Annual Meeting Research Digest No. 2002-7

a publication of the Pacific Earthquake Engineering Research Center

Factored Nonlinear Displacement Demand Estimation Methods for Probability-based Safety Assessment

A. Cornell[1] and F. Jalayer[2]

Overview

This digest presents a selection of methods for assessing nonlinear dynamic seismic demands in the context of probability-based safety checking formats. These formats, such as that used recently in the FEMA/SAC SMRF Guidelines (FEMA 351-353; Cornell et al., 2002), require estimation both of the median demand (e.g., maximum interstory drift angle) associated with a given level of ground motion intensity (e.g., spectral acceleration at the first-mode period), and of a “demand” factor which depends on certain additional parameters, such as the “dispersion” or record-to-record variability of these demands. Here it is assumed that these parameters will be estimated by nonlinear dynamic analysis. The objective may be to assess the safety of a specific structure or to develop generic results for use by code drafting committees (e.g., Foutch et al., 2002).

Applicability

The demand estimation methods presented here are applicable to practical seismic safety assessment of individual buildings and for use in developing guidelines. They also play a role in research. Some have been used, for example, in the development of the FEMA/SAC Guidelines (FEMA 351-353).

Factored Demand

The formal objective of probability-based safety checking is to confirm that the annual probability of some limit state, such as global collapse due to instability, is less than a specified or ‘target’ value. In an LRFD-like format this implies comparing a factored demand with respect to a factored capacity. We focus here on the factored demand, which we presume to be a peak dynamic displacement during a random future seismic event.

It has been shown in our prior work (e.g., Cornell et al., 2002) that, if one wants to confirm that the annual limit state probability of a structure is less than P_0 the factored demand one need estimate is:

$$\text{Factored Demand} = \hat{\theta}_{\theta_{max}|^{P_0} S_a} \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2_{\theta_{max}|S_a}} \qquad \text{Equation 1}$$

[1] Professor, Pacific Earthquake Engineering Center, Stanford University

In which

$\hat{\theta}_{\theta_{max}|S_a}$ Median demand, e.g., interstory drift angle, θ_{max}, for a given level of ground motion intensity, measured here by first-mode spectral acceleration, S_a

$\beta_{\theta_{max}|S_a}$ Dispersion measure for demand given the spectral acceleration level, S_a. This measure is the standard deviation of the natural log, or roughly the coefficient of variation.

P_0 A tolerable limit state probability level

${}^{P_0}S_a$ The spectral acceleration associated with the annual frequency of exceedance, P_0

$\hat{\theta}_{\theta_{max}|{}^{P_0}S_a}$ Median demand for a spectral acceleration level equal to ${}^{P_0}S_a$

$\frac{k}{b}$ A sensitivity factor that relates a change in structural demand to the change in spectral acceleration hazard

The spectral acceleration one obtains from the site's hazard curve. The coefficient *k* is the local "log slope" of that curve, i..e, the sensitivity of the annual probability to a change in the spectral acceleration. The coefficient *b* is the local "log slope" or sensitivity of the (median) drift to a change in the spectral acceleration.

In principle this coefficient, the median drift $\hat{\theta}_{\theta_{max}|S_a}$ and the dispersion $\beta_{\theta_{max}|S_a}$ must all be estimated by conducting appropriate dynamic analyses of the structure. The subject here is just which runs should be made and how should their results be processed to estimate these three values.

Four Methods

The figures below illustrate four approaches to estimating these demand parameters so that the factored demand itself can be evaluated. In Figure 1 one simply conducts n dynamic analyses of the structure with an appropriate set of accelerograms scaled to the common level, ${}^{P_0}S_a$, a so-called "single stripe" analysis. (In this case the results are shown for a model of the Van Nuys Holiday Inn - in the three-bay direction - at a spectral acceleration level of 0.94g. with n = 30.) The median of the n drift values is estimated by any appropriate method (e.g., ranking and finding the "middle" value), and standard deviation of the (natural) logs of the drifts is also obtained, providing two of the three parameters needed to estimate the factored demand. The parameter b can not be determined from such a method, however. One may simply adopt a value of b = 1, implying median drift is proportional to spectral acceleration, which is consistent with linear analysis or the equal displacement rule. This may overestimate the factored demands at large drift levels when *b* is typically larger than 1.

---

[2] Graduate Student Researcher, Pacific Earthquake Engineering Center, Stanford University

Figure 2 shows the "cloud" approach, when all records are not scaled to the same spectral acceleration level. For example, these may be a suite of accelerograms, exactly as recorded, from a common scenario, e.g., very similar magnitudes and distances. In this case a (log linear) regression analysis provides the three parameters, *b* being the slope of the line on log-log paper. In Figure 3 records are run at a suite of spectral acceleration stripes (perhaps the same records as here, or perhaps different records at different levels, representing different scenarios). In this case the single stripe results (median and dispersion) above are repeated for each level, and the medians and medians plus/minus the dispersion are connected level to level forming approximate functional relations between, for example, the median drift and spectral acceleration. The coefficient *b* is just the local slope of this line (in log-log terms). Finally, in Figure 4 one sees the incremental dynamic analysis (IDA) representation (Vamvatsikos and Cornell, 2002) of drift demand estimation. Each individual IDA is the result of connecting (smoothly, preferably, e.g., by spline fitting) the drift results from a sequence of scalings of a specific record. The IDA's of the suite of n records can then be analyzed at any proposed spectral acceleration level, e.g., ${}^{P_0}S_a$ to estimate the median and the dispersion, and a local slope, *b*, of the median drift versus spectral acceleration can be determined. While in this case the results in Fig. 4 were obtained from the multiple stripes in Fig. 3, it need not be the case that the same spectral accelerations levels are used for all records. This is convenient when one is exploring different intensity measures from the same nonlinear runs. While both the multi-stripe and IDA methods require considerable (although repetitive and easily automated) computation, they do provide factored demands for any probability level checking, and hence, for probability analysis. (That is, when the factored demand for a specific spectral acceleration level is found to equal the factored capacity, the probability associated with that spectral acceleration is the annual probability of exceeding the limit state.)

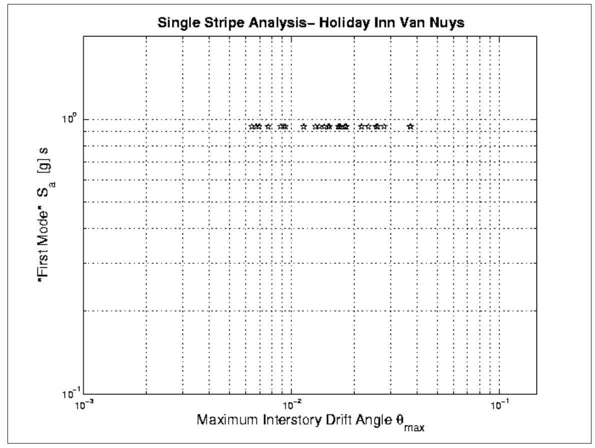

Figure 1 –Demand Assessment by a Single Stripe

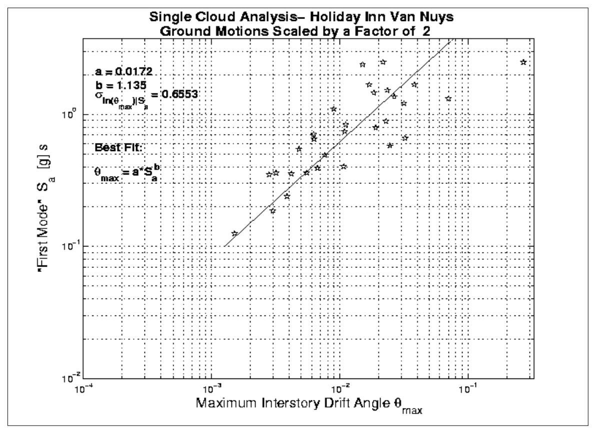

Figure 2 –Demand Assessment by a Cloud Analysis

The relative merits of these various methods are in some cases obvious, e.g., the single stripe method requires the least computation, but it provides no estimate of *b*, which may or may not cause inaccuracy. Subtler features include the fact that the cloud method can be accomplished without scaling records. These methods and variations on them are the subject of continuing investigation.

For further information

Recent papers, manuscripts and theses are available at http://www.stanford.edu/group/rms. For further information contact: cornell@ce.stanford.edu.

Acknowledgment
This work is supported in part by the Pacific Earthquake Engineering Research Center through the Earthquake Engineering research Centers Program of the National Science Foundation under Award Number EEC-9701568.

References

Jalayer, F. and Cornell, C. A.; "Alternative Nonlinear Demand Estimation Methods for Probability-Based Seismic Assessment", Paper in preparation, 2002.

Vamvatsikos, D. and Cornell, C. A. (2002). "Incremental Dynamic Analysis", Accepted for publication, *Earthquake Engineering and Structural Dynamics*, 2002.

Cornell, C.A., Jalayer, F, Hamburger, R.O. and Foutch, D.A., "The Probabilistic Basis for the 2000 SAC/FEMA Steel Moment Frame Guidelines", Accepted for publication, *ASCE Structural Journal*, 2002.

Yun, S-Y., Hamburger, R. O., Cornell, C. A., and Foutch, D. A., "Seismic Performance for Steel Moment Frames", Accepted for publication, *ASCE Structural Journal*, 2002

Keywords
Probabilistic seismic safety, intensity measures, nonlinear dynamic analysis, median drift, dispersion

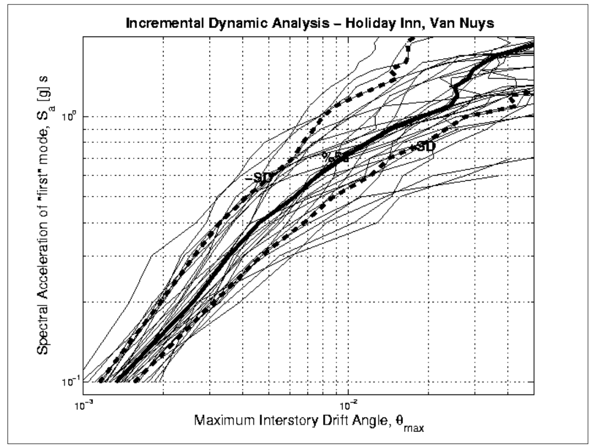

Figure 4 –Demand Assessment by Incremental Dynamic Analysis

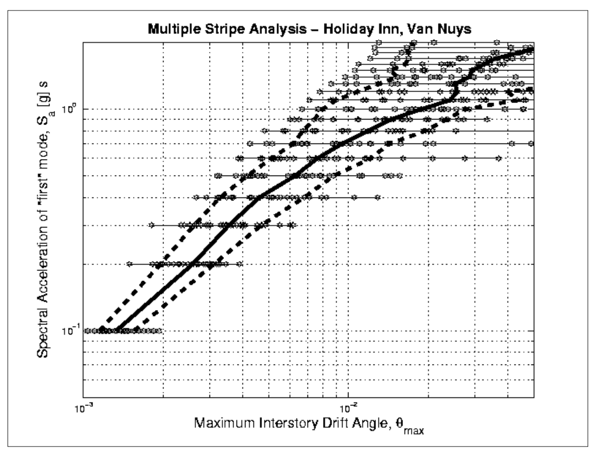

Figure 3 –Demand Assessment by Multiple Stripes