DESIGN PARAMETER SENSITIVITY IN BRIDGE PROBABILISTIC SEISMIC DEMAND MODELS

K. Mackie¹ and B. Stojadinovic²

ABSTRACT

The continuing development of performance-based earthquake engineering will soon enable accurate probabilistic quantitative evaluation of structural performance in a given seismic hazard environment. The Pacific Earthquake Engineering Research Center (PEER) is developing a probability-based framework comprising several models, particularly, a seismic demand model. The objective of this paper is to develop probabilistic demand models for typical highway overpass bridges. Demand models relate ground motion Intensity Measures, such as spectral acceleration, to bridge Demand Parameters, such as displacement ductility. These models will then be used to assess their sensitivity to variations in bridge design parameters, such as column height and diameter. The variation of parameters is achieved using a parametric finite element model, representing an entire array of bridge designs. Probabilistic seismic demand models were formulated by statistical analysis of the data produced during analyses of all bridge portfolio-ground motion bin combinations. Relations for each design parameter, and resulting trends, can be developed, giving bridge designers the power to quantitatively evaluate their design choices on structural performance.

Introduction

In the evolving field of Performance-based earthquake engineering, designers and owners are motivated to engineer structures to fulfill predetermined performance levels or objectives. Previously, performance-based design frameworks have addressed only the probabilistic evaluation of seismic hazards (FEMA-273 (FEMA 1996) and Vision 2000 (SEAOC 1995)). The resulting graduated arrays of performance levels are based on deterministic estimates of structural performance. The recent SAC Steel Project (FEMA 2000) provided a probabilistic extension to the performance side, enabling simultaneous consideration of uncertainties in both the demand and capacity.

To achieve a consistent probability-based framework that is substantially more general than that of the SAC project, the Pacific Earthquake Engineering Research Center (PEER) is

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developing a probabilistic framework for performance-based design and evaluation. Performance objectives are defined in terms of annual probabilities of socio-economic Decision Variables ($DV > z$) being exceeded. While useful to owners and designers, to address the problem rigorously, the outcome is de-aggregated into several interim models involving measures of damage ($DM > y$), structural performance ($EDP > x$), and seismic hazard ($IM > w$). Thus, the mean annual frequency of a $DV$ exceeding limit value $z$ is (Cornell 2000):

$$
ν_{DV} = \int \int \int G_{DV|DM}(z | y) dG_{DM|EDP}(y | x) dG_{EDP|IM}(x | w) dλ_{IM}(w)
$$

Utility of Eq. 1 is based on the mutual independence of the interim models. Relationships between the intermediate variables ($DM, EDP, IM$) should be chosen such that probability conditioning is not carried over from one model to the next. Additionally, the uncertainties generated at each stage need to be systematically addressed and propagated, making the selection of each interim model critical to the process. Previous research was aimed at determining optimal IM-EDP pairs in the demand model $G_{EDP|IM}(x | w)$ (Mackie 2002). Utilizing these optimal pairs, design parameter sensitivity is investigated in this paper.

**Probabilistic Seismic Demand Model**

The Probabilistic Seismic Demand Model (PSDM hereafter) is the outcome from a Probabilistic Seismic Demand Analysis (PSDA) (Luco 2001). PSDA typically involves five steps. First, a set of ground motions, representative of regional seismic hazard, is selected or synthesized. Intensity Measures (IM) descriptive of their content are computed. Second, the class of structures to be investigated is defined. Associated with this class are a suite of Engineering Demand Parameters (EDP) which can be extracted from analysis to assess structural performance under the considered motions. Third, a nonlinear finite element analysis model is generated to model the class of structures selected, with specific allowances for varying the design through design parameters. Fourth, dynamic analyses are performed until all motions and bridge model combinations have been exhausted. Finally, a demand model is formulated between resulting ground motion IMs and structural EDPs.

**PSDA Ground Motions**

The PSDA method used to formulate the PSDMs explicitly involves the ground motion bin approach. The bin approach (Shome 1999) is used to subdivide ground motions into arbitrary bins based on moment magnitude ($M_w$), closest distance ($R$), and local soil type. The use of magnitude and distance allows parallels between standard attenuation relationships and existing PSHA. An advantage of using bins is the ability to assess the effect of generalized earthquake characteristics on structural demands.

The bins in this study are delineated at a magnitude of 6.5 and a distance of 30 km. Four bins with 20 ground motions each were obtained from the PEER Strong Motion Database (peer.berkeley.edu/smcat), and are characteristic of non-near-field motions ($R > 15$ km) recorded in California. The specific records selected are the same as those used by Krawinkler (Medina 2001) in companion building research in PEER. All three components of the ground motion
records were scaled by a factor of two (amplitude scaling only).

**PSDA Class of Structures and Design Parameters**

Typical new California highway overpass bridges are selected as the class of structures. A class is characterized by geometry, components, and methods of design. Ideally, each of these parameters can be investigated in a parameter sensitivity study from the resulting PSDMs. The bridges presented in this paper are designed according to Caltrans Bridge Design Specification and Seismic Design Criteria (Caltrans 1999) for reinforced concrete bridges. Consistent with the displacement-based design approach outlined by Caltrans, the assumption is made regarding bridges, such that columns develop plastic hinges in flexure rather than experience shear failure. Configurations in this study are limited to two-equal-span overpasses with seat-type abutments on either end. Common to all bridges is a single column bent continuing below grade into a Type I integral pile foundation.

Seven design parameters were initially selected to describe variations of bridge design with respect to a base bridge configuration. They are listed in Table 1, along with limits on the parameter variations used in this study. The base bridge configuration includes two 18.2 m (60 ft) spans, a single-column bent 7.6 m above grade (30 ft), with a 1.6 m (5.25 ft) diameter circular column, 2% longitudinal reinforcement, and 0.7% transverse reinforcement. As the bridge parameters are varied (Table 1), their values relative to each other are intended to cover the complete spectrum of possible bridge designs even if particular bridge instantiations are uncommon in design practice. The base bridge is on a USGS class B (NEHRP C) soil site. Only one parameter was varied from the base configuration at a time.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Span length</td>
<td>$L$</td>
<td>18-55 m (60-180 ft)</td>
</tr>
<tr>
<td>Span-to-column height ratio</td>
<td>$L/H$</td>
<td>1.2-3.5</td>
</tr>
<tr>
<td>Column-to-superstructure dimension ratio</td>
<td>$D_c/D_s$</td>
<td>0.67-1.33</td>
</tr>
<tr>
<td>Longitudinal reinforcement ratio</td>
<td>$\rho_{s,\text{long}}$</td>
<td>1-4%</td>
</tr>
<tr>
<td>Transverse reinforcement ratio</td>
<td>$\rho_{s,\text{trans}}$</td>
<td>0.4-1.1%</td>
</tr>
<tr>
<td>Pile soil stiffness</td>
<td>$K_{\text{soil}}$</td>
<td>USGS A, B, C, D</td>
</tr>
<tr>
<td>Additional bridge dead load</td>
<td>$W_t$</td>
<td>10-150% self-weight</td>
</tr>
</tbody>
</table>

**PSDA Model**

The PEER OpenSees (www.opensees.org) platform was selected as the nonlinear finite element engine for modeling. Bridge columns and pile shafts are modeled using three-dimensional flexibility-based beam-column elements with fiber cross-sections. This element is limited to axial-flexural interaction, hence the assumption made earlier regarding shear failure. P-Δ effects were included for the column, but no other methods of softening were incorporated into the model. The circular column cross-sections have perimeter longitudinal reinforcing bars and spiral confinement.
Soil-structure interaction was modeled using bilinear $p-y$ springs placed at varying depths along the length of the pile shafts. The $p-y$ spring properties were determined using soil parameters corresponding to assumed properties of USGS soil groups (API 1993). The bridge deck was designed as a typical Caltrans reinforced concrete box girder section for a three-lane roadway. During the nonlinear analyses, the deck was assumed to remain elastic, therefore input into the model using elastic elements and cracked stiffness. For this particular study, abutments were not included in the model. Instead, the bridge ends were supported on rollers.

**PSDA Intensity Measures and Demand Parameters**

In this study, the IMs were limited to the spectral quantities and Arias Intensity only. First mode spectral displacement ($SdT_1$), $SaT_1$, and $SvT_1$ are used interchangeably, as the dispersions in the PSDMs are independent of the choice of spectral quantity. While calibrated for buildings that exhibit definite in-plane mode shapes, it was determined that the IM proposed by Luco (Eq. 2) also generates effective fits for the bridge model utilized (Luco 2001). This IM contains terms to account for both elastic response and inelastic response by extending to a longer period. The extended period factor ($2T_1$) is effective in capturing linear fits in the higher intensity range as it can be related to $\sqrt{\mu_\Delta}$.

$$Sa_{Luco} = Sa(T_1) \left[ \frac{Sa(2T_1)}{Sa(T_1)} \right]$$

The bridge EDPs were chosen from the PEER database of experimental results for concrete bridge components (Hose 2000). The database details specific discrete limit states for each of the EDPs considered. By mirroring the component database, it is possible to directly evaluate damage in a bridge, given the analysis demands. The measures range from global, such as drift ratio, to intermediate, such as cross-section moment, to local, such as stresses. Two global EDPs, used in current bridge design practice, are the column drift ratio ($\Delta$) and displacement ductility ($\mu_\Delta$). These are two kinematically dependent measures that can be used interchangeably in the PSDMs. The other EDPs that yield optimal PSDMs are maximum column moment ($M_{max}$), and intermediate EDP, and steel material stress ($\sigma_{steel}$), a local EDP.

**PSDA Analysis**

Nonlinear models were generated for each bridge configuration and analyzed using OpenSees. Modal analysis of different parameterized bridge models yields the periods of the first two elastic vibration modes (Table 2). The first mode shape is dominated by longitudinal deck motion accompanied by small deck rotations near the abutments. The second mode shape is dominated by simple transverse motion of the deck. Nonlinear time-history analyses were performed on all bridge configurations, using earthquakes in all bins. The final step in PSDA is to combine all the analyses into PSDMs, which relate ground motion specific IMs to class-specific EDPs. Given the wide array of IMs and EDPs for every analysis, the optimal models were chosen. Optimal is defined as practical, sufficient, effective, and efficient, as verified for the optimal models used below (Mackie 2002).
Table 2. Bridge first and second mode periods for sample bridge configurations.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$T_{1,\text{longitudinal}}$</th>
<th>$T_{2,\text{transverse}}$</th>
<th>Configuration</th>
<th>$T_{1,\text{longitudinal}}$</th>
<th>$T_{2,\text{transverse}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base bridge</td>
<td>0.64</td>
<td>0.55</td>
<td>$\rho_{s,\text{long}} = 0.01$</td>
<td>0.66</td>
<td>0.58</td>
</tr>
<tr>
<td>$L = 27$ m (1080)</td>
<td>1.18</td>
<td>1.00</td>
<td>$\rho_{s,\text{long}} = 0.03$</td>
<td>0.62</td>
<td>0.53</td>
</tr>
<tr>
<td>$L = 37$ m (1440)</td>
<td>1.87</td>
<td>1.58</td>
<td>$\rho_{s,\text{long}} = 0.04$</td>
<td>0.61</td>
<td>0.51</td>
</tr>
<tr>
<td>$L = 55$ m (2160)</td>
<td>3.92</td>
<td>3.28</td>
<td>$K_{\text{soil}} = A$</td>
<td>0.62</td>
<td>0.53</td>
</tr>
<tr>
<td>$L/H = 3.5$</td>
<td>0.48</td>
<td>0.41</td>
<td>$K_{\text{soil}} = C$</td>
<td>0.67</td>
<td>0.59</td>
</tr>
<tr>
<td>$L/H = 1.8$</td>
<td>0.82</td>
<td>0.72</td>
<td>$K_{\text{soil}} = D$</td>
<td>0.79</td>
<td>0.70</td>
</tr>
<tr>
<td>$L/H = 1.2$</td>
<td>1.21</td>
<td>1.08</td>
<td>$W_t = 0.25$</td>
<td>0.68</td>
<td>0.59</td>
</tr>
<tr>
<td>$D_c/D_s = 0.67$</td>
<td>0.72</td>
<td>0.64</td>
<td>$W_t = 0.5$</td>
<td>0.74</td>
<td>0.64</td>
</tr>
<tr>
<td>$D_c/D_s = 1.0$</td>
<td>0.49</td>
<td>0.39</td>
<td>$W_t = 0.75$</td>
<td>0.80</td>
<td>0.69</td>
</tr>
<tr>
<td>$D_c/D_s = 1.3$</td>
<td>0.39</td>
<td>0.30</td>
<td>$W_t = 1.50$</td>
<td>0.96</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Effectiveness and efficiency of a demand model is defined here for investigation of parameter sensitivity. Effectiveness describes the data fit. It is assumed the EDPs follow a log-normal distribution (Shome 1999). An equation describing the demand model can be written (Eq. 3), to which a linear regression in log-log space can be applied to determine the coefficients. Demand models lending themselves to this form allow closed form integration of Eq. 1 and casting of the entire framework in an LRFD type format. This was accomplished for steel moment frames in the SAC project (FEMA 2000). Efficiency describes the scatter about the fit. The measure used to evaluate efficiency is the dispersion, defined as the standard deviation of the logarithm of the demand model residuals. A more efficient demand model requires a smaller number of nonlinear time-history analyses to achieve a desired level of confidence.

$$ EDP = a(IM)^b $$

In each of the following models, the data is plotted on a log-log scale, with the EDP on the abscissa and the IM on the ordinate. This is the standard method for plotting any IM-EDP relationship, however, the intensity is still regarded as the dependent variable. A regression analysis of the data yields linear equations (Eq. 4), where $A$ and $B$ are the coefficients of a linear, or piecewise-linear regression in the log-log coordinate space. The dispersion values for each regression analysis are listed in each corresponding figure window. In this particular study, a bilinear least squares fit was made to reduce dispersion, as necessary. Each demand model is constructed in the longitudinal and the transverse direction independently.

$$ \log(EDP) = A + B \log(IM) $$

**Design Parameter Sensitivity**

The sample PSDMs formulated in this section make use of spectral IMs and the EDPs defined above. The consequence of this choice of demand model is the period dependence of the IM. Since the initial elastic stiffness is used to compute the measure of $SdT_1$, design parameters that vary the stiffness of the bridge system will cause intensity shifts in the demand models for a given earthquake. Alternate IMs, which introduce more dispersion, are Arias Intensity and PGV.
To alleviate the IM period dependence, these IMs can be used when comparing demand measures. The result is a single line of constant intensity parallel to the EDP axis.

Specifically, parameters used in this study are $L/H$, $D_c/D_s$, $L$, and $K_{s\text{oil}}$. They are chosen as they affect the period of the bridge, evident in the period differences within parameter groups (Table 2). The initial stiffness of the bridge is only minimally influenced by the amount of reinforcement (longitudinal and transverse) in the reinforced concrete column section. The only parameter that alters the mass, and accordingly the spectral values, is the additional dead weight, $W_t$. In order to understand the effect of design parameters, constant intensity lines have been added to the figures below.

![Intensity Measure vs Damage Measure (Parameters)](image1)

Figure 1. $L$ sensitivity, $SdT_1-\mu_A$.

![Intensity Measure vs Damage Measure (Parameters)](image2)

Figure 2. $L$ sensitivity, $SaT_1-M_{\text{max}}$.

Two optimal models for the span length parameter ($L$) sensitivity are shown in Figs. 1 and 2, respectively. Also shown on the figures are sample design trends when evaluating performance by varying the span length. Reducing the bridge span directly reduces the $\mu_A$ demand. The same is true of $\Delta$ as they are related by properties of the column. The fits in Fig. 1 provide desirable relationships, as they are linear, leading to simplified design equations. Fig. 2 is typical of the reduced dispersion obtained by a bilinear least squares fit. Also of note is the trend toward reduced dispersion in more flexible structures, given $L$, $\mu_A$, and $Sd$. This trend is opposite to many other models observed elsewhere (Mackie 2002).

The span- to column height-ratio ($L/H$) parameter models exhibit the same behavior as span length models when considering $M_{\text{max}}$ and $\Delta$. The span is held constant while column heights are varied to the specification of the parameter. Shown in Fig. 3 is the model pairing $\Delta$ with $Sa_{Luco}$ using a linear fit. Increasing stiffness results in reduced demand at a given intensity. Fig. 4 shows the trends in models that use local demand quantities, such as $\sigma_{\text{steel}}$ in this instance. This figure indicates that increased stiffness does not result in improved performance for all intensity levels, and local EDP models exhibit high efficiency. The slope of the fits corresponds directly to the rate of change of performance (demand). Steep slopes result in minimal demand changes, while shallow slopes produce large demand reductions for small variations in design parameter values. This will be performed later in a separate publication with more data.
Variation of column-to-superstructure dimension ratio ($D_c/D_s$) is based on varying the column diameter as the superstructure depth and properties are held fixed throughout. Therefore, for higher stiffness columns (larger diameter), a larger performance gain is realized (Figs. 5 and 6).

The design parameter (and thus period) dependent change in dispersions between a period-dependent (Fig. 5) and period-independent (Fig. 6) IM can be assessed directly. Piecewise linear fits with negative slopes are merely a function of fewer data points in the high intensity range and subsequent attempts by the bilinear fit algorithm to reduce dispersion.

The steel reinforcement ratio ($\rho_s$) is one of the parameters not causing intensity shifts in spectral plots. Using the optimized linear IM, $S_{d_{Luco}}$ the reduction in demand can be immediately correlated to the design strength (Fig. 7). A linear increase in reinforcement does not correspond to a linear decrease in demand, however. The sensitivity of performance at lower reinforcing ratios is more pronounced. Similarly for forces, increasing the reinforcing amount has a drastic affect on the amount of moment attracted to the column (Fig. 8). Evident are two linear regimes for each design parameter, as may be expected of an idealized bilinear moment-curvature relation of the column cross-section.
As seen in Table 2, the small change in period attributed to bridge mass results in a similarly small moment demand reduction. However, when a large mass is considered (150%), significant P-Δ effects cause an increase in moment demand, as shown in Fig. 9. In the context of the response spectrum, larger mass can be equated with a flexible structure, or longer period. Similar to Fig. 2 then, dispersions decrease with longer period design parameter models, however, the period ranges covered by these parameters are significantly longer than any of the other parameters considered. This phenomena is independent of whether a force or displacement based EDP is used, given the $L$ or $Wt$ design parameter.

Fig. 10 shows the relationships for variation of soil stiffness, $K_{soil}$. While there is a reduction in demand when considering either a rock or soft soil site, the difference between a USGS B or C site is small. Again, the negative bilinear fit slopes are a result of large dispersion in a few data points at high intensity levels.

**Design Trends**

Given the trends outlined in the models above, one model (Fig. 6) is selected in order to
develop a set of design equations using design parameter $D_c/D_s$ that can be used explicitly by designers without coupling to Eq. 1. In order to facilitate ease of use, a model with period independence is selected. This is done at the expense of efficiency, albeit eliminating $T$ as a design variable directly. The use of $Sd_T$ (Fig. 5) instead of Arias Intensity would have resulted in a decrease in dispersion of approximately 33%, more for low $T$.

Using the regression data and Eq. 4, the design equation can be written as Eq. 5.

$$\ln(EDP) = \left(5.84x^2 - 11.81x + 1.23\right) + \left(-1.08x^2 + 1.85x + 0.021\right)\ln(IM)$$  \hspace{1cm} (5)

with $x=D_c/D_s$, $EDP=\mu_\Delta$, and $IM=$Arias Intensity. For a particular case of $x=1.15$, the design equation reduces to Eq. 6.

$$\ln(EDP) = -4.623 + 0.721\ln(IM)$$  \hspace{1cm} (6)

For example, in order to reduce $\mu_\Delta$ from 3 to 2 (33% reduction), given an earthquake event with an Arias Intensity of 750 cm/s, the polynomial in Eq. 5 can be solved. The resulting design requires increasing the column diameter ratio from 0.65 to 0.90 (38% increase). Whereas, reducing $\mu_\Delta$ from 2.5 to 1.67 (33%) requires diameter increase from 0.78 to 0.99 (27%).

**Conclusions**

The PSDMs described in this paper can be used directly by designers as structural demand hazard curves. They allow assessment of the effects of structural design parameter variations on structural performance. Design decisions can be made on the trade-off between quantifiable performance levels and the resulting changes in design and material requirements, as shown by design Eq. 5 and 6. While these equations are predictive for all ranges, the modularity of the approach allows subsequent resolution refinement of both parameter ranges as well as the parameters themselves.

Additionally, cast as a component model in a performance-based earthquake engineering framework, such as Eq. 1, PSDMs provide the probabilistic relationship between ground motion IMs and structure-specific EDPs. The IM side can be coupled with hazard models and the EDPs to capacity models in order to compute probabilities of exceedance of such economic variables as repair cost. Armed with repair cost data, owners can establish their own performance criteria using economic considerations.

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