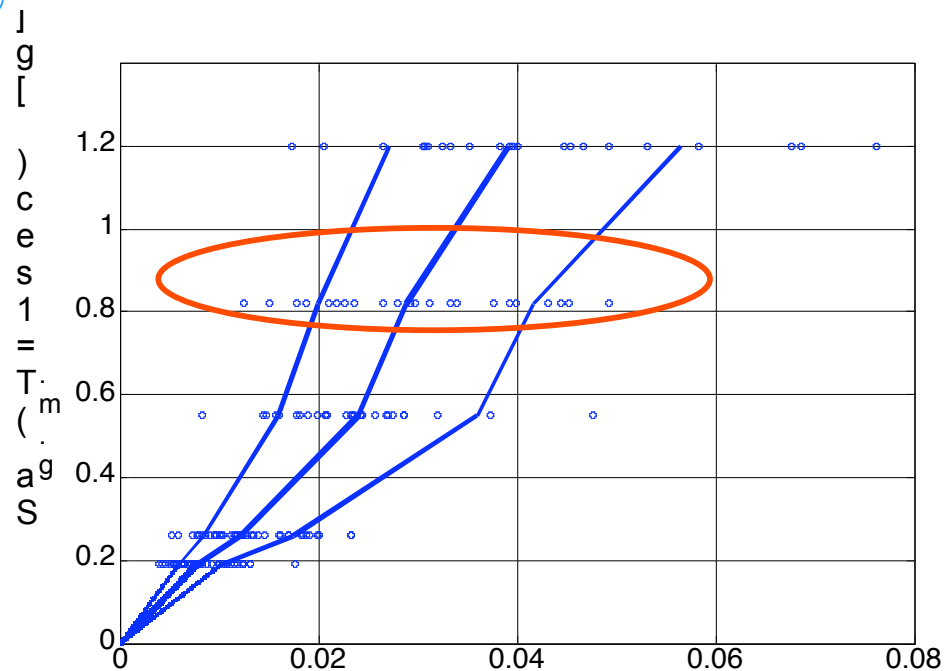
- Lumped plasticity beams, columns, and joints with strength/stiffness degradation
- Geometric NL (P- $\Delta$ )
- 20 ground motions (10 pairs)



## $\Theta_{pl,cap}$ of conforming members:

- **Columns (low axial)**  
Mean = 0.050 rad  
COV = 40%
- **Beams**  
Mean = 0.065 rad  
COV = 40%

# Peak Interstory Drift Demands



**At 2% in 50 year (MCE) Sa:**

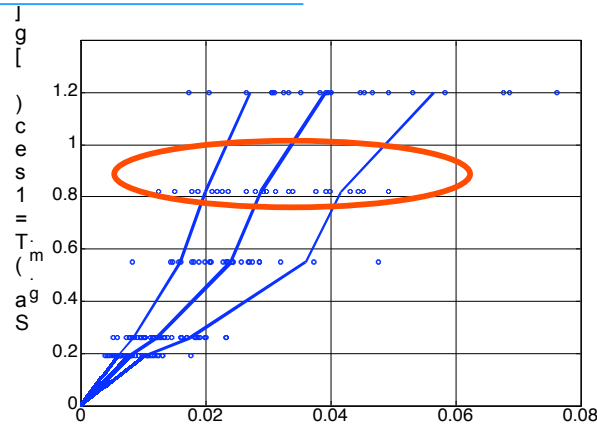
IDR<sub>max</sub> = 0.016 to 0.050

Mean IDR<sub>max</sub> = 0.028

COV = 37%

- 20 time history analyses at each of 5 hazard levels
- peak inter-story drift ratio from each time history analysis
- ground motions are scaled to hazard spectra over the region  $0.2T_1$  to  $1.5T_1$ .

# Probabilistic Measures of Drift Demand

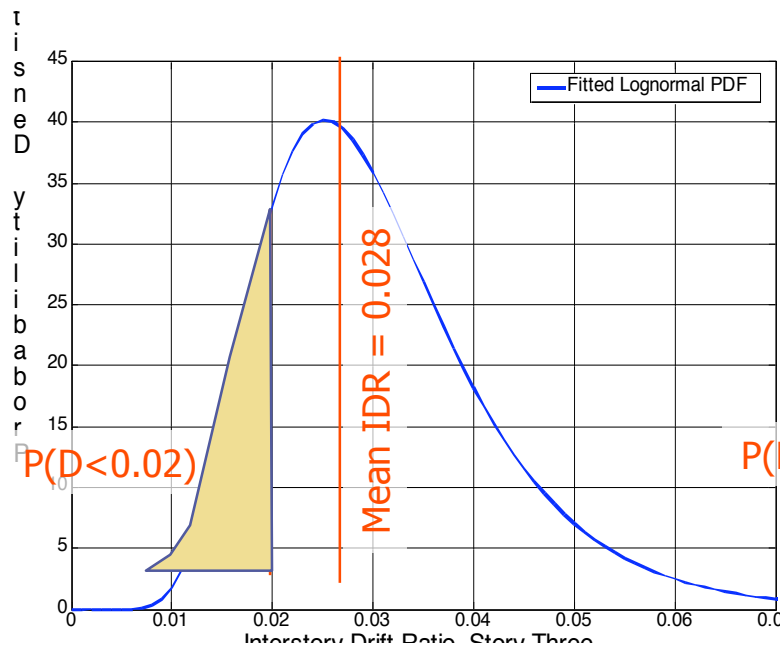


At 2% in 50 year (MCE) Sa:

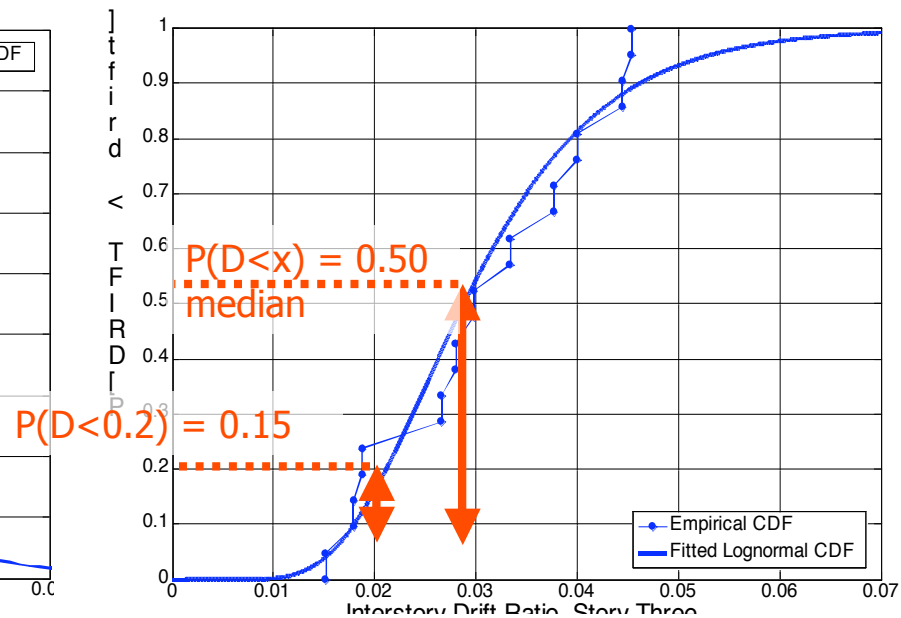
IDR<sub>max</sub> = 0.016 to 0.050

Mean IDR<sub>max</sub> = 0.028

COV = 37%

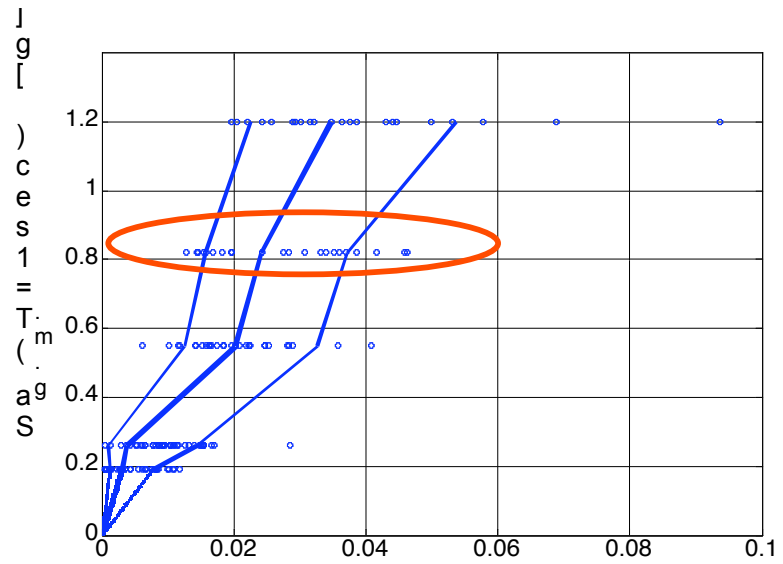


PDF (probability density function)



CDF (cumulative density function)

# Beam and Column Plastic Rotation Demands



**At 2% in 50 year (MCE) Sa:**

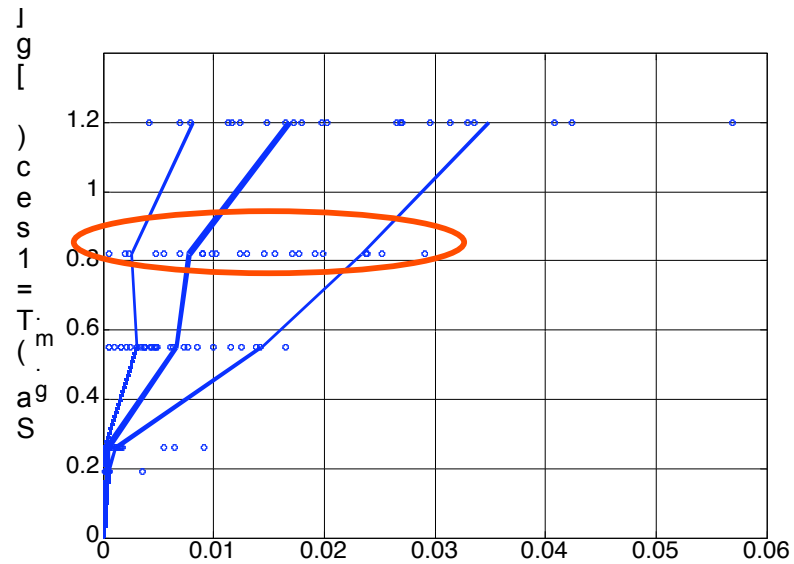
**Beams:**

$$\Theta_{p,max} = 0.012 \text{ to } 0.045$$

$$\text{Mean } \Theta_{p,max} = 0.025$$

$$\text{COV} = 43\%$$

*(vs. FEMA 356  $\Theta_{cp} < 0.025$ )*



**Columns:**

$$\Theta_{p,max} = 0 \text{ to } 0.03$$

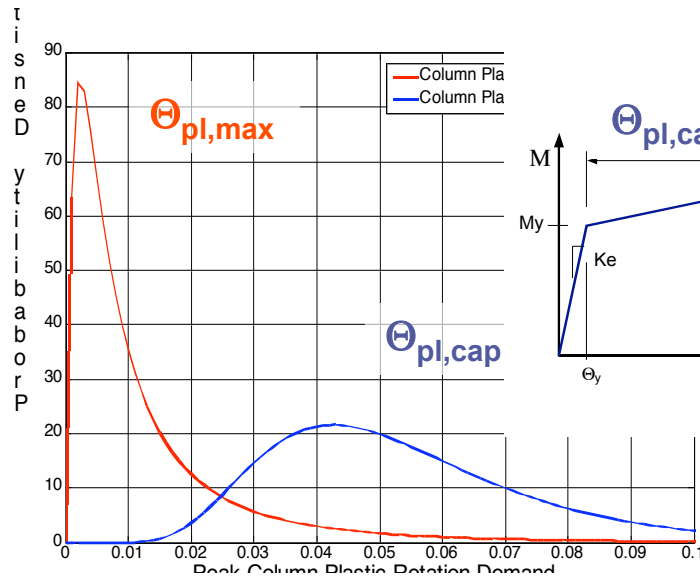
$$\text{Mean } \Theta_{p,max} = 0.010$$

$$\text{COV} = 110\%$$

*(vs. FEMA 356  $\Theta_{cp} < 0.020$ )*



# Probabilistic Limit State Assessment



	Demand ( $\Theta_{pl,max}$ )		Capacity ( $\Theta_{pl,cap}$ )	
	$\mu$	$\sigma_{ln}$	$\mu$	$\sigma_{ln}$
Beams	0.025	0.43	0.065	0.40
Col's	0.010	1.10	0.050	0.40

$$P[D \geq C] = 1 - \Phi\left(\frac{0 - \mu_{ln,z}}{\sigma_{ln,z}}\right)$$

where:  $\mu_{ln,z} = \mu_{ln,D} - \mu_{ln,C}$

$$\sigma_{ln,z}^2 = \sigma_{ln,D}^2 + \sigma_{ln,C}^2$$

$\Phi$  (standard normal table)

## Component Limit State Checks:

Beams  $P[D > C] = 5.6\%$

Columns  $P[D > C] = 5.6\%$

(just a coincidence that they turn out the same)

# Comments

- Advantages

- More transparent and rigorous assessment of component limit state criteria
- Framework to incorporate available test data (outside the scope of FEMA 356)

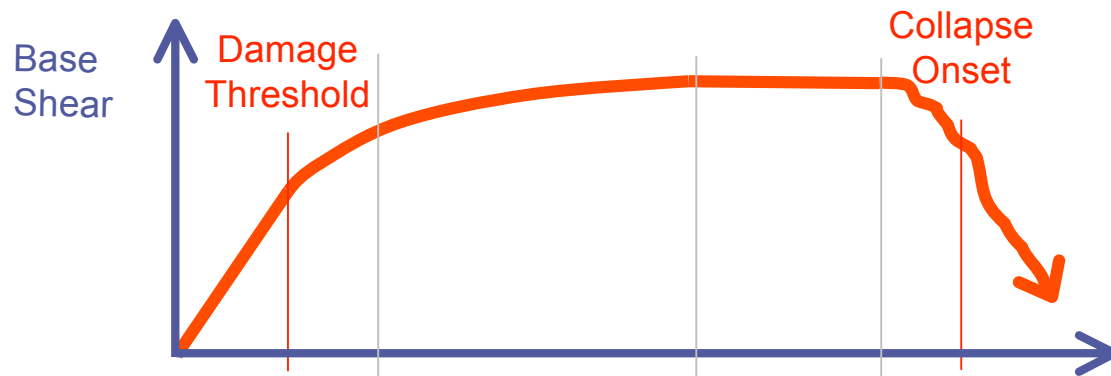
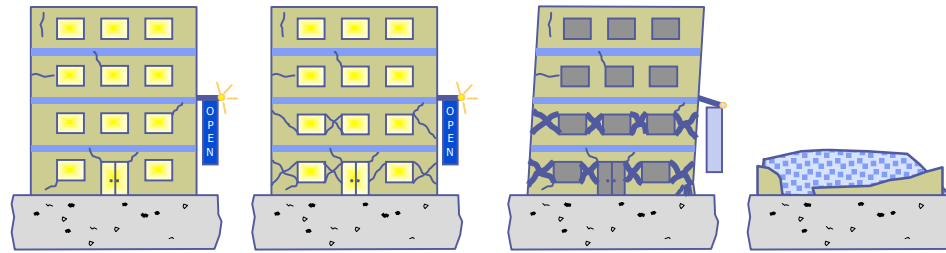
- Limitations and Issues

- Requires judgment to select appropriate limit states and the probabilistic acceptance criteria, i.e.,  $P[D > C]$  at some hazard level
- Still limited by assumptions between component and system performance.
- Does not incorporate variability and uncertainties in structural system behavior.

# Assessment Using Improved NLTH Analysis

- ◆ Nonlinear Component and System Modeling
- ◆ FEMA 356 Concepts with NLTH Analysis
- ◆ Preview of Comprehensive Collapse Simulation

# Performance-Based Earthquake Engineering



Deformation

FEMA 356 Performance Levels

PBEE today

PEER & ATC 58 vision

0 25% 50% 100%

\$, % replacement

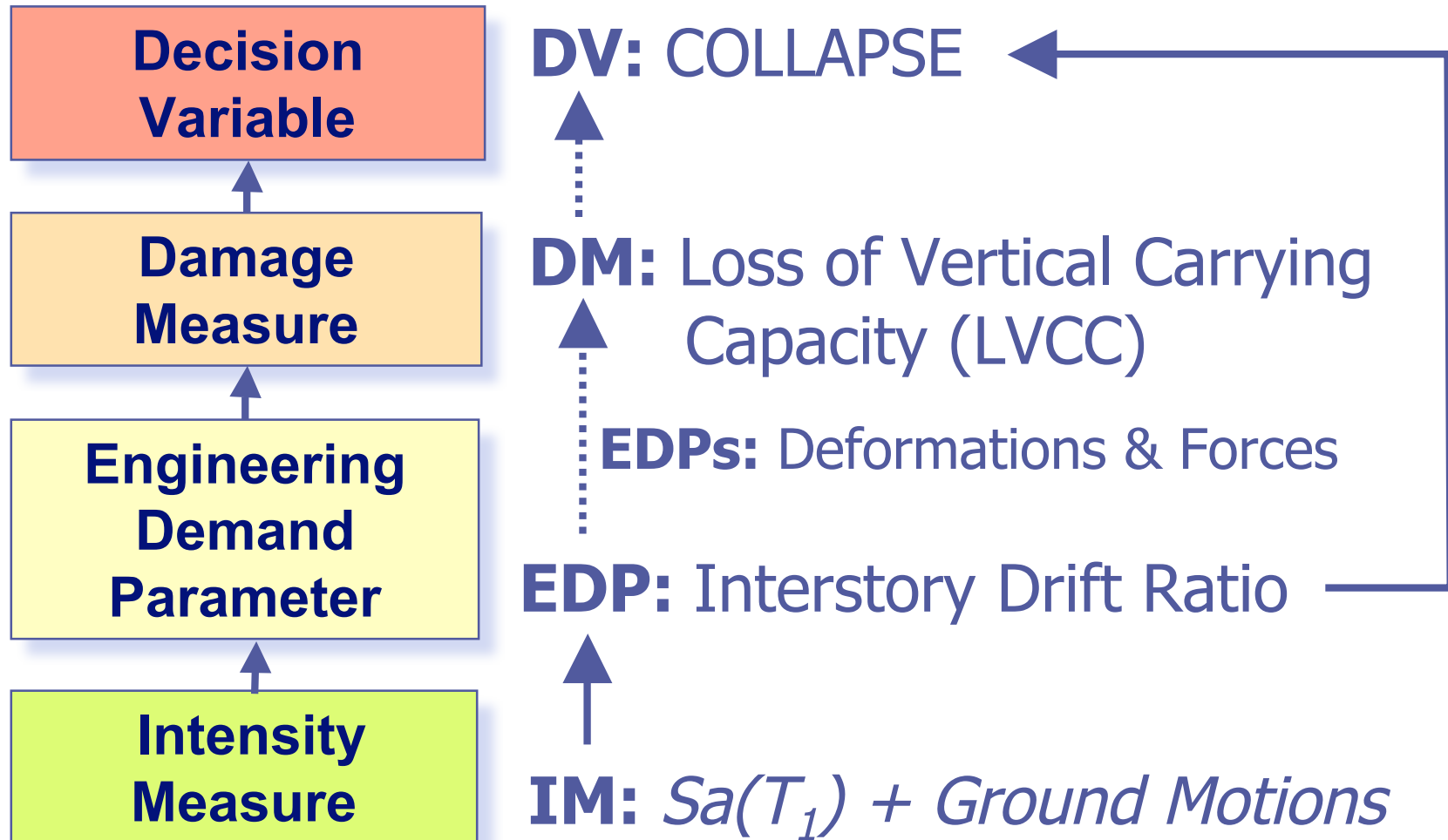
0.0 0.0001 0.001 0.01 0.25

Casualty rate

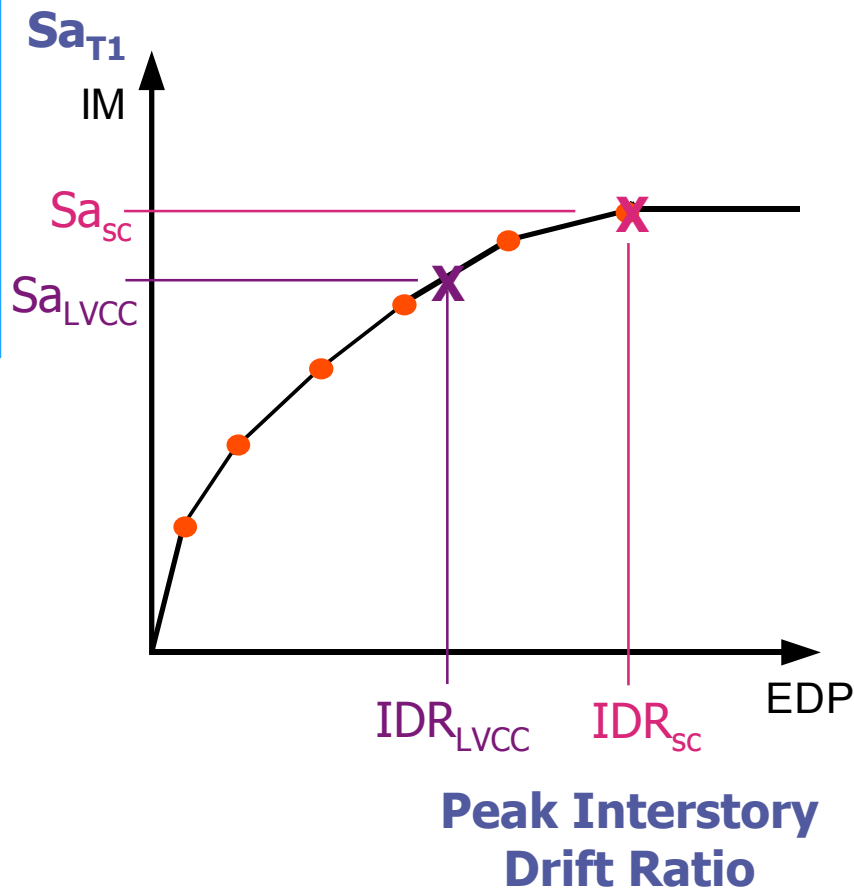
0 1 7 30 180

Downtime, days

# PBEE COLLAPSE (SAFETY) Assessment

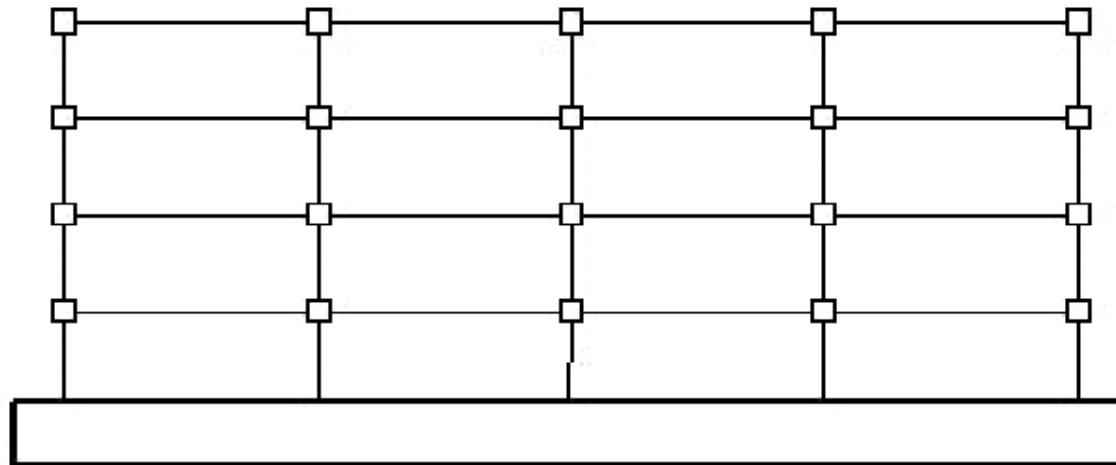


# Incremental Dynamic Analysis Concept



1. Given: Inelastic Analysis Model
2. Select and scale earthquake ground motion to specified earthquake intensity ( $IM$ )
3. Perform nonlinear time history analysis
4. Record and plot engineering demand parameter ( $EDP$ )
5. Repeat steps 2-5 until system collapse is observed through analysis
6. Perform check for local LVCC conditions that are not simulated in analysis

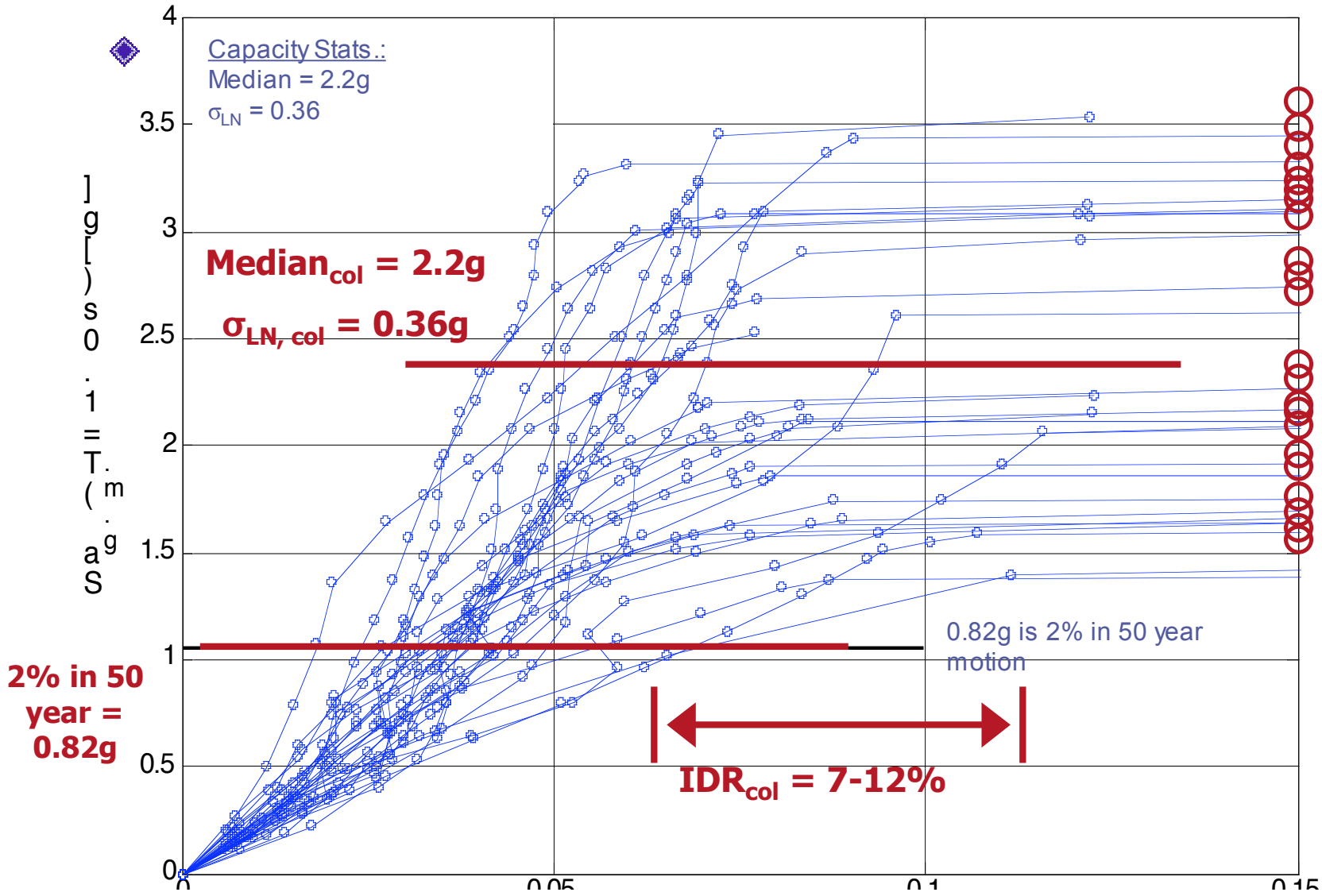
# Sideways Collapse Simulation – 4 Story SMF



Deierlein, Haselton, Liel; Stanford University

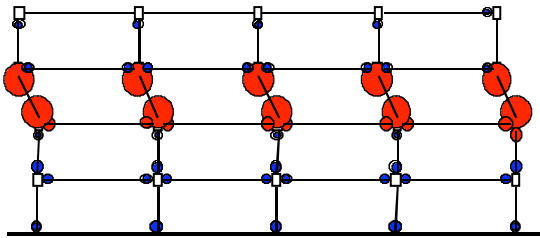
Selected\_2StoryMech\_Movie\_(DesA\_Buffalo\_v.9noGFrm\_grndDisp\_Corot)\_(AllVar)\_(0.00)\_(clough)\_EQ\_11122\_Sa\_2.55\_3

# Incremental Dynamic Analysis – Collapse

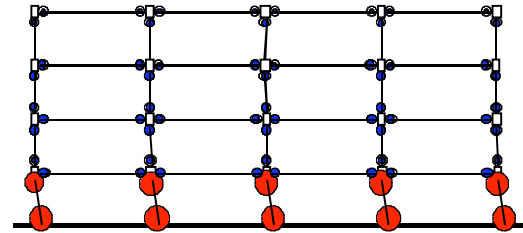




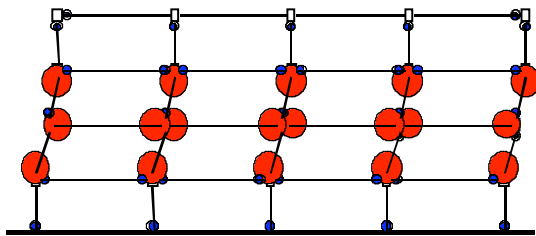
# Sideways Collapse Modes



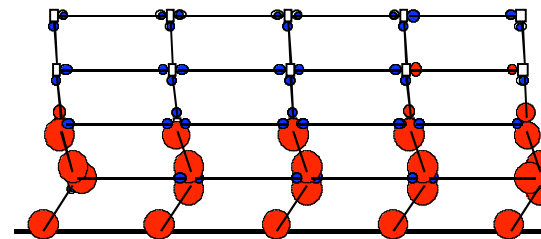
40% of collapses



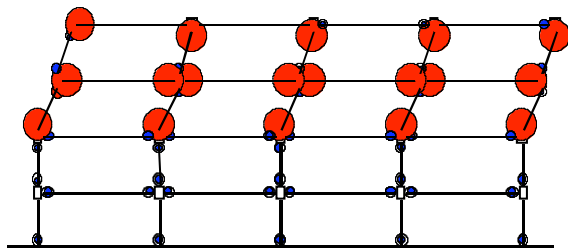
27% of collapses



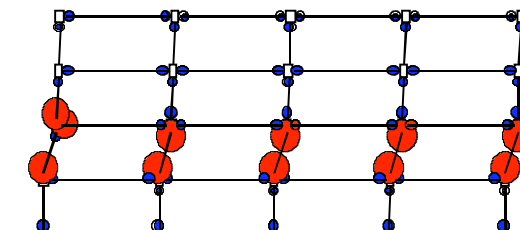
17% of collapses



12% of collapses

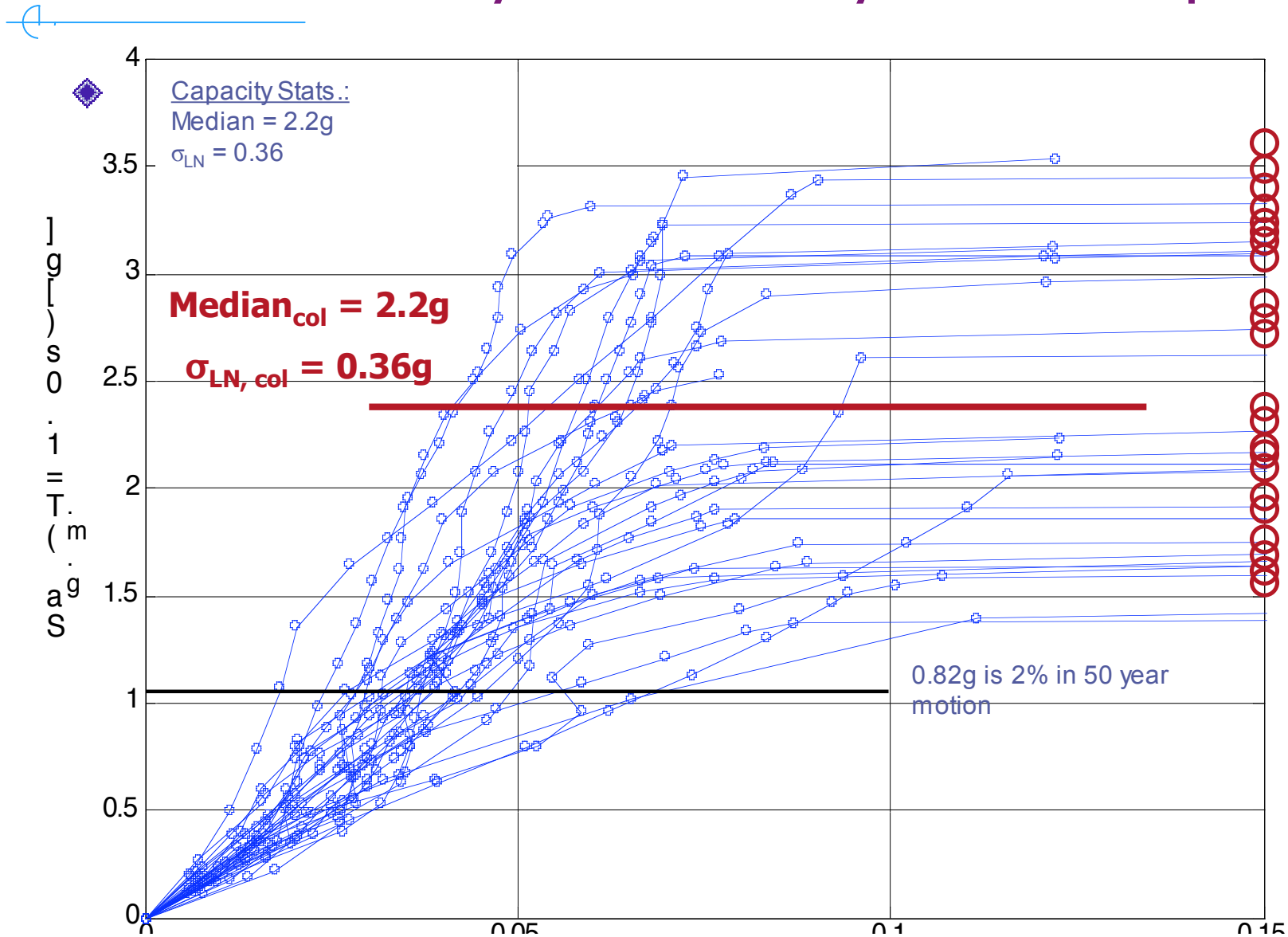


5% of collapses

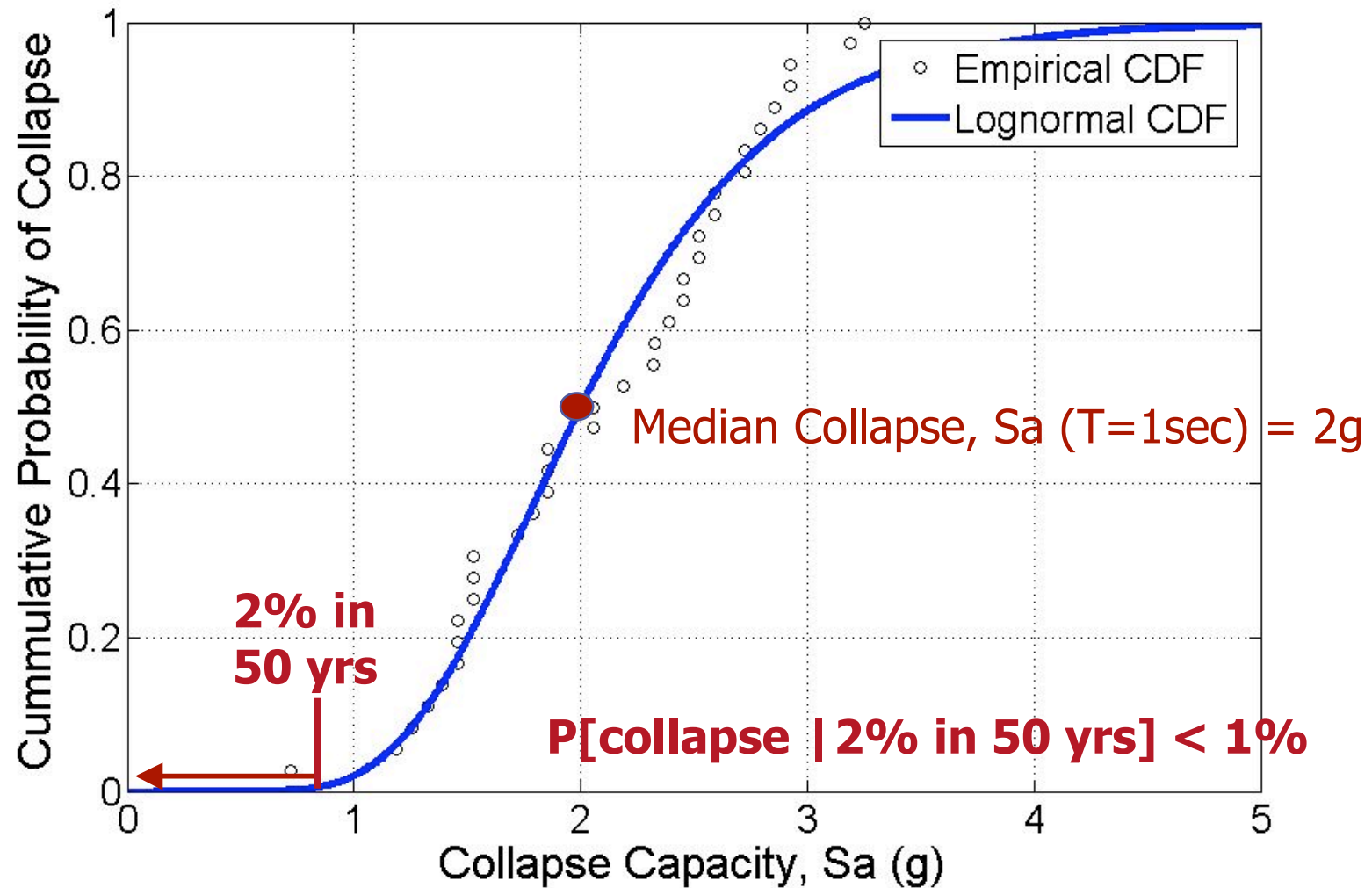


2% of collapses

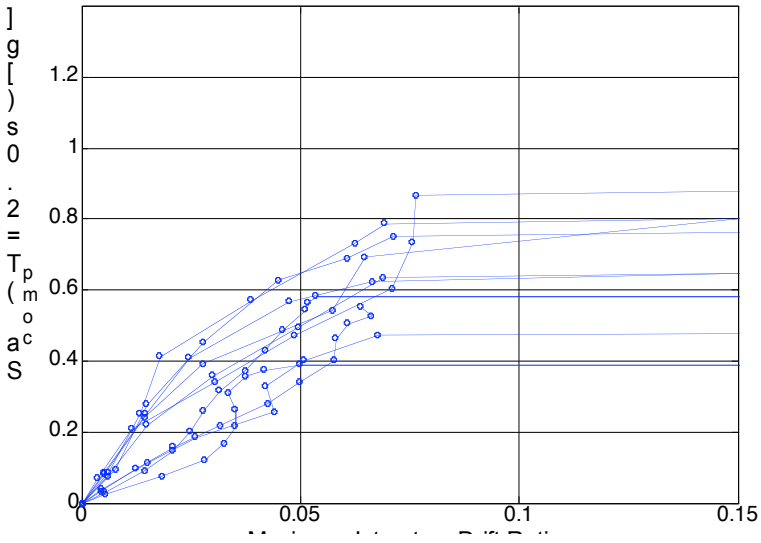
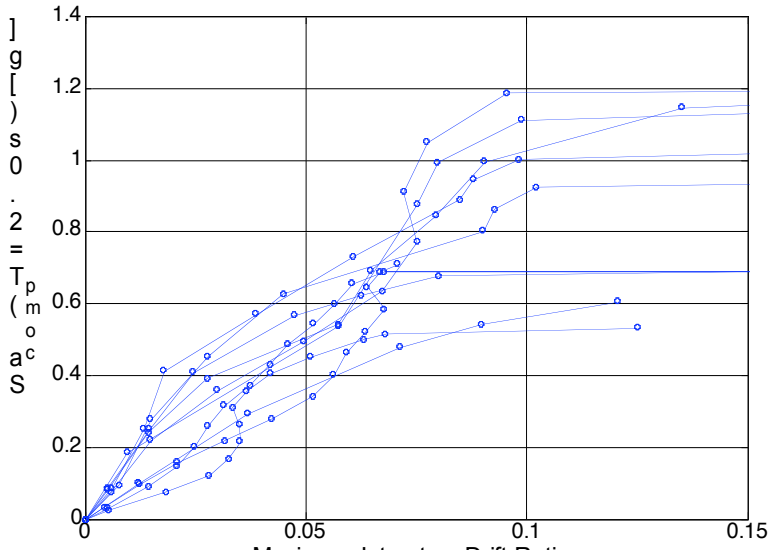
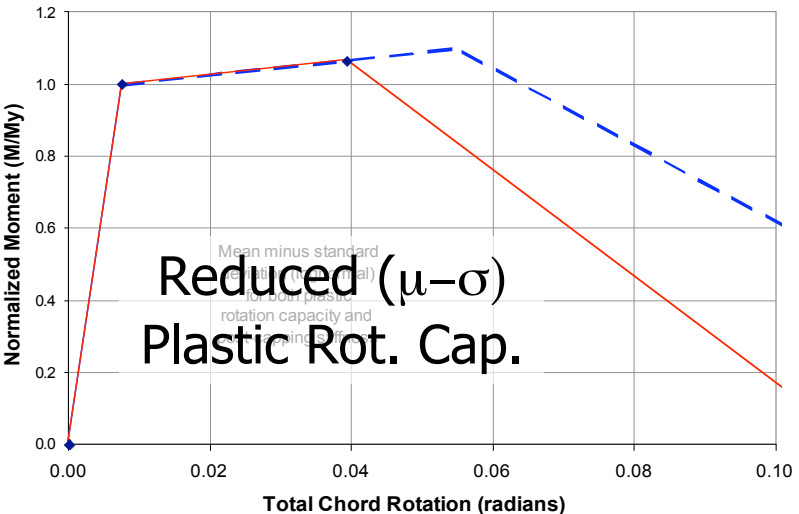
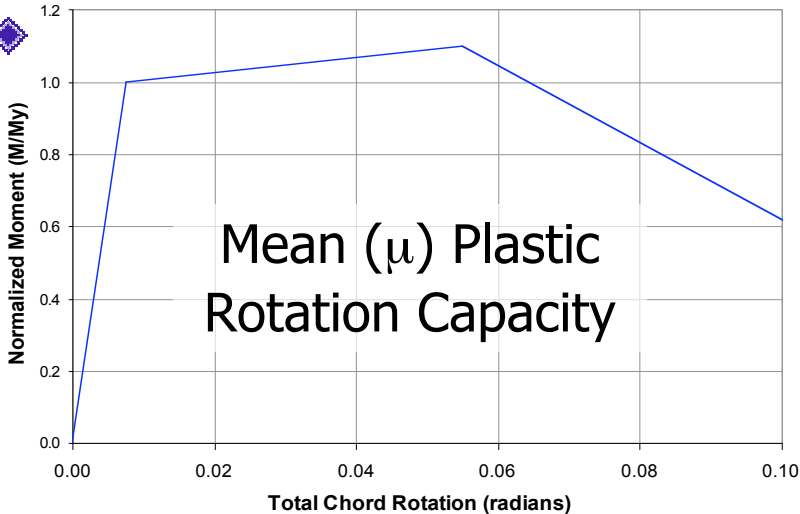
# Incremental Dynamic Analysis – Collapse



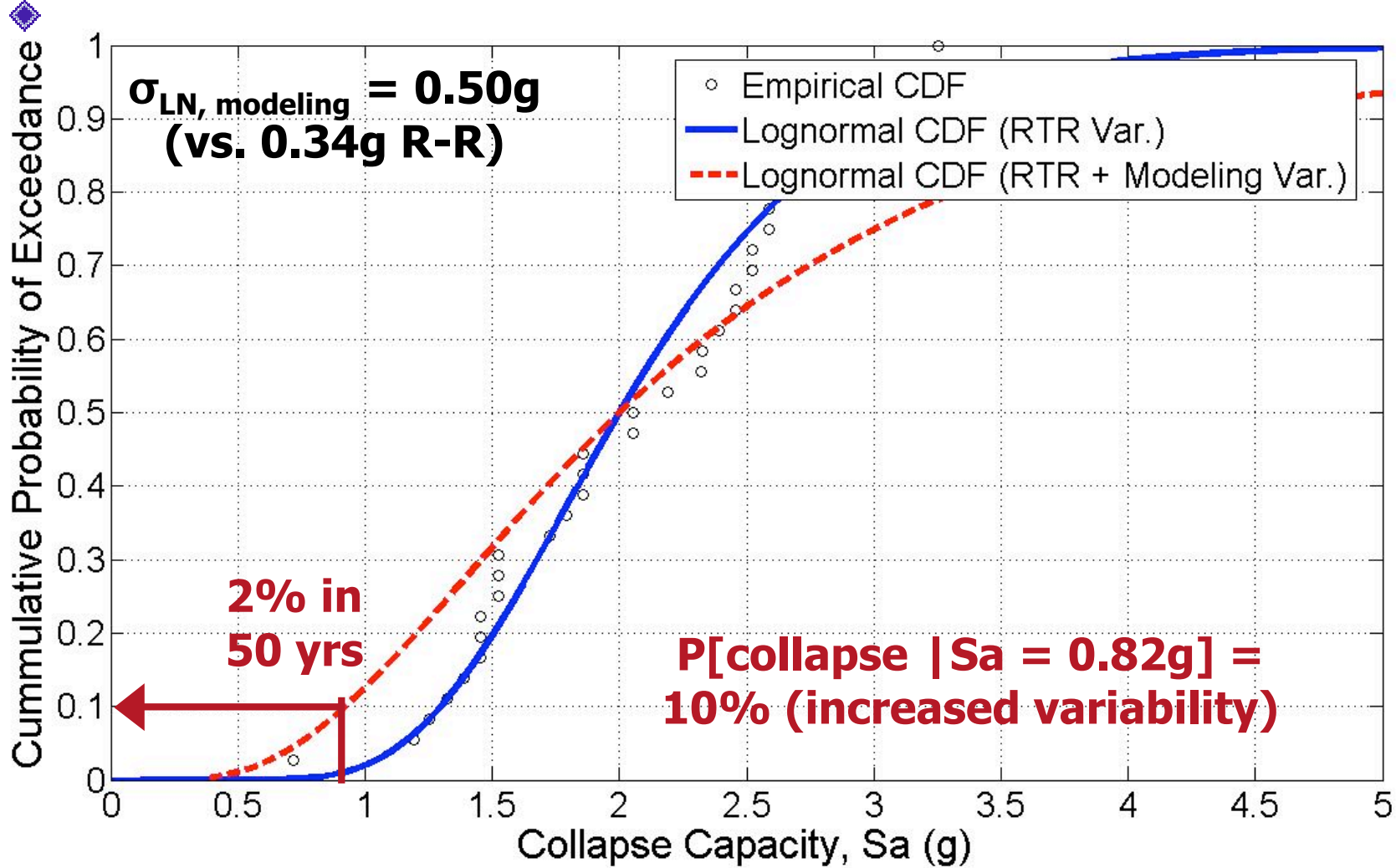
# Collapse Capacity – Empirical and Lognormal Fit



# Uncertainty – Plastic Rotation Capacity



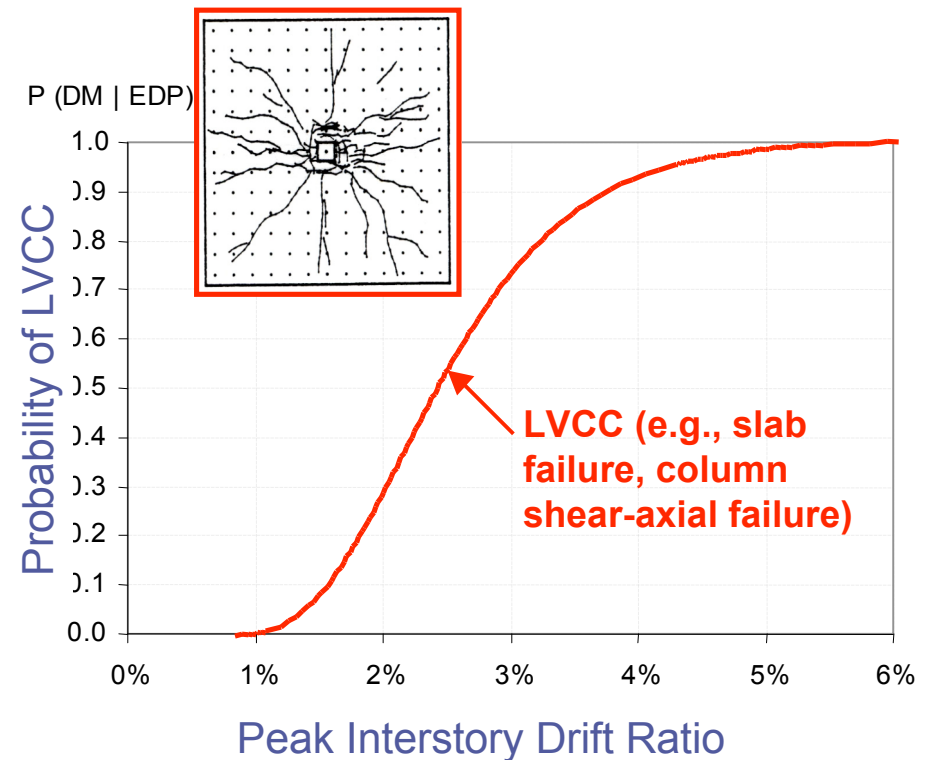
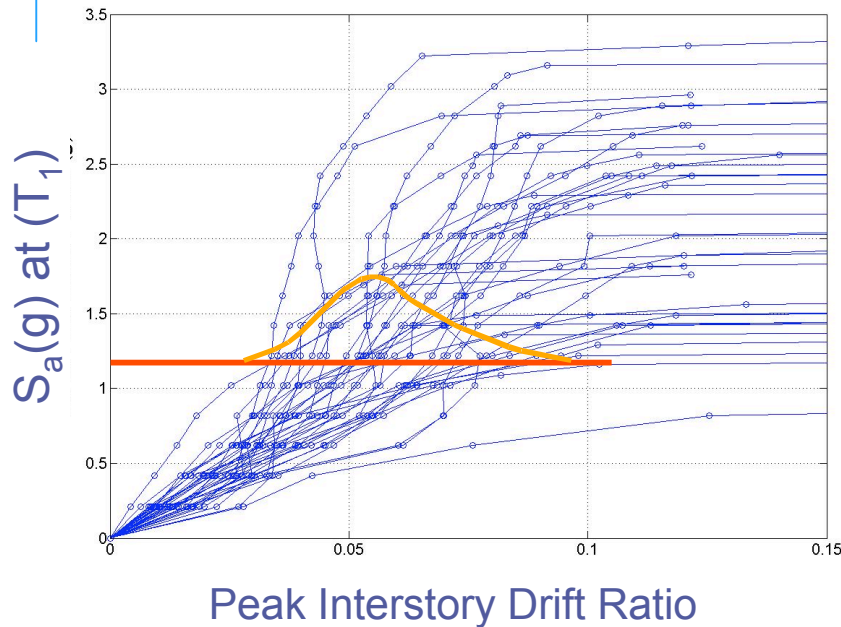
# Collapse Capacity – with Modeling Uncert.



# Combined Sidesway and Vertical (LVCC) Collapse

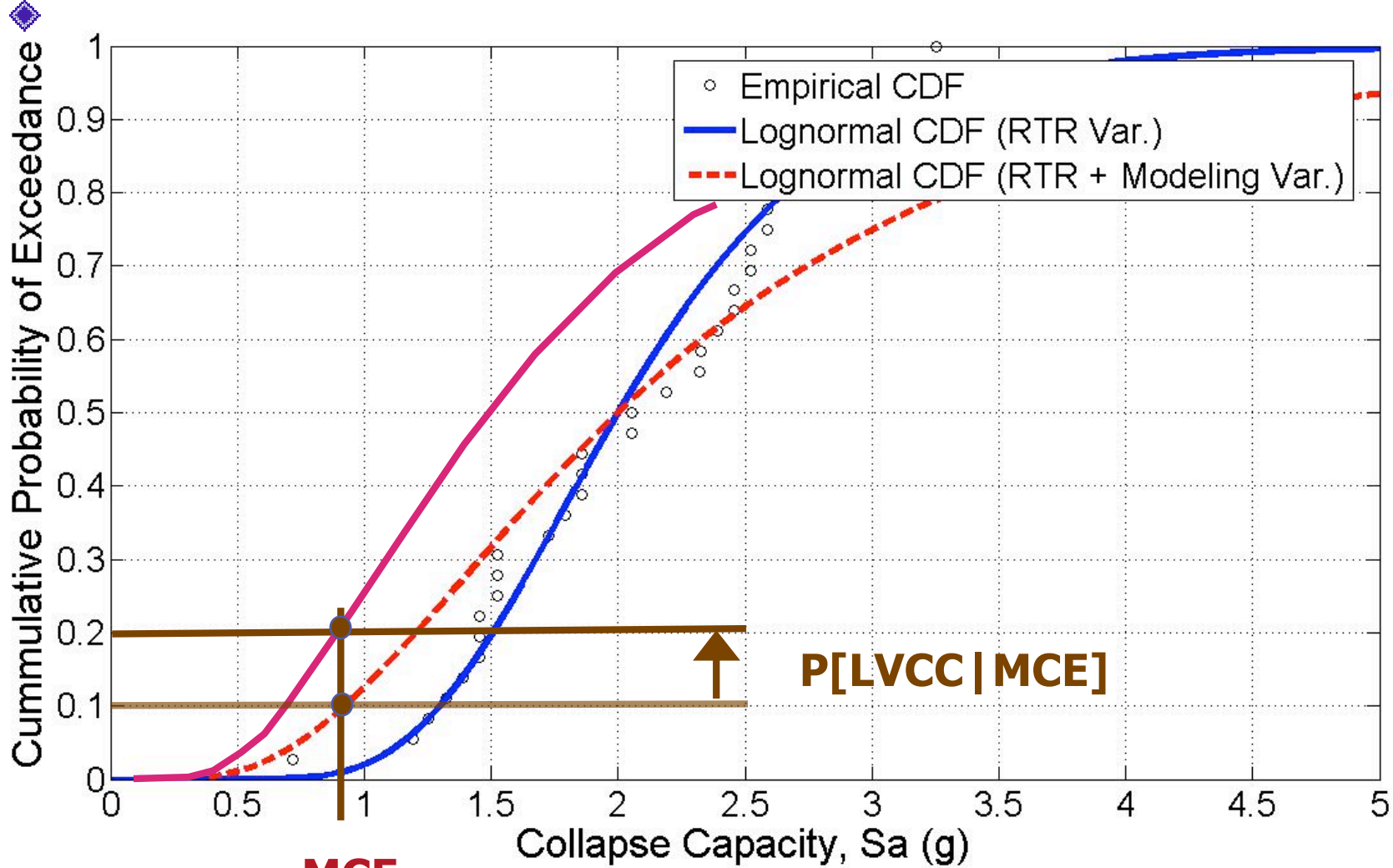
$$P[C | IM = im] = P[C_{SIM} | IM = im] + P[C_{DM} | NC_{SIM}, IM = im] \cdot P[NC_{SIM} | IM = im]$$

**Total Collapse Probability** = **Sidesway Collapse Probability at IM<sub>i</sub>** + **Probability of LVCC (given drift ratio)** × **Probability of no SS Collapse at IM<sub>i</sub>**



*Plot is shown for illustration purposes;  
not calibrated to test data.*

# Collapse Capacity – Simulation + LVCC



**MCE  
(2/50)**

*Plot is shown for illustration purposes;  
not calibrated to test or analysis data.*

# Concluding Remarks

## ◆ Benefits of Assessment by NLTH Analysis

- More explicit simulation of cyclic and dynamic effects
- Transparent and extendable to innovative systems and materials

## ◆ Challenges with NLTH Analysis

- Calibration/Validation of Hysteretic Component Models
- Selection and Scaling of Input Ground Motions
- Computational hurdles (convergence, runtime, post-processing)

## ◆ Standardization of Structural Component Models & Criteria

- Simulation & fragility models
- Statistically “neutral” models i.e., *mean* and *COV*
- Important role for material standards organizations (e.g., ACI)

## ◆ Future Vision -- Explicit Assessment of Collapse Risk



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