RELATION BETWEEN PROBABILISTIC SEISMIC DEMAND ANALYSIS AND INCREMENTAL DYNAMIC ANALYSIS

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ABSTRACT

A component of current probabilistic seismic design frameworks is a demand model relating the probability of exceeding certain demand measures given expected seismic hazard. This demand model can be formulated using two analysis methods, Probabilistic Seismic Demand Analysis and Incremental Dynamic Analysis. The first method attempts to represent seismicity through a wide selection of many ground motions, grouped into bins. The latter method achieves the same by stepwise incrementation of a select few ground motion records. Comparison of resulting Probabilistic Seismic Demand Models relating structural demand measures to quantified measures of intensity yield information as to the equivalency of the two methods and instances when they can be used interchangeably. The demand models developed in this paper are for typical California highway overpass bridges, using the variation of the column diameter design parameter as an illustrative case of the applicability of the methods.

Introduction

Probabilistic seismic demand analysis (PSDA) and Incremental dynamic analysis (IDA) are two of the tools used today to compute the probability of exceeding specified structural demand levels in a given seismic hazard environment. Such a relationship, termed a demand model, is an essential building block of a probabilistic performance-based seismic design framework developed in PEER (Cornell 2000). Probabilistic evaluation of performance given seismic uncertainties has been addressed in the SAC Steel Project (FEMA 2000). While the SAC project focused on buildings, the procedure remains analogous for any type of structure considered, in this case applied to overpass bridges. Methods of analysis inevitably introduce bias in demand models. The demand models that make use of either PSDA or IDA attempt to mimic more realistic nonlinear dynamic conditions to reduce this bias (Yun 2001).

PSDA uses a bin approach, where a portfolio of ground motions is chosen to represent

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the seismicity of an urban region. The intensities of the ground motions in the portfolio cover a range of seismic hazard Intensity Measures (IM), such as first mode spectral acceleration. Nonlinear time-history dynamic analyses are performed for each motion using a model of the structure to compute extreme values of structure-specific Engineering Demand Parameters (EDP).

IDA is the dynamic equivalent to a familiar static pushover analysis. Given a structure and a ground motion, IDA is done by conducting a series of nonlinear time-history analyses. The intensity of the ground motion, measured using an IM, is incrementally increased in each analysis. An EDP, such as global drift ratio, is monitored during each analysis. The extreme values of an EDP are plotted against the corresponding value of the ground motion IM for each intensity level to produce a dynamic pushover curve for the structure and the chosen earthquake record. To achieve comparison with an equivalent PSDA, IDA intensities must cover a similar range. Following either type of analysis, a regression of the data comprising maximum EDP and corresponding IM values produces a probabilistic seismic demand model.

A comparison of PSDA and IDA results for typical California two-span single-column-bent reinforced concrete highway overpass bridges is presented in this paper. The PSDA and IDA were conducted using the same portfolio of recorded ground motions. Probabilistic Seismic Demand Models (PSDMs) were computed in two ways:

1) For an interval of IM values, centered about a value used in IDA, EDPs were obtained from a PSDA done using a portion of the ground motion portfolio such that the ground motion IMs are in the chosen interval.
2) For a given IM, EDP data was obtained using IDA.

Such IDA-PSDA comparisons were done for a number of IM-EDP pairs. The differences between the two methods and the conditions that IMs and EDPs need to satisfy to ensure that IDA and PSDA results are statistically equivalent are discussed. The results of this paper can be used to streamline the process of developing probabilistic seismic demand models for a performance-based seismic design framework.

Class of Structures and Analysis Model

This study focuses on typical new California highway overpass bridges. The bridges presented in this paper follow standard Caltrans design criteria for reinforced concrete bridges. Configurations in this study are limited to two-equal-span overpasses including abutments on either end, with no skew. Common to all bridges is a single column bent anchored below grade by a Type I integral pile foundation. Bridge geometry and material property configurations are controlled by varying design parameters. The only variation presented here is the ratio of column diameter to superstructure depth (D_c/D_s). The PEER OpenSees (www.opensees.org) platform was selected as the nonlinear finite element engine for demand analysis. Complete details regarding the design and model are presented in (Mackie 2001).

The modeling of abutments is essential to a good finite element model of an overpass bridge, as the response of short- and medium-span bridges is expected to be largely dominated by abutment stiffness. In this study, simple elastic-plastic spring-gap elements were implemented. Spring stiffness values were determined from large scale abutments tests (Maroney 1994). A trilinear backbone is used for the longitudinal direction, taking into account bearing pads, initial gap, backwall, and piles. A simple elastic-plastic backbone is used in the transverse direction, accounting for shear keys, wing walls, and piles. Nonlinear models were generated for each bridge configurations and analyzed for each ground motion as dictated by the analysis method chosen. Each analysis involves a static pushover analysis, a modal analysis, and a dynamic time-history analysis. Determination of column yield deformations and forces is performed with zero abutment stiffness.

Demand Model

Probabilistic demand analysis is done in order to estimate the mean annual frequency (ν) of exceeding a given structural demand measure ($EDP > y$) in a postulated hazard environment ($IM = x$), Eq. 1. The results of such an analysis can be presented as either PSDMs, or structural demand hazard curves. PSDMs directly relate IMs to EDPs. Hazard curves couple Probabilistic Seismic Hazard Analysis (PSHA) to the PSDM (Luco and Yeo 2001), producing continuous rates of exceedance of the demand measure (Medina 2001). To emphasize the derivation of the IM-EDP relationship using a demand analysis tool, only PSDMs are shown herein.

$$\nu_{EDP}(y) = \int_x G_{EDP|IM}(y \mid x) \mid d\lambda_{IM}(x) \mid \qquad (1)$$

The procedure used to formulate the PSDMs of interest involves five steps. First, a set of ground motions, representative of regional seismic hazard, is selected or synthesized. Not only must the ground motions lend themselves to use in each of the analysis methods, but they must be categorized according to computable IMs descriptive of their content and intensity. Second, the class of structures to be investigated is defined. Associated with this class are a suite of EDPs which can be measured during analysis to assess structural performance under the considered motions. Third, a nonlinear finite element analysis model is generated to model the class of structures selected. Fourth, nonlinear dynamic analyses are performed using combinations of ground motions and structural configurations specific to the demand analysis tool. Finally, a PSDM is formulated between resulting ground motion IMs and structural EDPs.

Probabilistic Seismic Demand Analysis (PSDA)

The only variance between the two analysis tools presented is in the first step described above. The method for varying seismic demand using PSDA is to employ the bin strategy. Bins subdivide ground motions into groups based on moment magnitude (M_w), closest distance (R), and local soil type. Magnitude and distance are directly applicable to the prediction of earthquake IMs using standard attenuation relationships, as well as in existing PSHA. The effect of bin-specific earthquake characteristics can be evaluated on the resulting structural demands. An optimal matrix of bins contains an even distribution of M_w and R values at all levels of intensity, thus making it comparable to the spectrum of intensities generated during IDA. A sample PSDM generated using PSDA is shown in Fig. 1.

The bins used in this study are delineated at a magnitude of 6.5 and a distance of 30 km. Four bins with 20 ground motions each were obtained from the PEER Strong Motion Database (peer.berkeley.edu/smcat), and are characteristic of non-near-field motions ($R > 15$ km) recorded in California. The specific records selected are the same as those used by Krawinkler (Medina 2001). The ground motions in the bins are chosen such that the median spectral shapes (and dispersions) of all bins are roughly equivalent when scaled to a common spectral value. For PSDA, the acceleration amplitudes were all scaled by a factor of two, and the sampling period reduced to 0.02 seconds. The two horizontal ground motion components were arbitrarily oriented in the bridge longitudinal and transverse directions. These components may not have been recorded in perpendicular directions, nor correspond to the earthquake fault parallel and fault parallel directions. As the intention of the bin strategy is to vary the seismic demand, not capture near-field or directivity effects, the records were not rotated prior to use.

Incremental Dynamic Analysis (IDA)

The theory behind IDA lies in the ability to take a small subset of ground motions from a larger catalog or series of bins and reuse the same motion to mimic variable intensities. This variation in intensity is achieved by incrementally increasing the amplitude of the ground motion record (Vamvatsikos 2001). Careful selection of PSDA bins described above ensures that intensity values are clustered at approximately the same values as the increments used in IDA, allowing comparable demand results. Ideally, a ground motion could be scaled until the structure collapses, generating a dynamic pushover plot. However, given the limitations of the model used herein, the bridge continues to gain stiffness until numerical instability causes loss of convergence. Certain ground motions generate regions of intermediate instability that provide inaccurate (non-converged) measures of demand in these regions. Therefore, the region of interest has been purposely limited to the intensity ranges covered by PSDA.

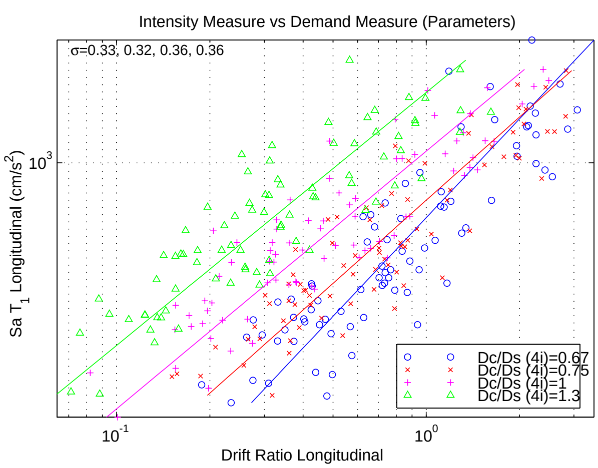

Figure 1. PSDA Sa_{T1}-Δ, D_c/D_s 4i.

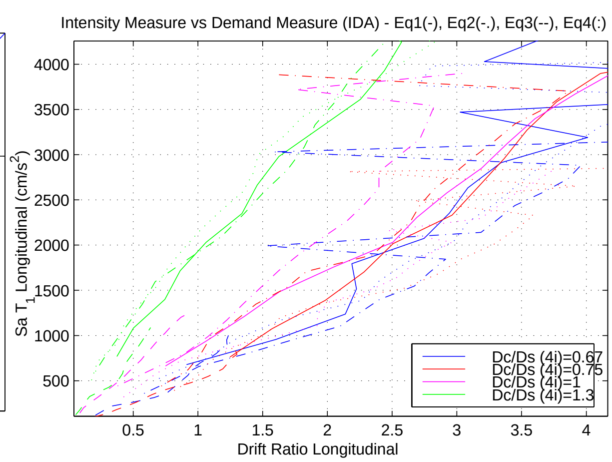

Figure 2. IDA Sa_{T1}-Δ, D_c/D_s 4i.

Table 1. Ground motions used for IDA.

Bin	Series "4i"	Series "4j"
LMSR	Northridge, 1994, Canyon County	Loma Prieta, 1989, Hollister City Hall
LMLR	Loma Prieta, 1989, SLAC Lab	Borrego Mtn., 1968, El Centro #9
SMSR	Livermore, 1980, Eastman Kodak	Imperial Valley, 1979, Chihuahua
SMLR	Imperial Valley, 1979, Delta	Morgan Hill, 1984, Capitola

Example IDA curves are shown in Fig. 2. Usually plotted in linear scale, each curve is for one ground motion as it is incremented. The use of bin scaling can be substantial in limiting the number of ground motions selected for analysis. Assuming the standard errors of estimation are the calculated dispersions divided by the root of number of records used (Shome 1999), the 1σ confidence band, given a dispersion of 0.15 and 4 records, is 7.5% wide. Accordingly, one ground motion has been selected from each of the PSDA bins, denoted as series "4i" and "4j" (Table 1).

Probabilistic Seismic Demand Models

Probabilistic seismic demand models are the result of either PSDA or IDA. Specifically, PSDMs relate structure specific EDPs to ground motion specific IMs. PSDMs have several inherent properties, where optimal PSDMs are selected based on practicality, sufficiency, effectiveness, and efficiency (Mackie 2002). Only those determined to be optimal are compared in this study. Of primary interest when using IDAs is the evolution of the median values (effectiveness), and dispersion (efficiency).

Effectiveness of a demand model describes how it lends itself to closed form solution of Eq. 1. EDPs are assumed to follow a log-normal distribution (Shome 1999), defining Eq. 2.

$$\log(EDP) = A + B\log(IM) \qquad (2)$$

to which a linear, or piecewise linear, regression in log-log space can be applied to determine the coefficients, A and B. PSDMs are presented in log-log scale, with the EDP on the abscissa and the IM on the ordinate. The IM remains the dependent variable, as in Eq. 2.

Efficiency is the amount of variability of an EDP given an IM. The measure used to evaluate efficiency is the dispersion, defined as the standard deviation of the logarithm of the demand model residuals (Shome 1999). An efficient demand model requires a smaller number of nonlinear time-history analyses to achieve a desired level of confidence. Only the aleatory variability attributed to the randomness in the source is measured herein. Additional epistemic uncertainty (Cornell 2001) derived from knowledge issues should also be included in Eq. 1.

PSDM Intensity Measures and Demand Parameters

The IM s used to describe ground motion characteristics are specifically limited in this study to first mode spectral acceleration (Sa_{T1}) and an optimized spectral acceleration measure (Luco and Cornell 2001) defined in Eq. 3. This measure is used to reduce the real dispersion (Fig. 5).

$$Sa_{Luco} = Sa(T_1)\sqrt{\frac{Sa(2T_1)}{Sa(T_1)}} \qquad (3)$$

The bridge EDP s were chosen from the PEER database of experimental results for concrete bridge components (Hose 2000). The database details specific discrete limit states for each of the EDP s considered. The measures range from global, such as drift ratio (Δ), to intermediate, such as maximum column curvature (ϕ_{max}), to local, such as column reinforcing steel stresses (σ_{steel}).

IDA-PSDA Comparison

Conditions under which IDA and PSDA can be used interchangeably in a demand analysis and PSDM formulation are investigated in this section. Initial comparisons use the same computational effort, as both incorporate 80 nonlinear analyses per bridge configuration. PSDA attains this from the four bins of 20 motions, while IDA scales four motions 20 times.

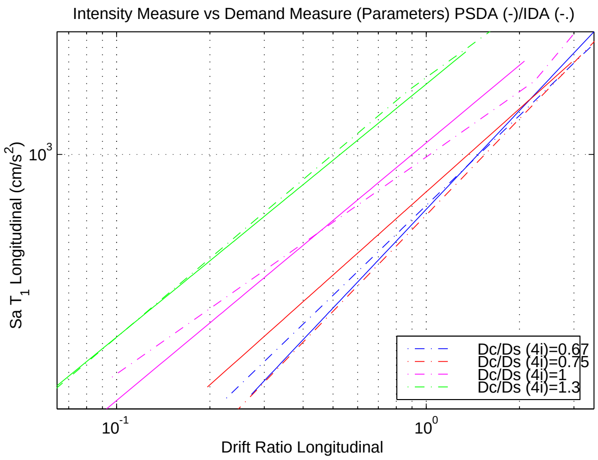

Figure 3. IDA/PSDA, D_c/D_s (Δ) 4i.

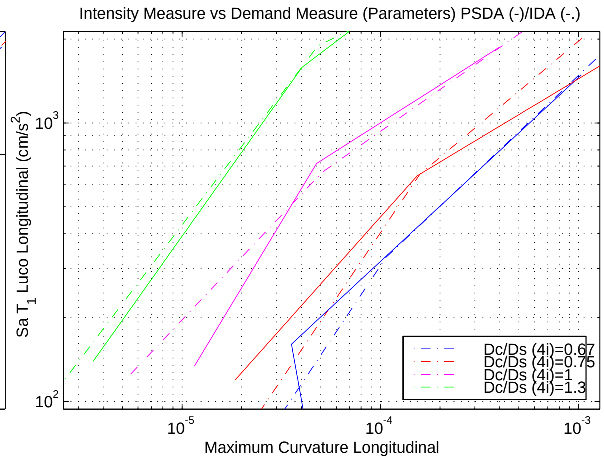

Figure 4. IDA/PSDA, D_c/D_s (ϕ_{max}) 4i.

Due to the "pushover" nature of IDA, it tends to capture demand values at the upper end of the spectrum. However, in order to facilitate comparison in the same data ranges, the IDA and PSDA ranges are reduced to the same interval. Considering the same PSDM developed above using both analysis methods, the data median and distribution are subsequently compared. Fig. 3 shows the median contrast plot. IDA analysis results are shown in dotted lines, PSDA in solid. Ideally, the median values would agree between methods. The differences between IDA and PSDA for this PSDM are, indeed, small, especially for stiffer bridges (higher D_c/D_s). As the location of the bilinear fit intersection is determined by the least squares algorithm to minimize dispersion, PSDMs may differ substantially, as shown in Fig. 4. This is, in particular, true of the PSDMs for intermediate (ϕ_{max} Fig. 4) and local EDPs (σ_{steel}). For the PSDM shown (Fig. 3), median lines agree for all but D_c/D_s =0.75, especially for higher stiffness bridge configurations.

More critical for PSDM use is dispersion. In Fig. 1, the resultant PSDA dispersions are 0.33, 0.33, 0.36, and 0.36. For initial IDA comparisons, these dispersions are assumed to be the real dispersions associated with the given PSDM. The IDA plot (Fig. 2), when cast as a PSDM, yields dispersions of 0.27, 0.32, 0.27, and 0.18, respectively. To compare dispersions directly, PSDMs from Fig. 3 are plotted using one sigma (μ+1σ or 84th percentile) stripes. It should be noticed that the 1σ distribution using IDA lies below the PSDA envelope of true dispersions for all parameter values (Fig. 5). Drawing from these results, the estimated dispersion for the model is under-predicted, even if the median values agree for this ground motion subset. The real dispersion, however, is reduced by using the optimized spectral acceleration quantity (Eq. 3).

Evolution of Median Value and Dispersion

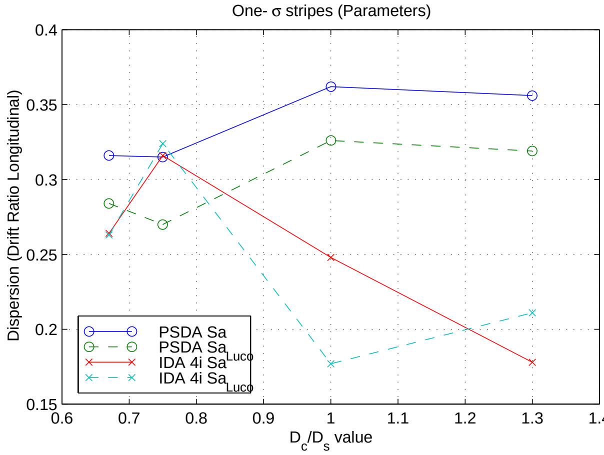

Figure 5. IDA/PSDA, D_c/D_s 4i 1σ stripes.

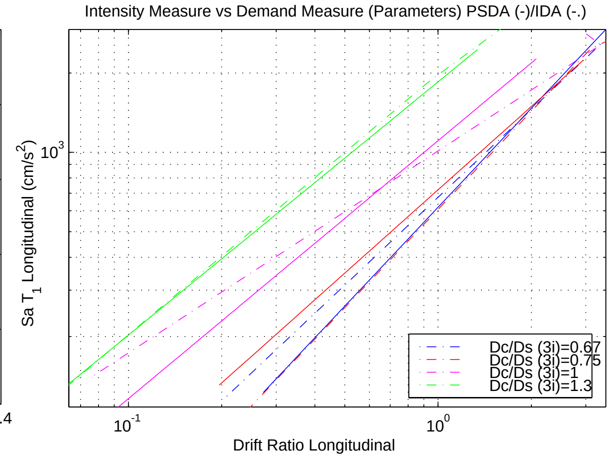

Figure 6. IDA/PSDA, D_c/D_s (Δ) 3i.

The worth of using IDA could be further increased then, if fewer analyses were required to attain the same level of confidence in the median which we associate with the PSDA method. Fig. 6 repeats the comparison of median values, except the IDA data uses only 3 ground motions ("3i" removes LMSR bin) amplified 20 times. This 25% reduction in analysis effort results in similar fits for high stiffness bridge configurations (D_c/D_s =1.3), but an increasing deviation for all others. Up to 50% deviation (between PSDA and IDA median lines) exists. Table 2 shows the evolution of dispersion with reduction of computation effort for the models shown. The significant reduction in the estimated dispersion for fewer analyses indicates a loss of confidence in the median. Therefore, reducing computational effort as described above provides uncertain results. A better IDA approach would be to use 5 motions amplified 16 times or 8 motions amplified 10 times each. As indicated in Table 2, the estimated dispersions approach the true dispersions as the number of records is increased.

Table 2. Dispersions of PSDA and IDA runs (Sa_{T1} vs Δ).

	Type	**D_c/D_s =0.67**	**D_c/D_s =0.75**	**D_c/D_s =1.0**	**D_c/D_s =1.3**
PSDA	4i	0.32	0.32	0.36	0.36
PSDA	16	0.33	0.32	0.36	0.35
IDA	4i	0.26	0.32	0.25	0.18
IDA	3i	0.23	0.31	0.26	0.21
IDA	2i	0.21	0.18	0.29	0.21
IDA	4j	0.29	0.31	0.35	0.25
IDA	8ij	0.30	0.36	0.31	0.27

Ground Motion Dependence

Another issue of contention when performing IDA is the selection of ground motions. The bin approach allows for incorporation of wide ranges of frequency content and amplitude variation, simply by choice of different earthquakes (by locations and site effects). However, if only one motion is selected from each bin, the results may become biased. To explore this, a second set of IDA ground motions was selected, group "j" (Table 1). The median and 1σ analysis plots were once again performed for this new subset of motions, and compared to the same PSDA results as in Fig. 3.

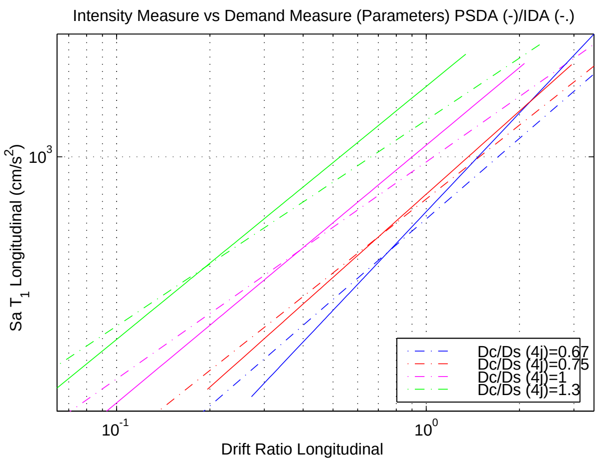

Figure 7. IDA/PSDA, D_c/D_s (Δ) 4j.

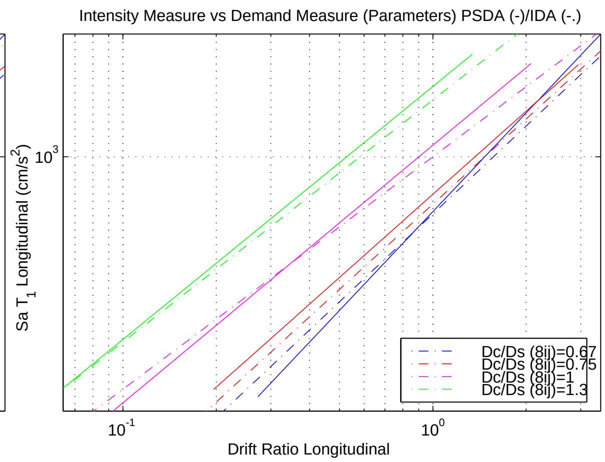

Figure 8. IDA/PSDA, D_c/D_s (Δ) 8ij.

Fig. 7 shows the change in median slope with the new motions. As the median lines intersect at intensity levels close to the median of the range of interest, the difference in demand for a given intensity is small. However, when used as predictors for performance at higher levels of shaking, the results will diverge substantially. As discussed, a better choice would be 8 motions ("8ij") amplified 10 times each (Fig. 8). The median values show better agreement than both Figs. 3 and 5. Corresponding dispersions for both "4j" and "8ij" are shown in Table 2 and the 1σ stripe plot (Fig. 10). Dispersion for the 8 motion IDA approaches the real dispersion.

Verification of PSDA

Thus far, all the IDA-PSDA comparisons have been addressed against a baseline PSDA set of results, assumed to correctly predict the median and dispersion. However, there is no guarantee that the number of motions used in the bin approach is sufficient to guarantee accurate representation of the median and dispersion. At the expense of computation time, it is possible to create a denser array of ground motions in the M_w, R bin matrix. This is accomplished with the addition of three ground motion bins created by Luco and Cornell (Luco and Yeo 2001) to the existing ones from Krawinkler (Medina 2001). Each of the near-field, intermediate-field, and far-field bins contain 75 ground motions, bringing the total number of motions considered to 250 (duplicate motions were removed). These bins provide a larger subset of information in the higher intensity (near-field) region, which may be useful in comparison with the upper reaches of the IDAs. However, as before, the regions of interest are standardized.

Figure 9 shows the median values for each of the bridge configurations. The original PSDA data is once again in solid lines while the expanded ground motion set is shown in dotted lines. The 1σ stripes are shown in Figure 10. Dispersions for both ground motion selections are also shown in Table 2. As can be seen, the 80 motion (four bin) subset provides a sufficiently accurate representation of expected behavior. On average, median values differ by less than 5%, and dispersions are roughly equivalent, verifying the initial PSDA assumptions.

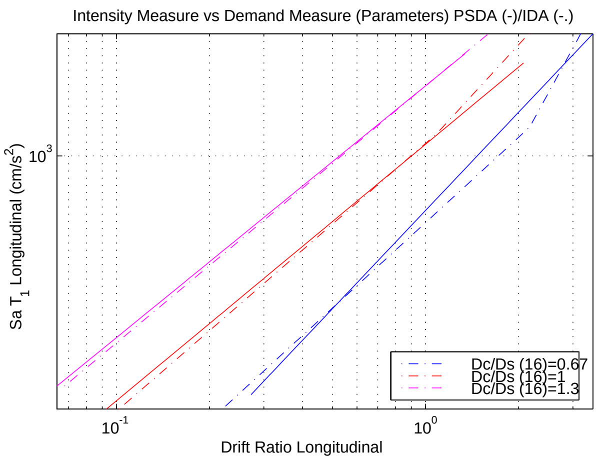

Figure 9. PSDA comparison, median.

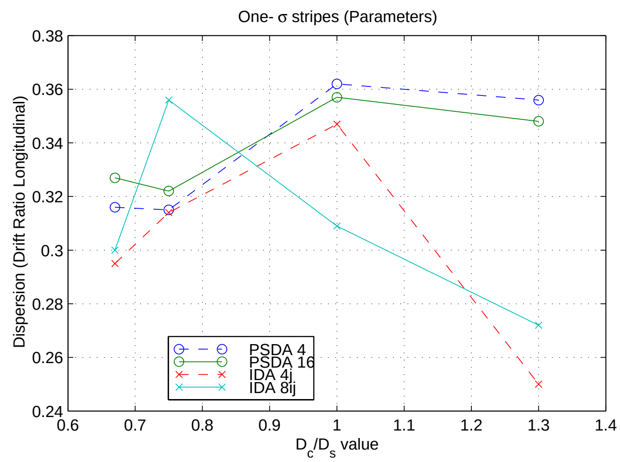

Figure 10. IDA/PSDA, D_c/D_s 1σ stripes.

Conclusions

Given the optimal PSDM formed from the IM-EDP pair Sa_{T1} and global Δ, the two demand analysis tools, PSDA and IDA, can be used interchangeably to compute the probability of exceeding specified structural demand levels, i.e. to formulate PSDMs for performance-based analysis. Similarly, other optimal PSDMs can be generated with either IDA or PSDA. Other IM-EDP combinations can be used to reduce the real dispersion of the model if efficiency is critical. With the same computational effort, the median and 1σ least square fits for both methods produce results with similar confidence levels in the median as long as an adequate number of ground motions are considered for the IDA. According to the PSDM presented in this study, even using 8 motions will slightly underpredict dispersion. Using more motions will cause the median and dispersion values between the two methods to converge. For stiffer bridge configurations, it is possible to reduce the number of IDAs used with a subsequent decrease in confidence of predicting the EDP. The IDA method is sensitive to the choice of ground motions, however, therefore it is recommended that the number of analyses not be reduced and a representative set of motions be carefully selected from the regional seismic hazard database of interest. Realistically, if more than 10 motions are required for dispersion agreement, or low confidence in IDA results is expected for bilinear PSDMs involving local and intermediate EDPs, the PSDA method should be used as it provides sufficient variation in ground motion content.

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