



## Effective Stiffness of Reinforced Concrete Columns

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### Overview

The assumed stiffnesses of the structural members of a building strongly influence the computed response of the building to ground shaking. For linear analysis, the member stiffnesses control predictions of the period of the structure, the distribution of loads within the structure, and the deformation demands. For nonlinear analysis, an accurate estimate of the member stiffness is required to reliably estimate the yield displacement, which in turn, affects the predicted displacement ductility demands. Practical, accurate procedures are needed to estimate the effective stiffness up to yielding of each structural component.

This research digest compares the measured effective stiffnesses of reinforced concrete columns from the PEER Structural Performance Database (Berry et al. 2004) with stiffnesses calculated following the Federal Emergency Management Agency (FEMA) 356 seismic rehabilitation guidelines (ASCE 2000). The FEMA 356 procedure substantially overestimates the stiffness of columns with low axial loads, in which there can be significant bar slip in the beam-column joints or footings. The digest provides practical recommendations for improving estimates of effective stiffness.

### Effective Stiffness Model

The yield displacement of a column can be considered as the sum of the displacements due to flexure, bar slip, and shear:

$$\Delta_y = \Delta_{flex} + \Delta_{slip} + \Delta_{shear} \quad [1]$$

Assuming the column is fixed against rotation at both ends and assuming a linear variation in curvature over the height of the column, the contribution of flexural deformations to the displacement at yield can be estimated as follows:

$$\Delta_{flex} = \frac{L^2}{6} \phi_y = \frac{L^2}{6} \frac{M_{0.004}}{EI_{flex}} \quad [2]$$

where  $L$  is the length of the column,  $\phi_y$  is the yield curvature, and  $M_{0.004}$  is the flexural moment at a maximum concrete compressive strain of 0.004. The effective flexural stiffness of the column,  $EI_{flex}$ , can be determined from the moment and curvature at first yield (Figure 1). For the purpose of this paper, the “first yield” of a column is defined as the first point at which either the first reinforcing bar yields in tension or the concrete reaches a maximum compressive strain of 0.002.

The displacement due to bar slip at yield can be estimated as (Elwood and Moehle, 2003):

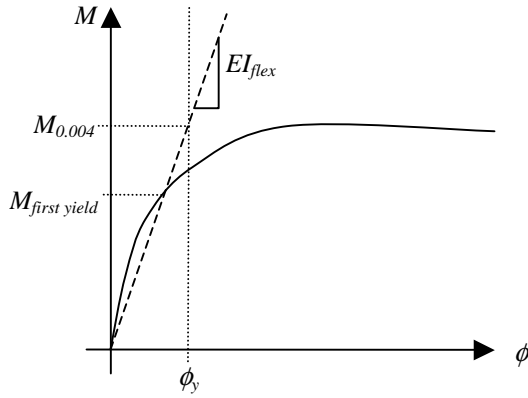
$$\Delta_{slip} = \frac{L d_b f_s \phi_y}{8u} \quad [3]$$

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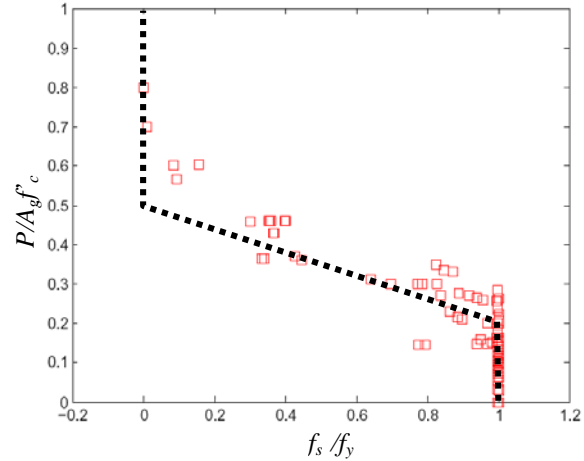
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where  $d_b$  is the diameter of the longitudinal reinforcement,  $f_s$  is the stress in the tension reinforcement, and  $u$  is the average bond stress between the longitudinal reinforcement and the footing or joint concrete. The bond stress was assumed to have a magnitude of  $6\sqrt{f'_c}$  (psi units) (Sozen et al., 1992).



**Figure 1: Moment-curvature relationship**



**Figure 2: Relationship between axial load and stress in the tension reinforcement**

At first yield, the stress in the tension reinforcement ( $f_s$  in Eq. 3) varies with the column axial load. For columns with low axial loads,  $f_s$  can be taken as equal to the yield stress,  $f_y$ . The tensile stress,  $f_s$ , decreases as the axial load increases, reaching zero when the depth of the neutral axis is equal to the effective depth of the column. The variation of  $f_s$  with axial load was investigated by considering 120 columns with normal-strength concrete ( $f'_c < 60$  MPa) from the PEER Structural Performance Database (Berry et al. 2004). Figure 2 shows that  $f_s$  can be approximated as equal to the yield stress for axial loads below  $P/A_g f'_c = 0.2$  and equal to zero for axial loads above  $P/A_g f'_c = 0.5$ , with a linear interpolation between these points.

The shear deformation, which often is negligible, can be estimated as:

$$\Delta_{shear} = \frac{2M_{0.004}}{(AG)_{eff}} \quad [4]$$

For engineering practice, the response of a column prior to yielding can be approximated as linear-elastic with a single effective stiffness,  $EI_{eff}$ :

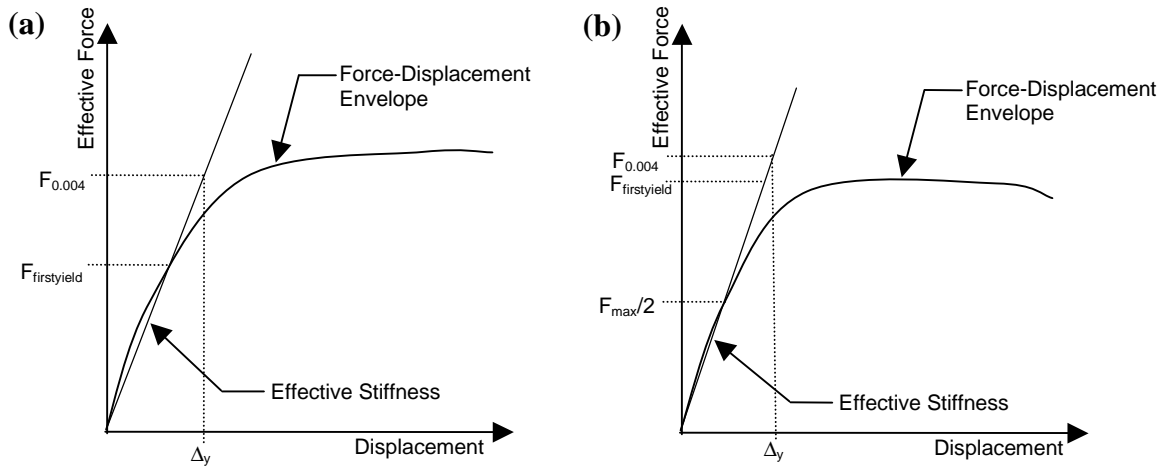
$$EI_{eff\,calc} = \frac{M_{0.004} L^2}{6\Delta_y} \quad [5]$$

where  $\Delta_y$  is given by equations 1, 2, 3, and 4.

### Comparison of Calculated and Measured Stiffnesses

For each column, the envelope of the measured lateral load-displacement relationship was extracted from the force-displacement history and corrected for P-delta effects. The yield displacement of the column was then determined as shown in Figure 3. For columns for which the maximum measured effective force,  $F_{max}$ , was at least 105% of the calculated force at first yield, the effective stiffness was defined based on the point on the measured force-

displacement envelope corresponding to the calculated force at first yield (Figure 3a). For columns not reaching this level of force, the effective stiffness was defined based on the point on the force-displacement envelope with an effective force equal to  $F_{max}/2$  (Figure 3b).

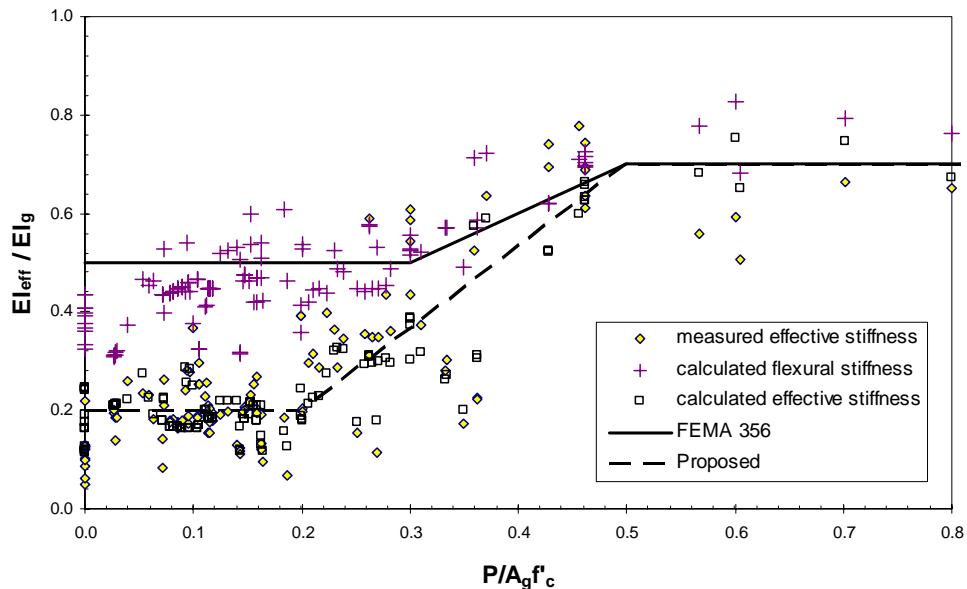


**Figure 3: Definition of measured yield displacement and effective stiffness**

The measured effective stiffness can be defined as (Figure 3):

$$EI_{eff\,meas} = \frac{F_{0.004} L^3}{12\Delta_y} \quad [6]$$

Figure 4 compares the measured and calculated effective stiffnesses for the 120-column dataset as a function of the normalized axial load. The effective stiffnesses calculated with Eqs. 1-5 provide a good estimate of the measured effective stiffnesses. The recommended effective stiffness values from FEMA 356 are consistent with the calculated flexural stiffnesses, but they greatly overestimate the measured effective stiffnesses for columns with axial loads below  $0.3A_g f'_c$ . At low axial loads, slip deformations appear to account for approximately half of the total deformation at yield, and cannot be neglected.



#### Figure 4: Comparison of calculated and measured effective stiffnesses

The following recommendations are proposed for estimating the effective stiffness of rectangular reinforced concrete columns with normal-strength concrete:

$$\begin{aligned}
 EI_{eff} / EI_g &= 0.2 & \frac{P}{A_g f'_c} &\leq 0.2 \\
 &= \frac{5}{3} \frac{P}{A_g f'_c} - \frac{4}{30} & 0.2 < \frac{P}{A_g f'_c} &\leq 0.5 \\
 &= 0.7 & 0.5 < \frac{P}{A_g f'_c} &
 \end{aligned}
 \quad [7]$$

As shown in Figure 4, Eq. 7 is consistent with the FEMA 356 recommendations for axial loads above  $0.5A_g f'_c$ , but reduces the stiffnesses for columns with lower axial loads.

Table 1 provides statistics for the ratio of the measured effective stiffness (Eq. 6) to the calculated effective stiffness for the modeling strategies discussed in this digest. The FEMA 356 recommendations overestimate the measured effective stiffnesses by nearly 100%, and the ratio has a coefficient of variation that exceeds 50%. Eq. 7 provides a much better estimate of the effective stiffnesses observed for the 120 columns considered in this study.

**Table 1: Statistics for the ratio of measured to calculated effective stiffness**

Calculated Stiffness Model	$E[ EI_{eff\ meas} / EI_{eff\ calc} ]$	$Cov[ EI_{eff\ meas} / EI_{eff\ calc} ]$
Flexural Stiffness (Eq. 2)	0.56	0.46
Total Stiffness (Eq. 5)	1.05	0.28
FEMA 356	0.52	0.53
Design Recommendations (Eq. 7)	0.99	0.35

#### References

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**Keywords**

Reinforced concrete, columns, stiffness, flexure, bond slip , FEMA 356