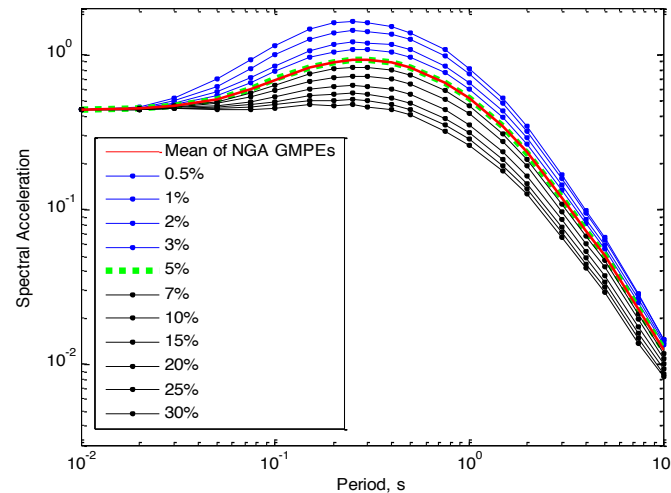




Damping Scaling Models for Elastic Response Spectra



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Outline

- ❑ Background
- ❑ Database
- ❑ Model Development: “Average” Horizontal Component
 - Median *DSF* (Damping Scaling Factor)
 - Variability
- ❑ Comparison with Existing Models
- ❑ Model for Vertical Component
- ❑ Deliverables / Products

- **Motivation:** GMPEs are traditionally developed for 5% damping. Real structures can have damping ratios other than 5%.

TABLE 3. RECOMMENDED DAMPING VALUES

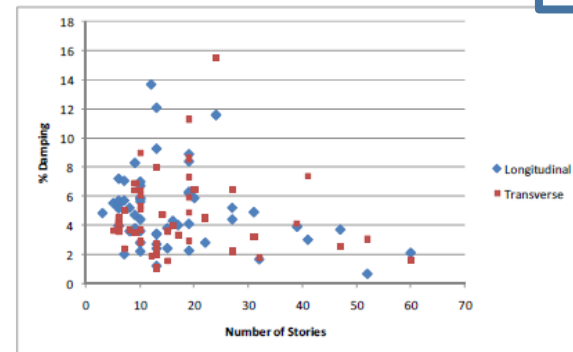
Stress Level	Type and Condition of Structure	Percentage Critical Damping
Working stress, no more than about ½ yield point	• Vital piping	1 to 2
	• Welded steel, prestressed concrete, well reinforced concrete (only slight cracking)	2 to 3
	• Reinforced concrete with considerable cracking	3 to 5
	• Bolted and/or riveted steel, wood structures with nailed or bolted joints	5 to 7
At or just below yield point	• Vital piping	2 to 3
	• Welded steel, prestressed concrete (without complete loss in prestress)	5 to 7
	• Prestressed concrete with no prestress left	7 to 10
	• Reinforced concrete	7 to 10
	• Bolted and/or riveted steel, wood structures, with bolted joints	10 to 15
	• Wood structures with nailed joints	15 to 20

Newmark and Hall 1982

2.4.4.1 Selection of Target Damping

In **linear-elastic response history analyses**, using either modal response history or direct integration, the magnitude of damping is chosen to represent, in an approximate sense, the amount of energy dissipation at the expected deformation levels. **At low deformation levels, prior to significant yielding or damage to structural components, damping values are typically in the range of 0.5% to 5% critical damping in the primary vibration modes. At higher deformation levels, damping values up to 20% of critical (or more) may be specified to approximate hysteretic effects that are not otherwise represented in the analysis.**

PEER/ATC-72-1 (2010)



(a) damping versus number of stories

Background

- **Goal:** Develop scaling factors to translate existing GMPEs for spectral ordinates at 5% damping to spectral ordinates at other damping ratios from 0.5 to 30%.
- **Definition:** $DSF = PSA(\beta) / PSA(5\%)$
- **Develop:** $\ln(DSF) = \mu(\beta, T, \text{earthquake, site, } \mathbf{b}) + \varepsilon; \quad \varepsilon \sim N(0, \sigma)$

Distribution? Predictor Variables?
Functional Form? Variability?

Background

- **Goal:** Develop scaling factors to translate existing GMPEs for spectral ordinates at 5% damping to spectral ordinates at other damping ratios from 0.5 to 30%.
- **Definition:** $DSF = PSA(\beta) / PSA(5\%)$
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→ Existing literature dating all the way back to Newmark and Hall 1982

→ Database of over 8,000 recorded motions

Literature Review

Extra

Reviewed and summarized over 25 related papers (see the PEER report).

Literature Review

Taken from the PEER report:

		Relation	Model	Notes
GMPEs for $\beta \neq 5\%$	(See Bommer and Mendis, 2005 for a review and comparison of these relations)	Akkar and Bommer, 2007	<p>Geometric mean elastic spectral displacement (SD):</p> $\log[SD(T, \beta)] = b_1 + b_2M + b_3M^2 + (b_4 + b_5M) \log \sqrt{R_{jb}^2 + b_6^2} + b_7S_S + b_8S_A + b_9F_N + b_{10}F_R$ <p>Regression coefficients b_i, $i = 1, \dots, 10$, and standard deviations are given in tables at specified periods for damping ratios of 2, 5, 10, 20, and 30%</p>	<ul style="list-style-type: none"> • Applicability: Periods up to 4s Magnitudes between 5 and 7.6 Distances up to 100km • Database: 532 accelerograms from Europe and the Middle East • Acknowledge dependence of DSF on magnitude, distance, and therefore duration.
		Berge-Thierry et al., 2003	GMPE for pseudo acceleration response spectrum (PSA) is provided for damping ratios of 5, 7, 10, and 20%	<ul style="list-style-type: none"> • Applicability: Periods up to 10s Magnitudes between 4 and 7.9 Distances up to 330km • Database: 965 horizontal and 485 vertical components from Europe (83%) and California (17%)
		Bommer et al., 1998	GMPE for relative displacement response spectrum (SD) is provided for damping ratios of 5, 10, 15, 20, 25, and 30%	<ul style="list-style-type: none"> • Applicability: Magnitudes between 5.5 and 7.9 Distances up to 260km • Database: 183 records from Europe
		Boore et al., 1993	GMPE for pseudo velocity response spectrum (PSV) is provided for damping ratios of 5, 10, and 20%	<ul style="list-style-type: none"> • Applicability: Periods up to 2s Magnitudes between 5.3 and 7.7 Distances up to 100km • Database: 271 records from western North America
		Trifunac and Lee, 1989	GMPEs for pseudo velocity response spectrum (PSV) are provided with regression coefficients tabulated at specified periods for damping ratios of 0, 2, 5, 10, and 20%	<ul style="list-style-type: none"> • Applicability: Periods between 0.04 and 14s • Database: 438 records from 104 earthquakes, mostly from California up to the year 1981

Literature Review

Taken from the PEER report:

GMPEs for $\beta \neq 5\%$	Faccioli et al., 2004	Propose a model for the displacement spectra. Consider damping ratios of 0 and 5% only.	<ul style="list-style-type: none"> • Applicability: Periods between 0.01 and 10s Magnitudes between 5.4 and 7.6 Distances up to 50km • Database: 253 records (3 components each) from Taiwan, Japan, Italy, and Greece • Conclude that the influence of damping ratio is limited at long periods
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	Relation	Model	Predictive Variables	Notes
Random Vibration Methods	White-Noise Assumption	$DSF \cong \sqrt{\frac{5}{\beta}}$	1. Damping ratio, β	<ul style="list-style-type: none"> • A white-noise process is wide-band and stationary, which is not necessarily true for a real earthquake ground motion
	Recommended method by McGuire et al., 2001 (NUREG/CR-6728)	<p>If $1 < f < 5$ Hz : Using Rosenblueth (1980) $SA(f, \beta)$ $= SA(f, 0.05) \left[\frac{1 + 4.9\beta f D}{1 + 4.9 \times 0.05 f D} \right]^{-0.41}$</p> <p>If $f \geq 5$ Hz : Using Vanmarcke (1976) $SA(f, \beta)$ $= \left\{ PGA^2 + [SA(f, 0.05)]^2 - PGA^2 \right\} \left[\frac{1 + 4.9\beta f D}{1 + 4.9 \times 0.05 f D} \right]^{-0.82} \cdot 0.5$</p>	1. Damping ratio, β 2. Frequency, f 3. Duration, D (distance dependent; use Abrahamson and Silva 1997 model for Western U.S.; and Atkinson and Boore 1997 model for Central Eastern U.S.)	<ul style="list-style-type: none"> • Applicability: Horizontal and vertical components $\beta = 0.5 - 20\%$ • The most theoretically consistent method • SA: spectral acceleration

Literature Review

Taken from the PEER report:

	Relation	Model	Predictive Variables	Notes
Empirical DSF Models	Hatzigeorgiou, 2010	$DSF = 1 + (\beta - 5)[1 + b_1 \ln(\beta) + b_2 (\ln(\beta))^2][b_3 + b_4 \ln(T) + b_5 (\ln(T))^2]$ <p>Regression coefficients $b_i, i = 1, \dots, 5$, are tabulated for different soil conditions for acceleration, velocity and displacement response spectra.</p>	<ol style="list-style-type: none"> 1. Damping ratio, β 2. Period, T 3. Soil conditions 	<ul style="list-style-type: none"> • Applicability: $T = 0.1 - 5s$ $\beta = 0.5 - 50\%$ Magnitudes between 5-8 Distances up to 60km • Database: 100 far-fault records, 110 near-fault records, 100 artificial accelerograms • States that fault distance has no impact on DSF • Performs nonlinear regression analysis test on about 8000 mathematical equations (i.e., DSF models)
	Stafford et al., 2008	$DSF = 1 - \frac{b_1 + b_2 \ln(\beta) + b_3 \ln(\beta)^2}{1 + \exp\{-[\ln(x) + b_4]/b_5\}}$ <p>x is a measure of duration and can be any of the following parameters: D_{5-75} : significant duration D_{5-95} : significant duration $N_{rr}(2.0)$: number of equivalent load cycles</p> <p>Regression coefficients $b_i, i = 1, \dots, 5$, and standard deviations are given in Table 2 for relative displacement spectra</p>	<ol style="list-style-type: none"> 1. Damping ratio, β 2. Duration, x <p>Data are averaged over periods of 1.5 to 3s.</p>	<ul style="list-style-type: none"> • Applicability: $T = 1.5 - 3s$ $\beta = 2 - 55\%$ Magnitudes between 4.2-7.9 Distances up to 300km • Database: 1699 records from NGA database excluding Chi-Chi and records with missing metadata • Confirm and quantify the strong dependence on Duration, which is strongly related to Magnitude. Mild dependence on Period is reported. • A modified logistic model is used in modeling.

Literature Review

Taken from the PEER report:

	Relation	Model	Predictive Variables	Notes
Building Codes	Eurocode 8, 2004 (see Bommer and Mendis, 2005, page 148; see Akkar and Bommer, 2007, page 1291)	$\left(\frac{10}{5 + \beta}\right)^{0.5} \geq 0.55$ Original form (1994): $\sqrt{\frac{7}{2+\beta}}$	1. Damping ratio, β	$T = 0.2 - 6s$ Unity at very low (i.e., 0s) and very high periods (i.e., 10s used in Bommer and Mendis, 2005) is imposed. Should not apply to β resulting in $DSF < 0.55$. Records represent European strong ground motions with magnitudes between 4.0 and 7.5 and distances up to 200 km.
	NEHRP, 2003 (FEMA 450) (see Table 13 of Cameron and Green 2007)	Tabulated for seismically isolated buildings and structures with damping devices.	For seismic isolation: (Table 13.3-1) 1. Damping ratio, β For damping devices: (Table 15.6-1) 1. Damping ratio, β 2. Period, T	
	Caltrans, 2001 (Reviewed in Bommer and Mendis, 2005)	$\frac{1.5}{0.4\beta + 1} + 0.5$	1. Damping ratio, β	For $\beta = 5 - 10\%$ on bridges. Based on Kawashima and Aizawa, 1986 for absolute acceleration.
	U.S. codes that are based on Newmark and Hall, 1982, and are reviewed in Naeim and Kircher, 2001.	<ul style="list-style-type: none"> • SEAOC Blue Book, 1990: based on Newmark and Hall, 1982 • 1991 UBC (ICBO 1991): based on 1990 Blue Book Tabulated for base-isolated buildings (velocity domain) • ATC 1996, NEHRP/FEMA 1997: Extension to both velocity and acceleration domains: Tabulated B_L and B_1 for long T, and B_S for short T. (B_L is for base-isolated buildings. B_S is for buildings with damper systems and for nonlinear pushover analysis using capacity-spectrum method). ATC-40, 1996 FEMA 273, 1997 • 1997 UBC (see ICBO 1997), 2000 IBC (see ICC 1999): Based on NEHRP1997 /FEMA 1998 (See Table 2 of Naeim and Kircher, 2001, or Table II of Lin et al., 2005) 		

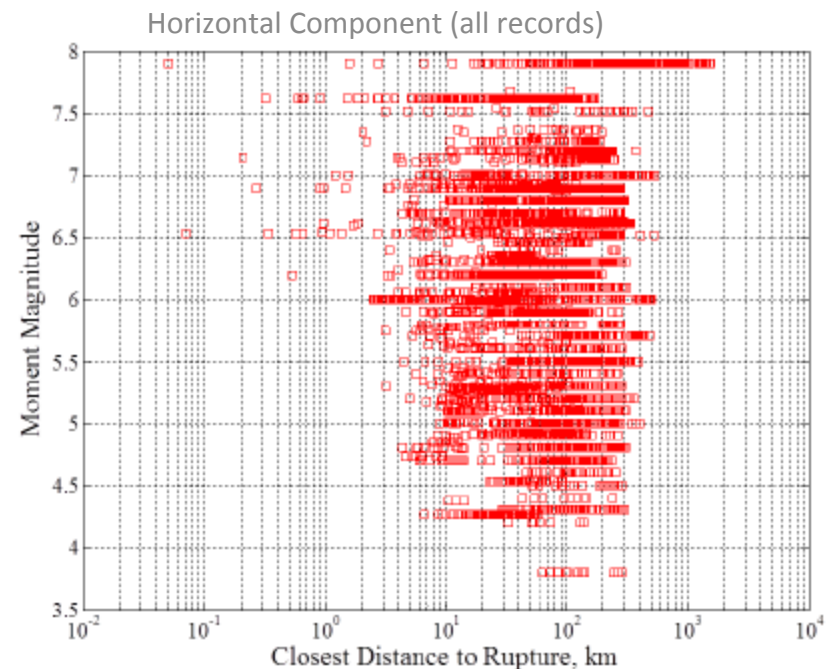
Database : Shallow crustal earthquakes in active tectonic regions

4/21

- Elastic Response Spectrum calculated for horizontal (RotD50, GMRotI50) and vertical components: over 8,000 records each

$$DSF = PSA(\beta) / PSA(5\%)$$

- 11 damping ratios: 0.5, 1, 2, 3, 5, 7, 10, 15, 20, 25, and 30%
- 21 NGA periods: 0.01 – 10 s



Database : Shallow crustal earthquakes in active tectonic regions

4/21

- Elastic Response Spectrum calculated for horizontal (RotD50, GMRotI50) and vertical components: over 8,000 records each

$$DSF = PSA(\beta) / PSA(5\%)$$

- 11 damping ratios: 0.5, 1, 2, 3, 5, 7, 10, 15, 20, 25, and 30%
- 21 NGA periods: 0.01 – 10 s

Selected records used in modeling:

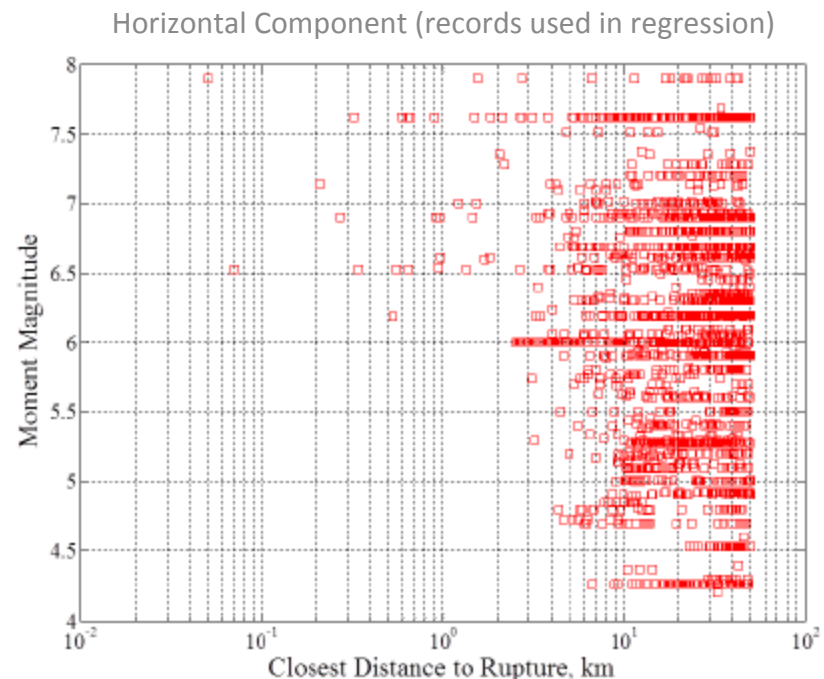
- $0 \leq R_{rup} < 50$ km
- $4.2 \leq M \leq 7.9$
- $116 \leq V_{S30} \leq 2016$ m/s

Horizontal:

- 2,250 records
- $0.25 \leq D_{5-75} \leq 59.32$ s

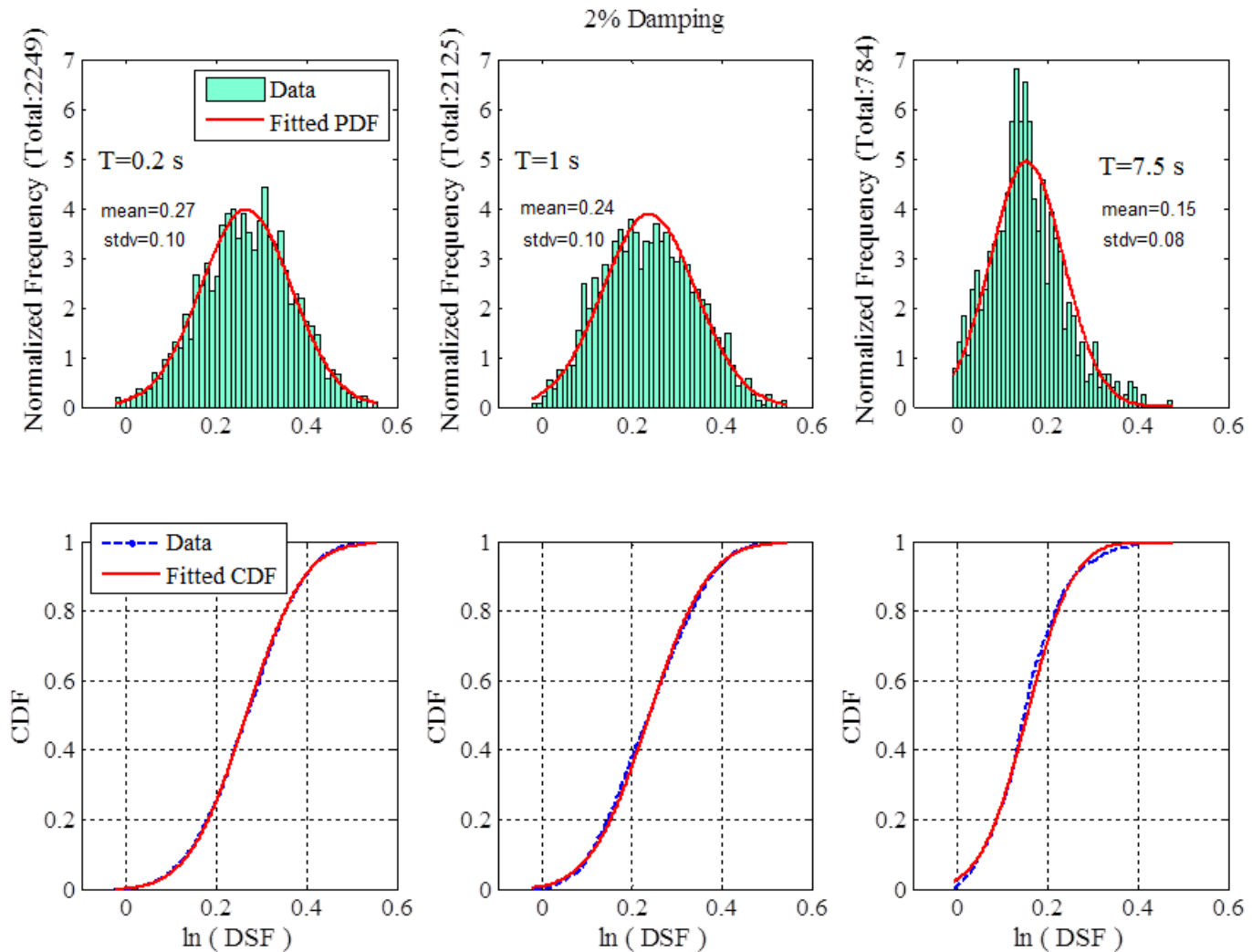
Vertical:

- 2,229 records
- $0.48 \leq D_{5-75} \leq 89.29$ s



Distribution of *DSF*

- Lognormal at a given period and damping (with few exceptions)



Predictor Variables (in literature)

- Existing models have looked at dependence of *DSF* on

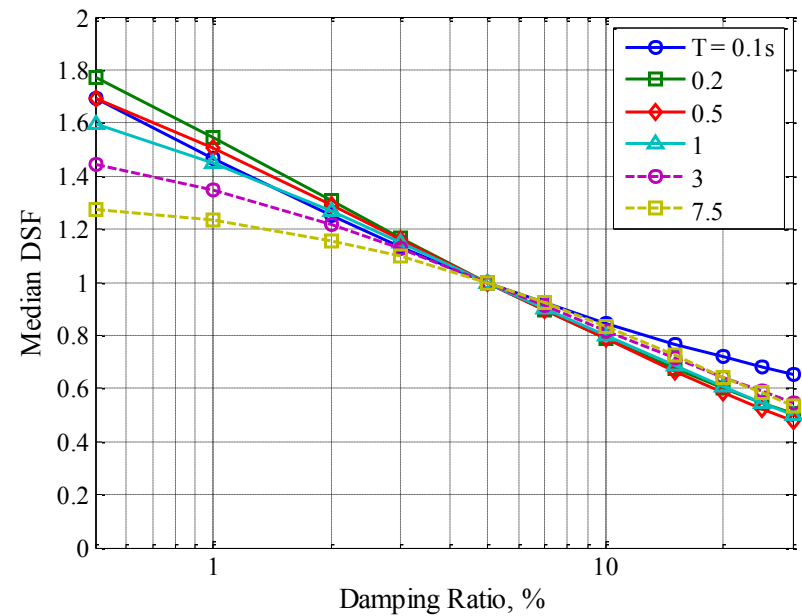
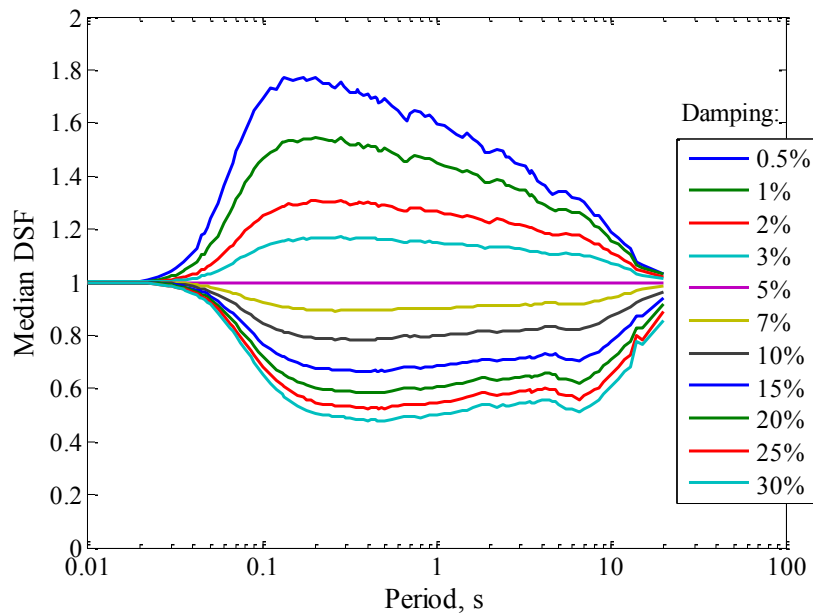
β : damping
T: period (frequency) } → Common factors in modeling

D: duration
M: magnitude
R: distance
Site class
Tectonic setting (WUS, CEUS) } → Very few models

Predictor Variables (in our database)

7/21

- Influence of damping and period

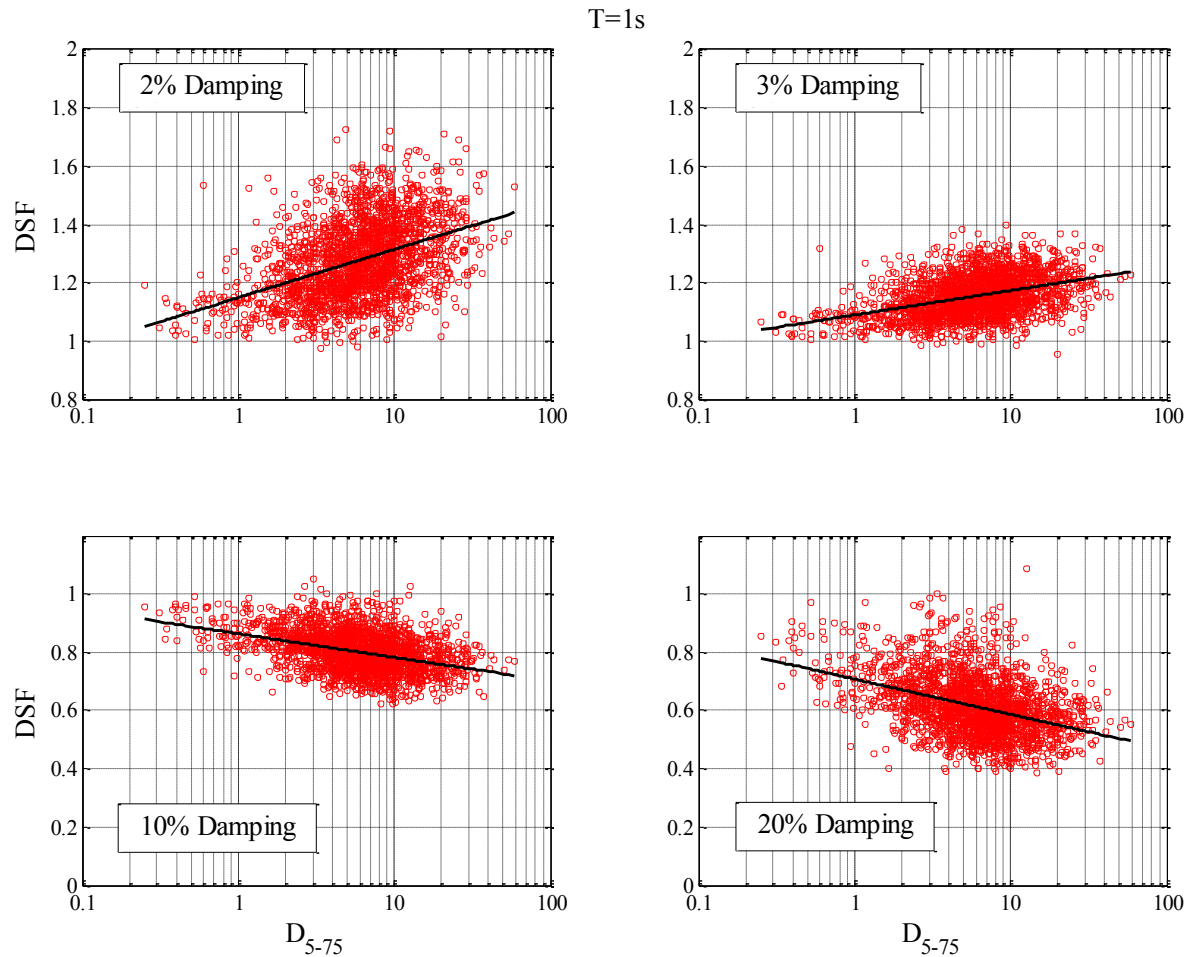


Figures are for data w/ R<50km

Predictor Variables (in our database)

8/21

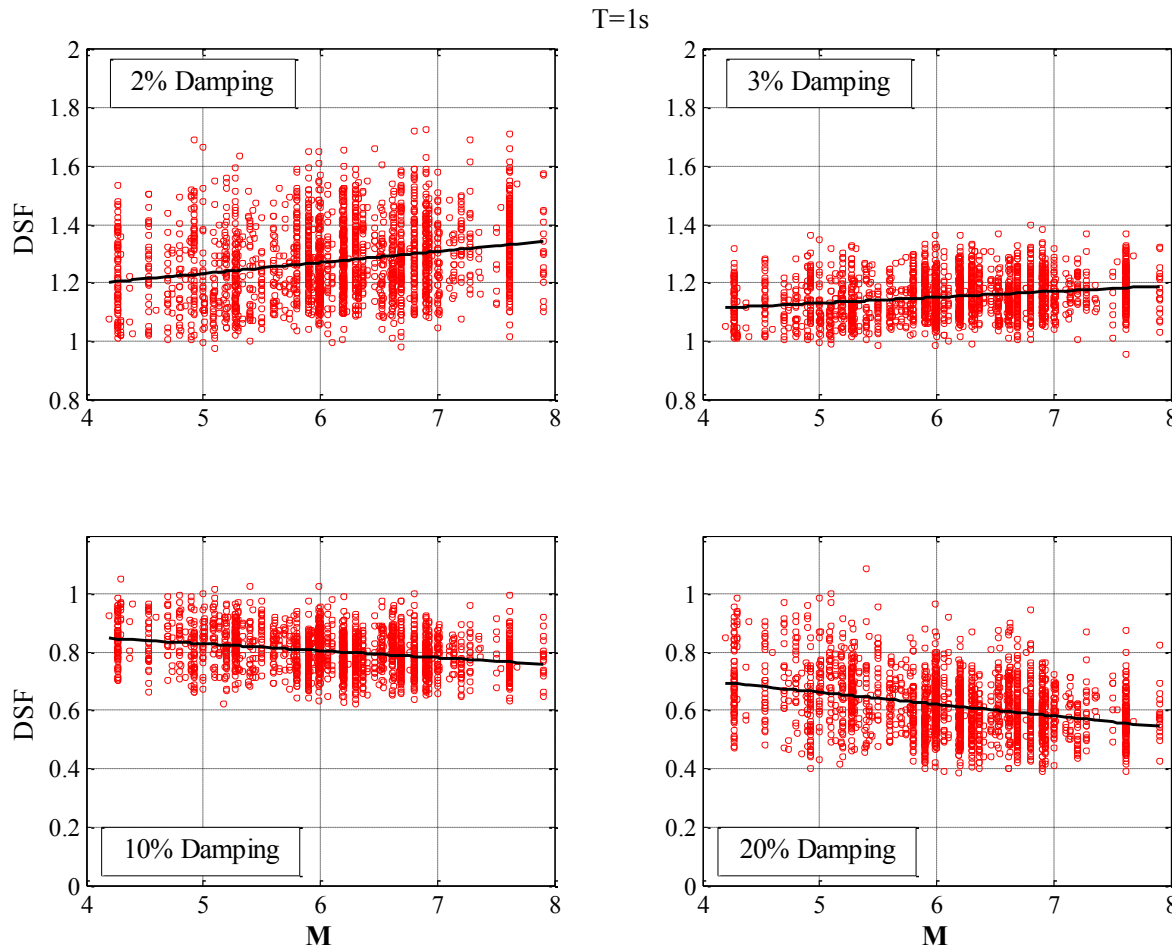
- Influence of duration, magnitude, and distance



Figures are for data w/ R<50km

Predictor Variables (in our database)

- Influence of duration, magnitude, and distance

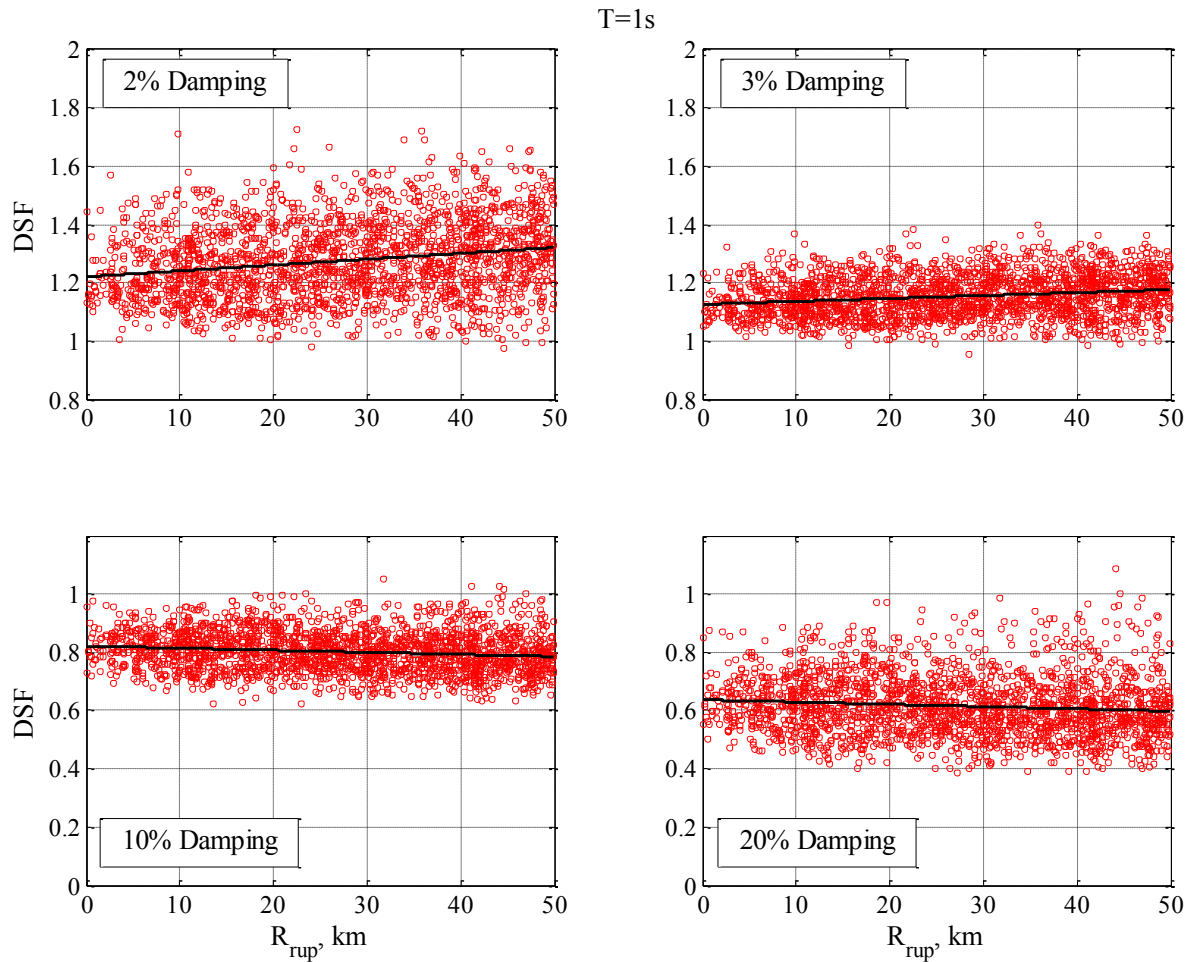


Figures are for data w/ R<50km

Predictor Variables (in our database)

10/21

- Influence of duration, magnitude, and distance



Figures are for data w/ $R < 50$ km

Model Development

Extra

Step 0:

$$\ln(\text{DSF}) = c_0 + \varepsilon ; \varepsilon \sim N(0, \sigma)$$

Step 1:

$$\ln(\text{DSF}) = c_0 + c_1 M + \varepsilon$$

Step 2:

$$\ln(\text{DSF}) = c_0 + c_1 M + c_2 M^2 + \varepsilon$$

Step 3:

$$\ln(\text{DSF}) = c_0 + c_1 M + c_2 \ln(R_{rup} + 1) + \varepsilon$$

Express c_0 , c_1 , and c_2 in terms of β

$$\ln(\text{DSF}) = c_0 + c_1 M + c_2 \ln\{(R_{rup}^2 + c_3^2)^{1/2}\} + \varepsilon$$

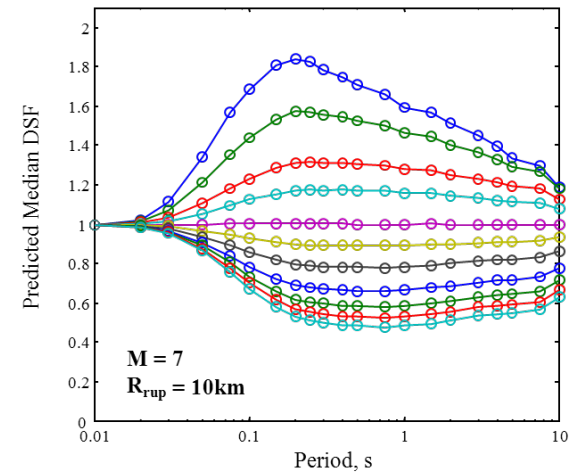
Step 4:

$$\ln(\text{DSF}) = c_0 + c_1 \ln(D_{5-75}) + c_2 \ln(D_{5-75})^2 + \varepsilon$$

Proposed Model

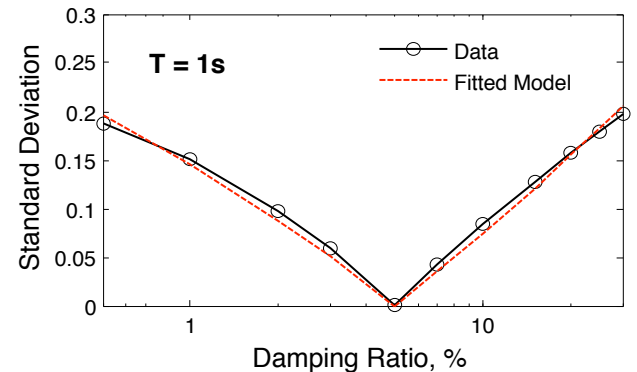
- Median *DSF*

$$\ln(DSF) = b_0 + b_1 \ln(\beta) + b_2 (\ln(\beta))^2 + [b_3 + b_4 \ln(\beta) + b_5 (\ln(\beta))^2] M + [b_6 + b_7 \ln(\beta) + b_8 (\ln(\beta))^2] \ln(R_{rup} + 1) + \epsilon$$



- Variability of *DSF*

$$\sigma_{\ln(DSF)} = |a_0 \ln(\beta/5) + a_1 \ln(\beta/5)|$$



Proposed Model

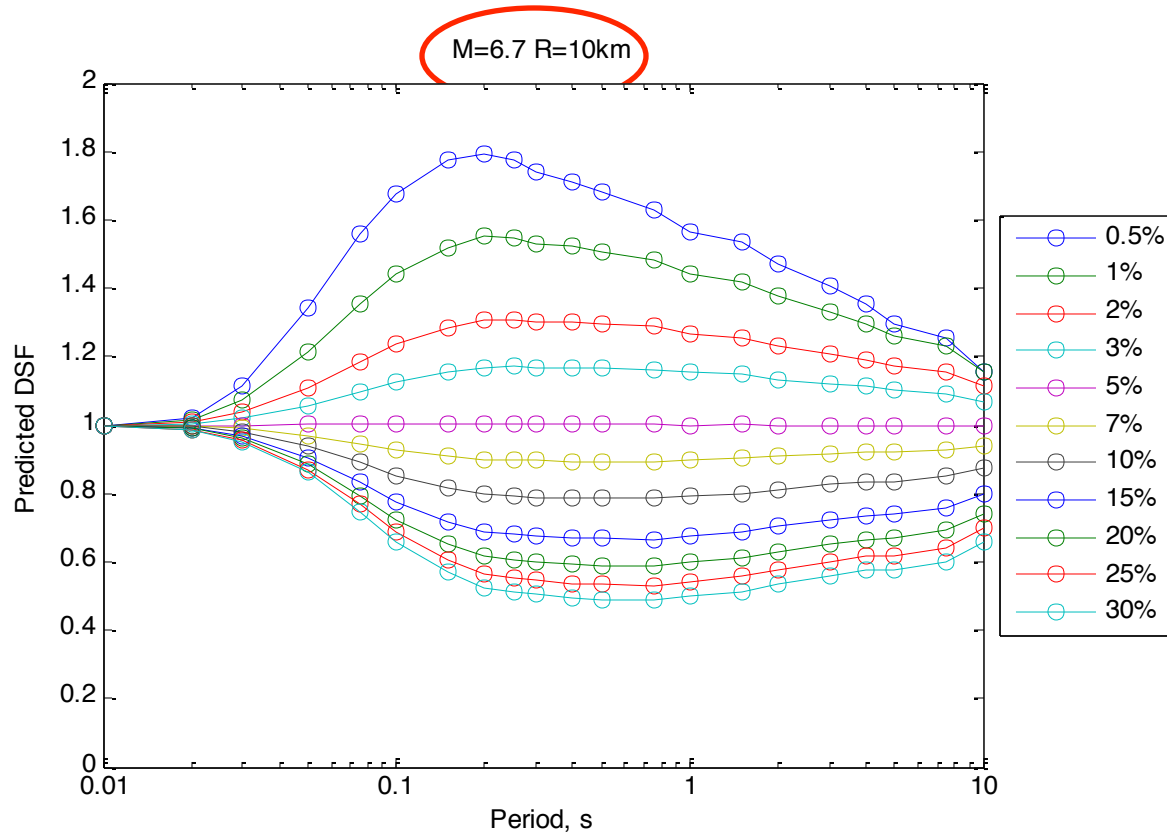
Taken from Rezaeian et al. 2012 (PEER report):

Table 1 Regression coefficients for the horizontal component RotD50.

<i>T, s</i>	b0	b1	b2	b3	b4	b5	b6	b7	b8	a0	a1	<i>SE(σ)*</i>
0.01	1.73E-03	-2.07E-04	-6.29E-04	1.08E-06	-8.24E-05	7.36E-05	-1.07E-03	9.08E-04	-2.02E-04	-3.70E-03	2.30E-04	1.88E-04
0.02	5.53E-02	-3.77E-02	2.15E-03	-4.30E-03	3.21E-03	-3.32E-04	-4.75E-03	2.52E-03	2.29E-04	-2.19E-02	2.11E-03	4.99E-04
0.03	1.22E-01	-7.02E-02	-2.28E-03	-3.21E-03	6.91E-05	9.82E-04	-1.30E-02	7.82E-03	2.27E-04	-5.21E-02	4.60E-03	1.04E-03
0.05	2.39E-01	-1.06E-01	-2.63E-02	-8.57E-04	-7.43E-03	4.87E-03	-1.69E-02	8.08E-03	1.71E-03	-9.57E-02	1.31E-03	4.70E-03
0.075	3.05E-01	-7.32E-02	-7.29E-02	2.02E-04	-1.64E-02	1.03E-02	-9.26E-04	-6.40E-03	4.42E-03	-1.21E-01	-5.79E-03	4.60E-03
0.1	2.69E-01	4.18E-03	-1.07E-01	5.80E-03	-2.49E-02	1.34E-02	2.35E-02	-2.37E-02	5.84E-03	-1.24E-01	-1.08E-02	3.80E-03
0.15	1.41E-01	1.00E-01	-1.18E-01	3.01E-02	-4.09E-02	1.41E-02	3.16E-02	-2.47E-02	3.15E-03	-1.15E-01	-1.14E-02	3.97E-03
0.2	5.01E-02	1.45E-01	-1.11E-01	4.69E-02	-4.77E-02	1.18E-02	3.10E-02	-2.29E-02	2.41E-03	-1.08E-01	-8.85E-03	4.64E-03
0.25	2.28E-02	1.43E-01	-9.73E-02	5.20E-02	-4.70E-02	9.47E-03	2.71E-02	-2.02E-02	1.31E-03	-1.04E-01	-7.35E-03	4.66E-03
0.3	-1.58E-02	1.48E-01	-8.83E-02	5.21E-02	-4.36E-02	7.33E-03	3.87E-02	-2.66E-02	1.76E-03	-1.01E-01	-6.90E-03	5.31E-03
0.4	2.24E-02	1.03E-01	-7.41E-02	4.63E-02	-3.58E-02	4.65E-03	3.63E-02	-2.45E-02	1.18E-03	-1.02E-01	-6.71E-03	6.21E-03
0.5	3.19E-02	7.04E-02	-5.57E-02	4.25E-02	-2.94E-02	1.88E-03	3.87E-02	-2.47E-02	3.13E-04	-1.01E-01	-6.22E-03	7.13E-03
0.75	1.04E-02	5.33E-02	-3.72E-02	4.47E-02	-2.40E-02	-2.40E-03	3.47E-02	-2.59E-02	2.90E-03	-1.01E-01	-5.86E-03	6.85E-03
1	-8.84E-02	8.92E-02	-2.14E-02	4.98E-02	-2.36E-02	-4.70E-03	5.02E-02	-3.43E-02	2.32E-03	-1.02E-01	-7.31E-03	6.66E-03
1.5	-1.57E-01	9.33E-02	3.28E-03	5.85E-02	-2.36E-02	-8.02E-03	4.81E-02	-3.30E-02	2.10E-03	-1.02E-01	-8.75E-03	6.66E-03
2	-2.96E-01	1.50E-01	2.09E-02	7.30E-02	-2.96E-02	-9.95E-03	5.24E-02	-3.32E-02	6.86E-04	-1.03E-01	-9.22E-03	6.04E-03
3	-4.07E-01	1.97E-01	3.28E-02	8.35E-02	-3.54E-02	-1.01E-02	5.57E-02	-2.91E-02	-3.17E-03	-9.63E-02	-1.07E-02	6.03E-03
4	-4.49E-01	2.07E-01	4.42E-02	8.75E-02	-3.59E-02	-1.14E-02	5.07E-02	-2.43E-02	-4.67E-03	-9.83E-02	-1.37E-02	3.37E-03
5	-4.98E-01	2.17E-01	5.36E-02	9.03E-02	-3.48E-02	-1.29E-02	5.19E-02	-2.30E-02	-5.68E-03	-9.42E-02	-1.53E-02	2.99E-03
7.5	-5.25E-01	2.06E-01	7.79E-02	9.88E-02	-3.76E-02	-1.51E-02	2.91E-02	-4.93E-03	-9.02E-03	-8.95E-02	-1.63E-02	2.59E-03
10	-3.89E-01	1.43E-01	6.12E-02	7.14E-02	-2.36E-02	-1.30E-02	2.33E-02	-5.46E-03	-5.92E-03	-6.89E-02	-1.43E-02	1.94E-03

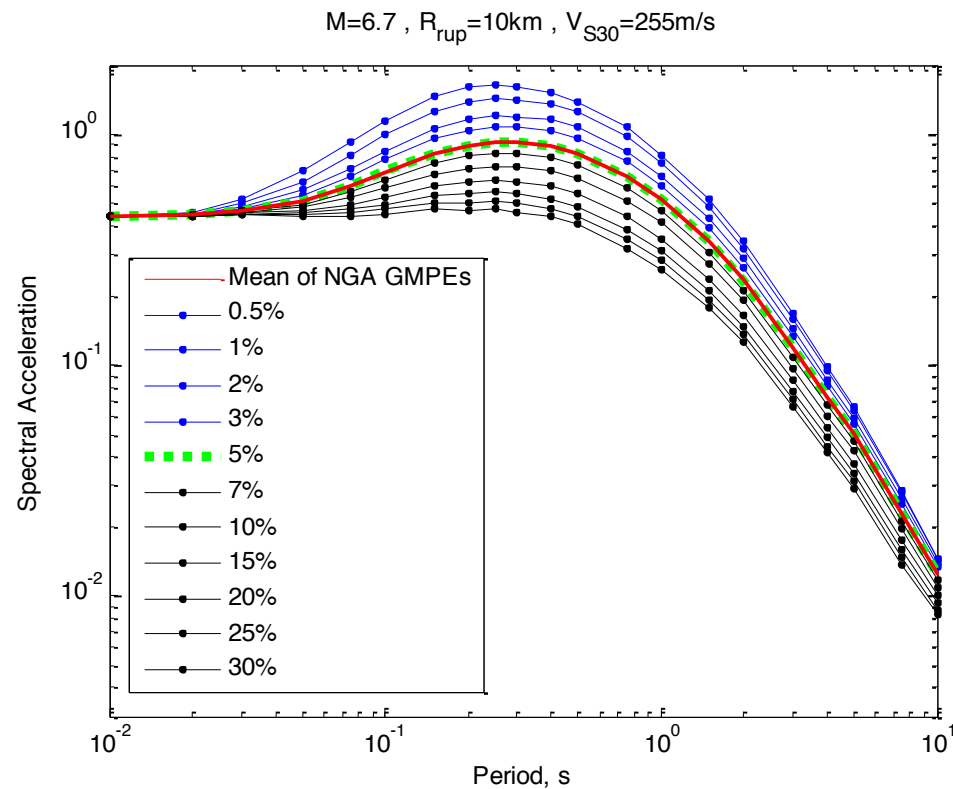
* Standard error in modeling σ according to Equation (12).

Proposed Model: Example Application

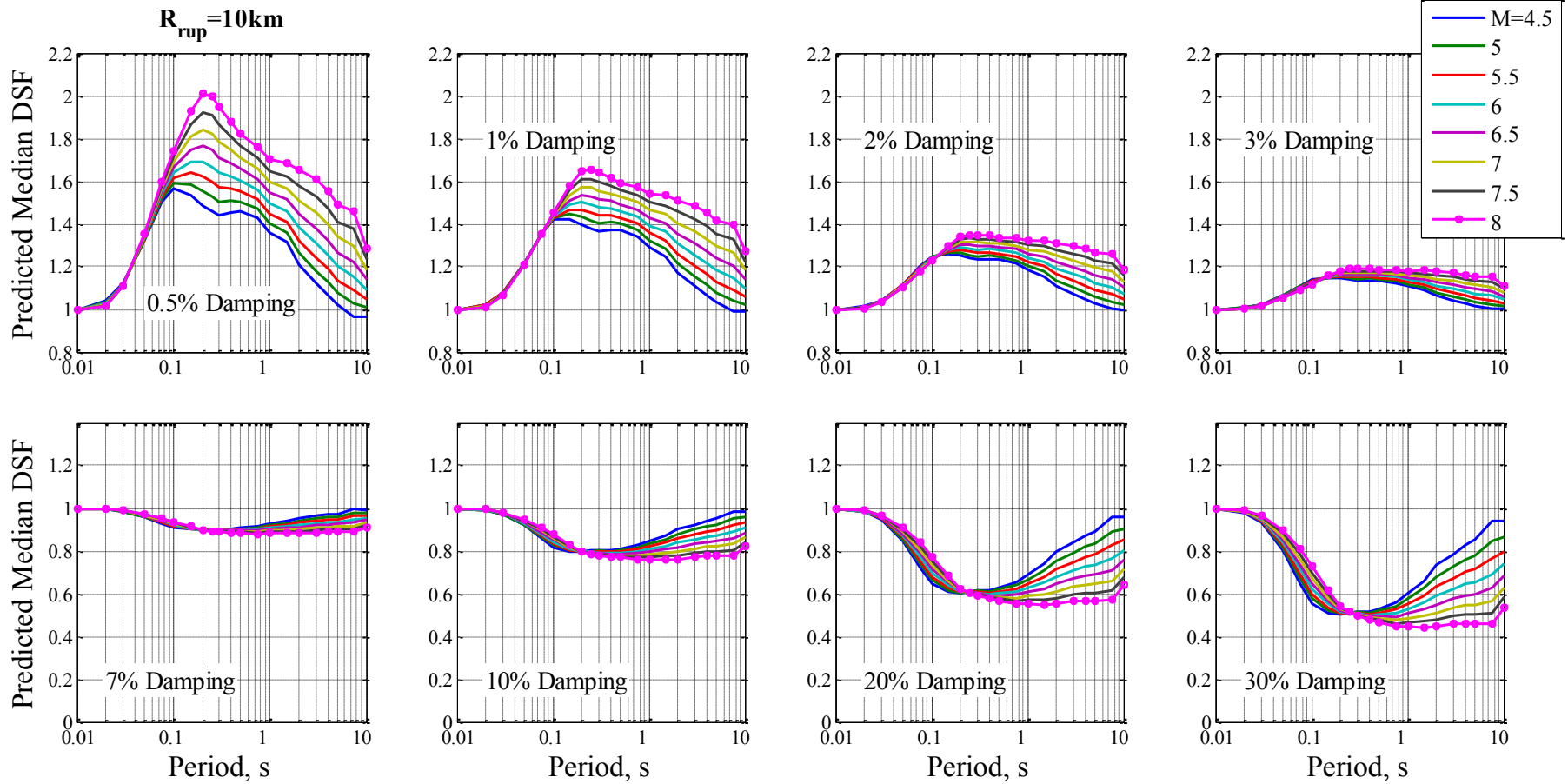


Proposed Model: Example Application

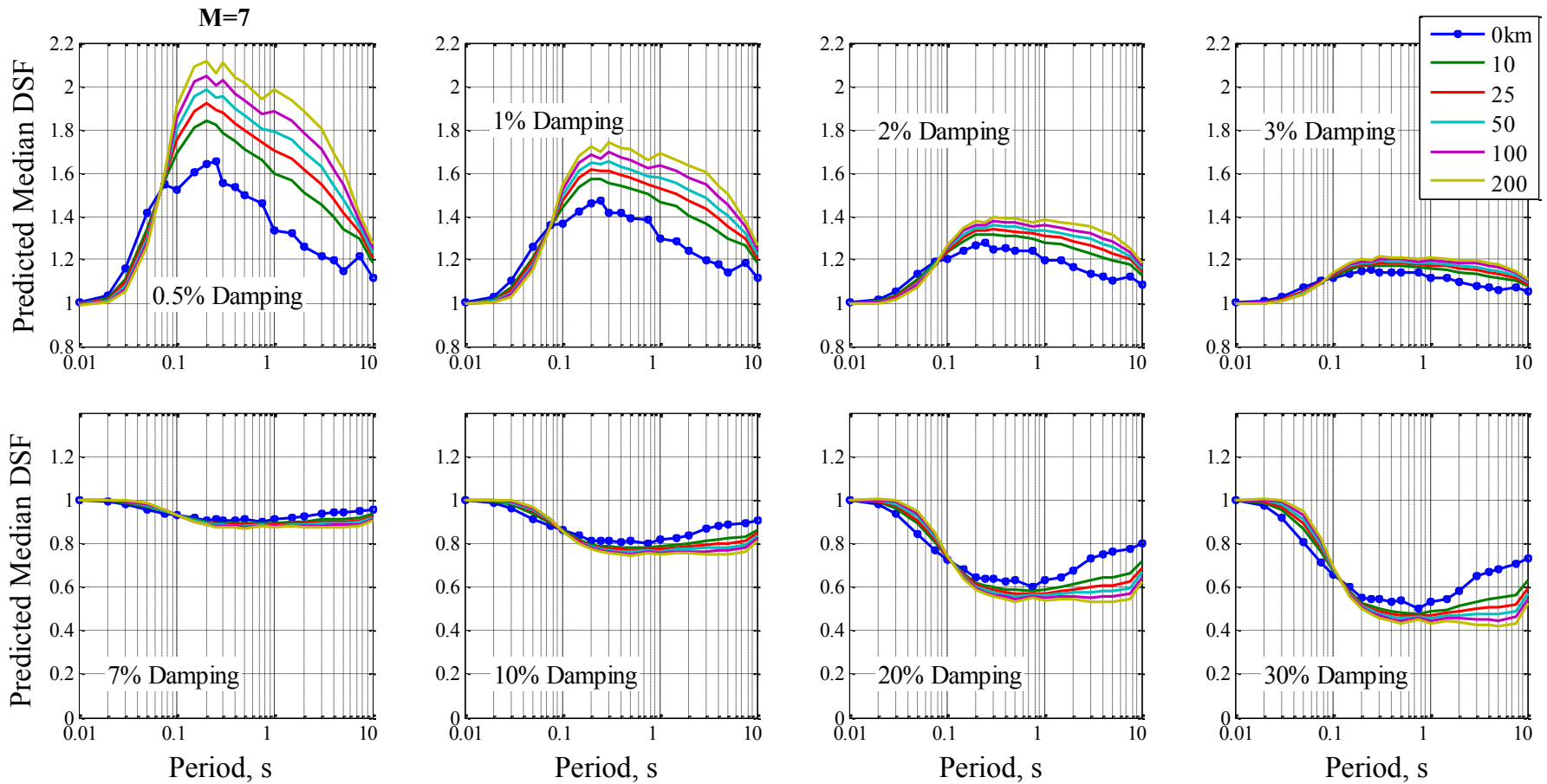
The geometric mean of the five NGA-West1 GMPEs (red) is scaled to adjust for various damping ratios from 0.5 to 30%. The model for RotD50 component is used. Assumptions to estimate the NGA-GMPEs: reverse fault, dip = 45°, hanging wall, fault rupture width = 15km, Rjb=0km, Rx=7km.



Variation with Magnitude



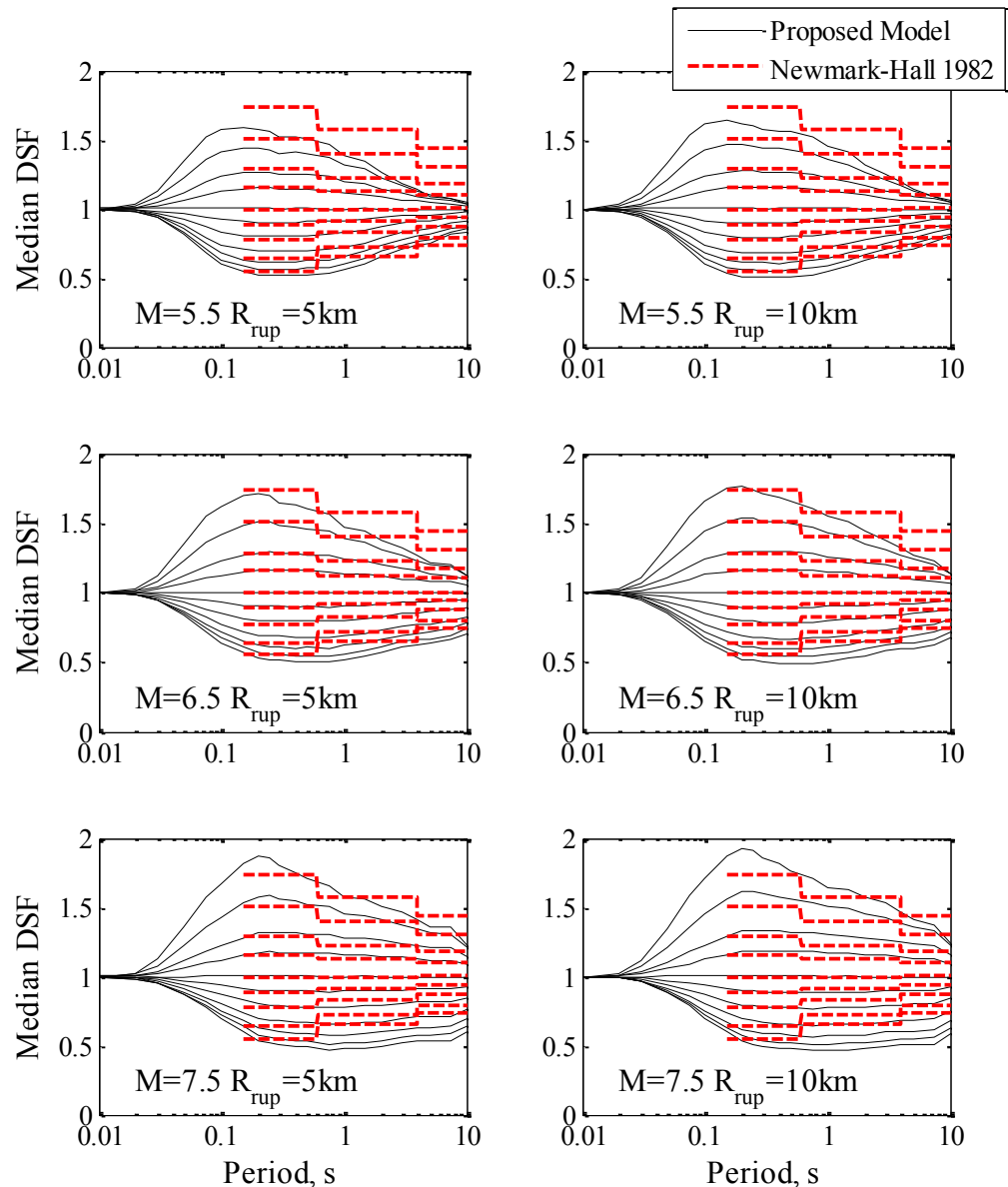
Variation with Distance



Comparison with Existing Models

The proposed model is plotted for all 11 damping ratios from 0.5 to 30%.
(0.5,1,2,3,5,7,10,15,20,25,30)

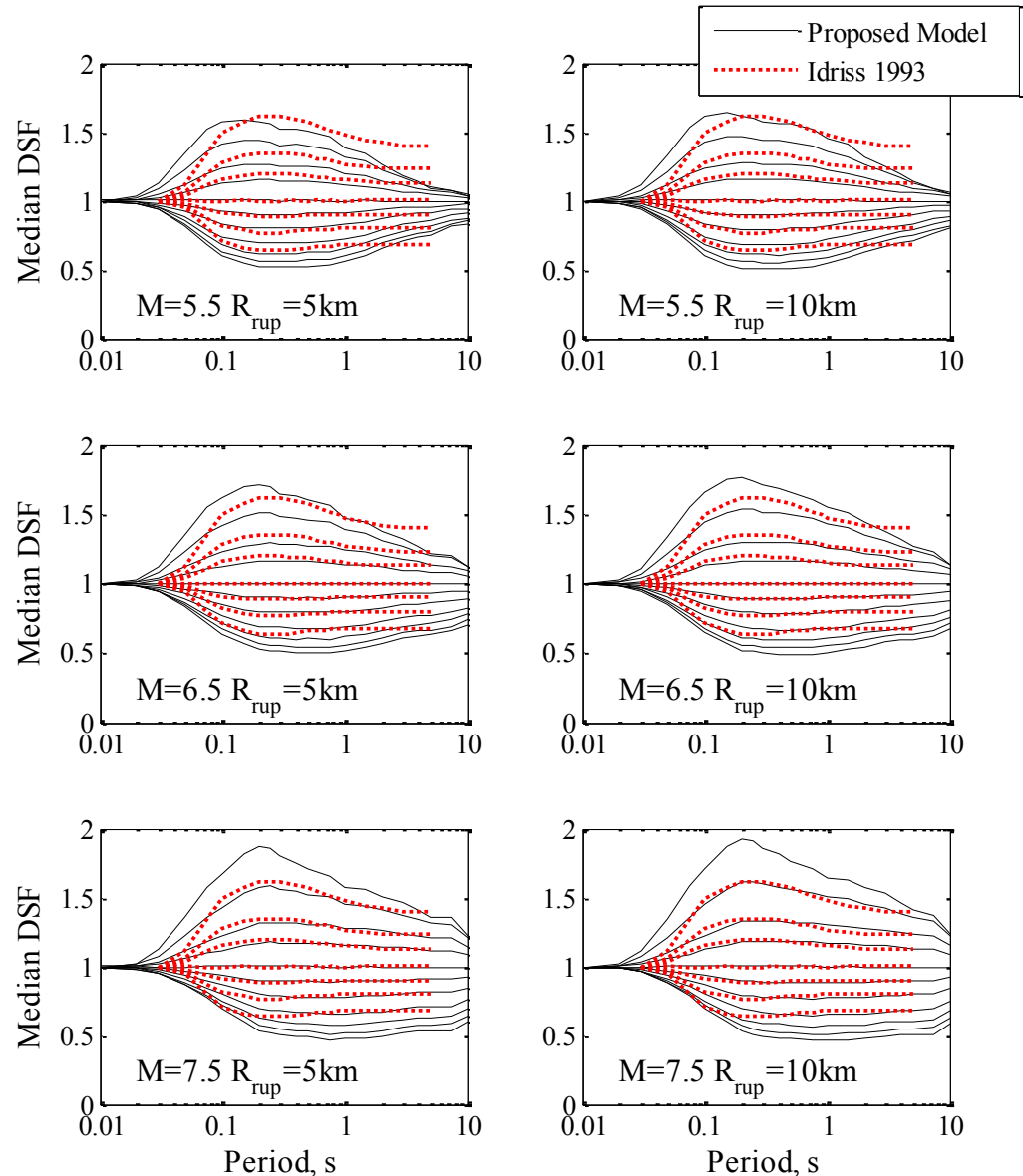
Newmark and Hall (1982) is plotted for $\beta=0.5,1,2,3,5,7,10,15,20\%$. It is applicable to $T=0.125-10s$, and is not a function of M or R_{rup} .



Comparison with Existing Models

The proposed model is plotted for all 11 damping ratios from 0.5 to 30%. (0.5,1,2,3,5,7,10,15,20,25,30)

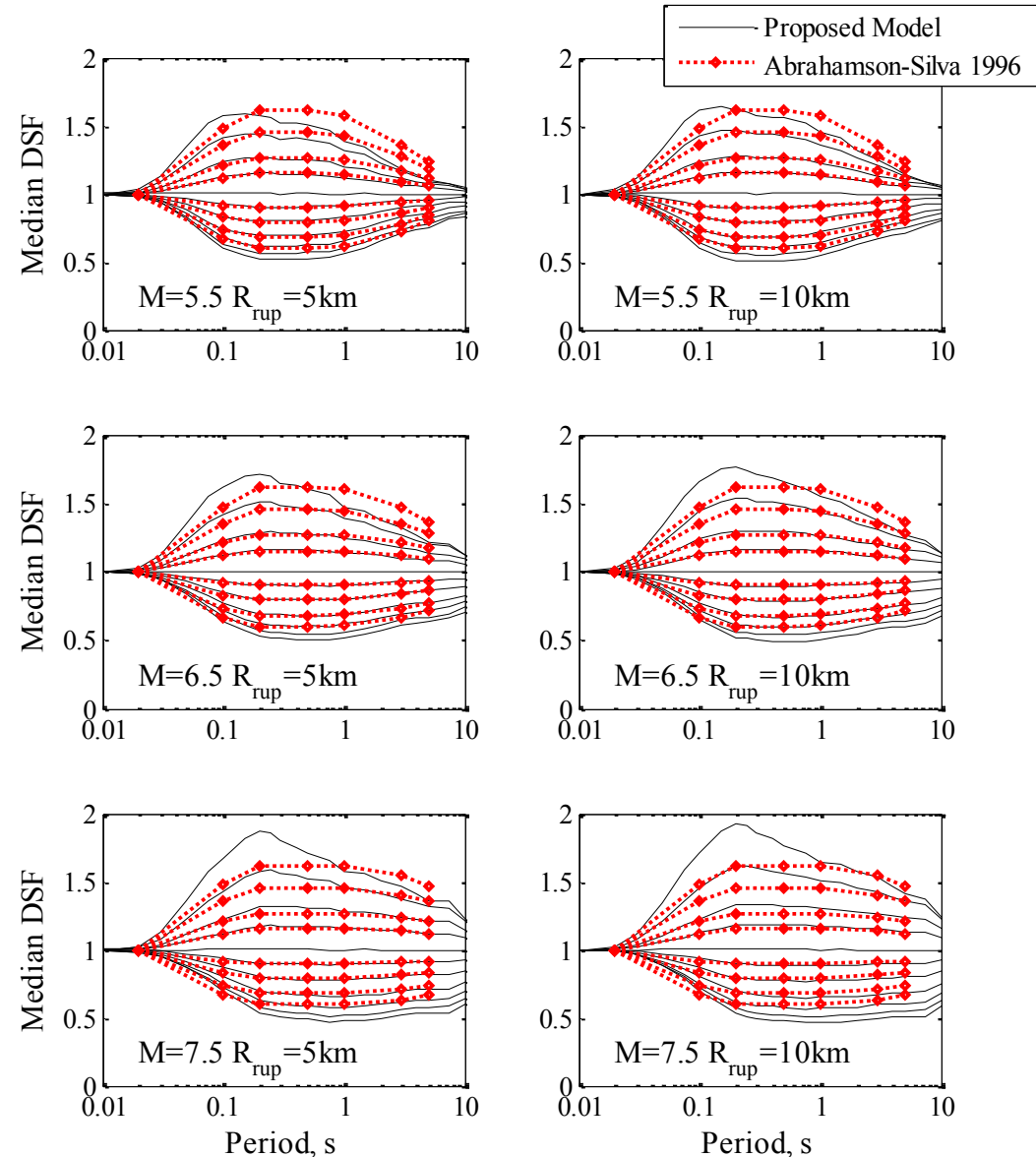
Idriss (1993) is plotted for $\beta=1,2,3,5,7,10,15\%$. It is applicable to $T=0.03-5s$, and is not a function of M or R_{rup} .



Comparison with Existing Models

The proposed model is plotted for all 11 damping ratios from 0.5 to 30%.
(0.5,1,2,3,5,7,10,15,20,25,30)

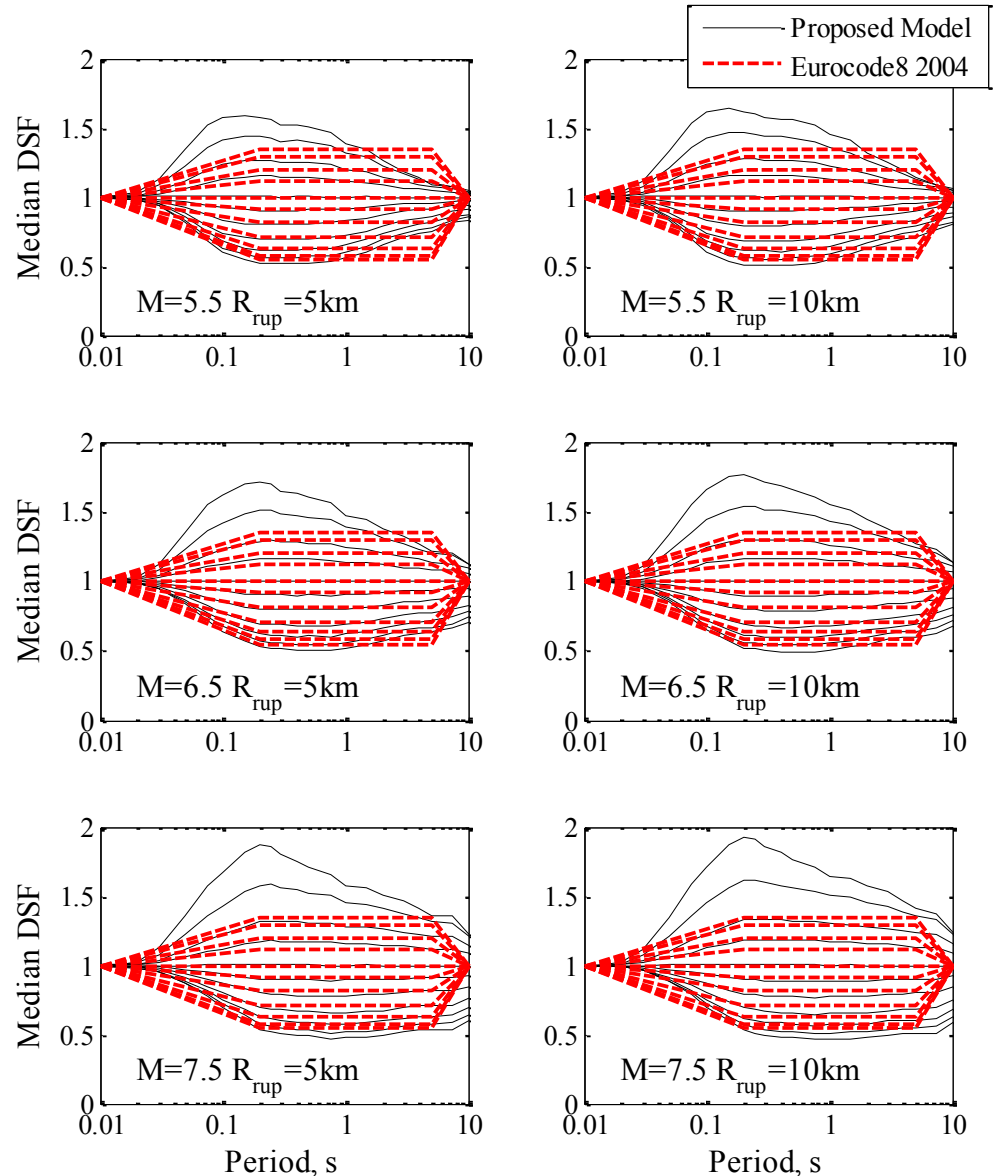
Abrahamson and Silva (1996) is plotted for $\beta=0.5,1,2,3,7,10,15,20\%$. It is applicable to $T=0.02-5s$, and is a function of M , but not R_{rup} .



Comparison with Existing Models

The proposed model is plotted for all 11 damping ratios from 0.5 to 30%.
(0.5,1,2,3,5,7,10,15,20,25,30)

The model by Eurocode 8 (2004) is plotted for all 11 damping ratios. It is not a function of M or R_{rup} .



Vertical Component

- Functional forms are similar to those of the horizontal component

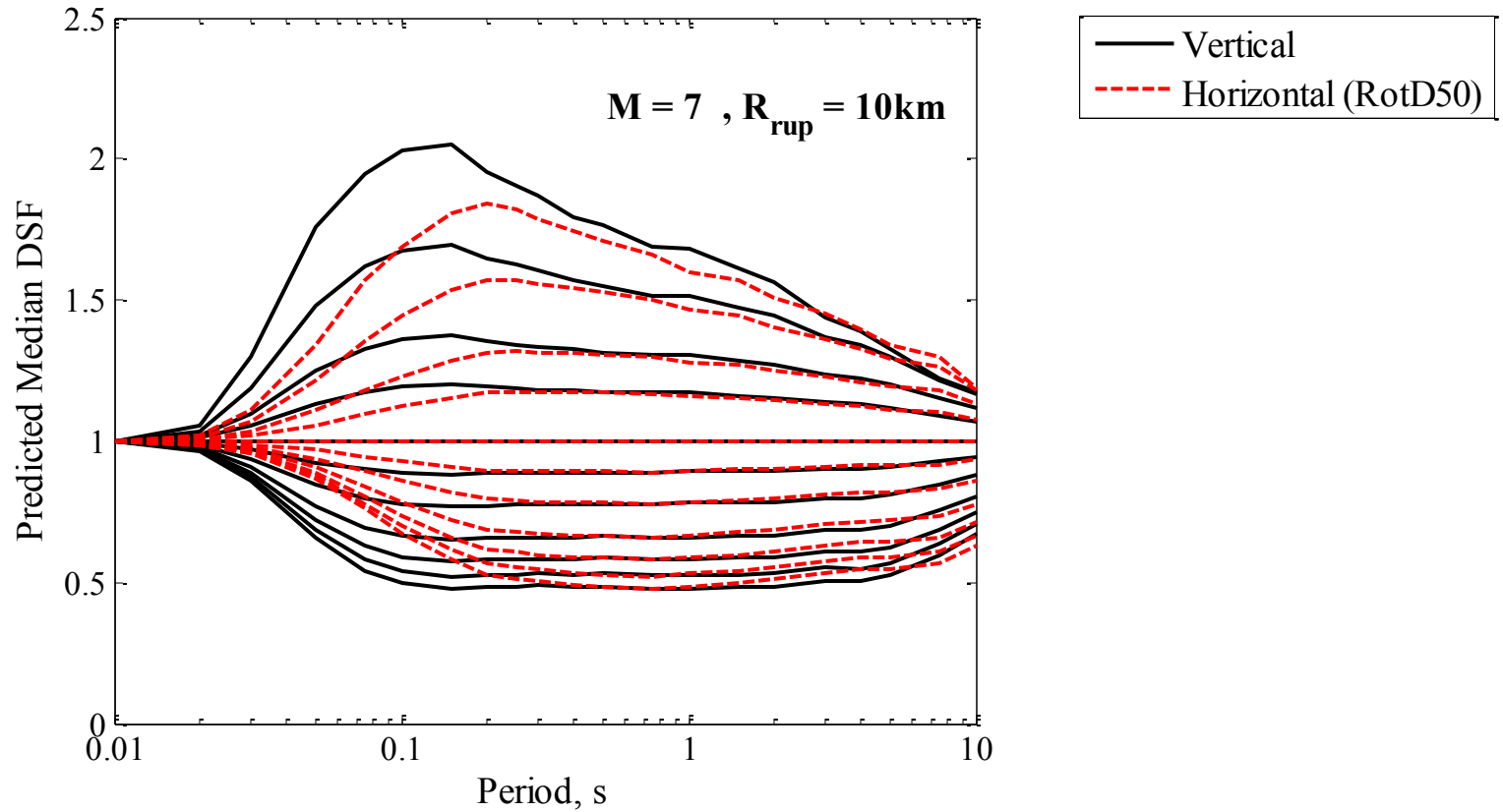
Taken from the PEER

Table 3 Regression coefficients for the vertical component.

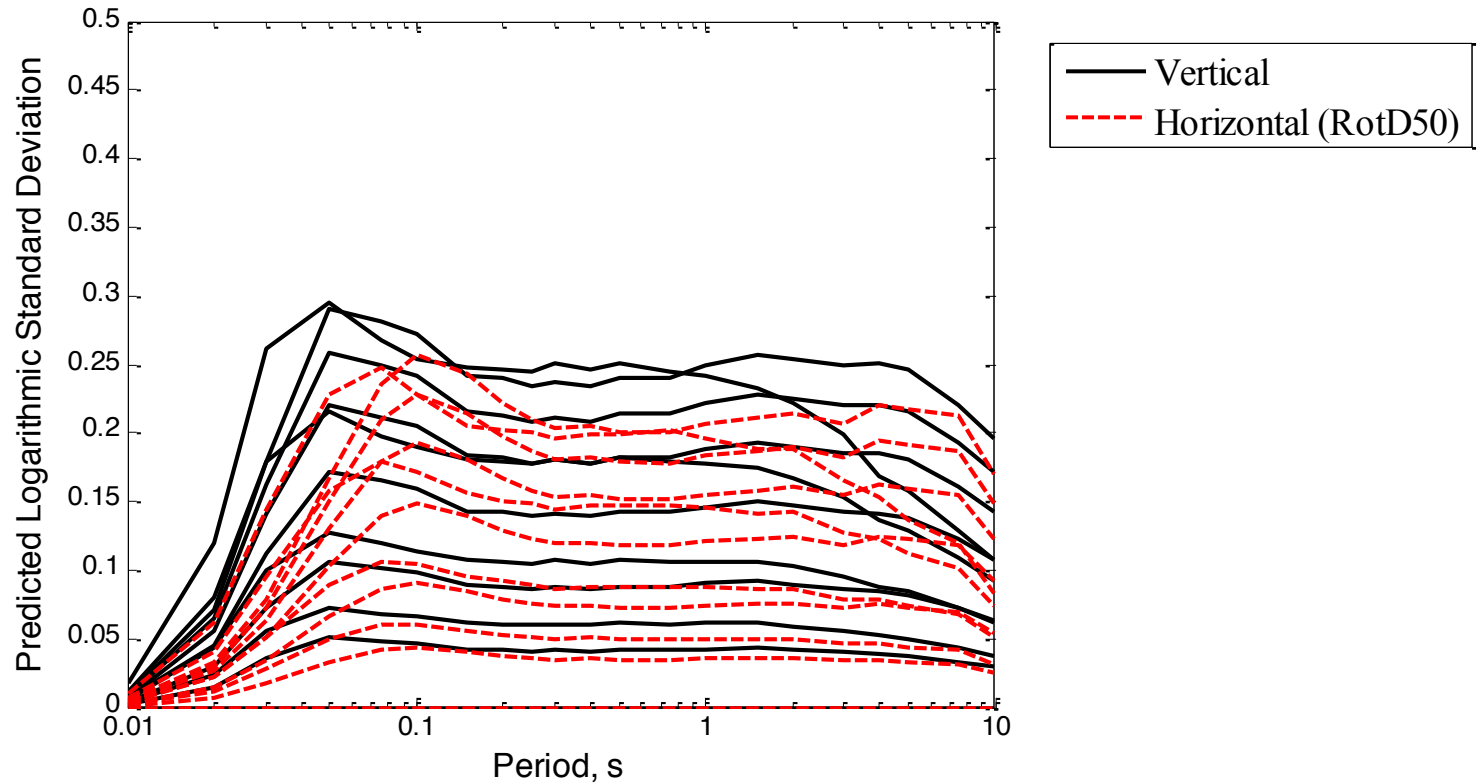
T, s	b0	b1	b2	b3	b4	b5	b6	b7	b8	a0	a1	$SE(\sigma)^*$
0.01	5.82E-03	-3.31E-03	-3.64E-04	-3.81E-04	2.15E-04	2.92E-05	-1.82E-03	1.54E-03	-2.48E-04	-6.15E-03	5.21E-04	4.17E-04
0.02	1.36E-01	-8.77E-02	1.65E-03	-1.02E-02	6.91E-03	-2.83E-04	-1.23E-02	6.98E-03	3.60E-04	-4.50E-02	3.16E-03	5.64E-04
0.03	3.49E-01	-1.94E-01	-1.19E-02	-1.61E-02	6.48E-03	1.95E-03	-2.59E-02	1.22E-02	2.19E-03	-1.06E-01	3.16E-03	4.25E-03
0.05	4.34E-01	-1.68E-01	-6.08E-02	-1.15E-03	-1.01E-02	6.59E-03	-1.37E-02	-3.18E-03	6.97E-03	-1.47E-01	-8.28E-03	8.02E-03
0.075	3.48E-01	-6.40E-02	-9.47E-02	1.69E-02	-2.37E-02	8.31E-03	6.22E-03	-1.97E-02	9.83E-03	-1.39E-01	-9.96E-03	6.85E-03
0.1	3.06E-01	-3.80E-02	-9.44E-02	2.63E-02	-2.96E-02	8.20E-03	1.14E-02	-1.80E-02	6.93E-03	-1.34E-01	-1.02E-02	8.38E-03
0.15	1.87E-01	6.67E-02	-1.16E-01	4.32E-02	-4.50E-02	1.15E-02	1.66E-02	-1.73E-02	4.82E-03	-1.23E-01	-6.66E-03	8.44E-03
0.2	1.86E-01	4.16E-02	-9.66E-02	3.55E-02	-3.56E-02	8.37E-03	2.73E-02	-2.37E-02	4.13E-03	-1.22E-01	-6.52E-03	9.09E-03
0.25	1.21E-01	7.76E-02	-9.75E-02	4.13E-02	-3.96E-02	8.98E-03	3.10E-02	-2.22E-02	1.97E-03	-1.20E-01	-5.99E-03	8.70E-03
0.3	1.41E-01	5.39E-02	-8.91E-02	3.79E-02	-3.61E-02	7.91E-03	2.76E-02	-1.85E-02	1.02E-03	-1.22E-01	-5.78E-03	9.76E-03
0.4	1.72E-01	1.29E-02	-7.08E-02	2.97E-02	-2.58E-02	4.42E-03	2.93E-02	-2.13E-02	1.05E-03	-1.20E-01	-5.74E-03	8.83E-03
0.5	2.21E-01	-3.86E-02	-6.00E-02	2.18E-02	-1.90E-02	3.21E-03	2.72E-02	-1.64E-02	-2.29E-04	-1.23E-01	-6.08E-03	1.03E-02
0.75	1.68E-01	-2.35E-02	-5.40E-02	2.49E-02	-1.57E-02	6.34E-04	3.10E-02	-2.21E-02	2.01E-03	-1.22E-01	-6.75E-03	9.14E-03
1	8.65E-02	2.28E-02	-5.28E-02	3.47E-02	-2.11E-02	4.55E-04	3.53E-02	-2.43E-02	1.75E-03	-1.24E-01	-8.33E-03	9.33E-03
1.5	-3.62E-02	7.02E-02	-3.20E-02	4.82E-02	-2.57E-02	-2.44E-03	3.63E-02	-2.24E-02	2.93E-04	-1.25E-01	-1.04E-02	8.14E-03
2	-8.29E-02	9.13E-02	-2.57E-02	5.37E-02	-2.64E-02	-4.34E-03	3.16E-02	-2.30E-02	2.38E-03	-1.22E-01	-1.11E-02	8.20E-03
3	-2.26E-01	1.21E-01	1.05E-02	6.50E-02	-2.59E-02	-8.86E-03	3.45E-02	-2.00E-02	-9.44E-04	-1.16E-01	-1.29E-02	6.07E-03
4	-4.08E-01	2.02E-01	3.12E-02	8.61E-02	-3.44E-02	-1.19E-02	4.15E-02	-2.23E-02	-2.25E-03	-1.11E-01	-1.63E-02	4.96E-03
5	-2.54E-01	1.11E-01	2.96E-02	6.37E-02	-2.13E-02	-1.15E-02	2.86E-02	-1.34E-02	-2.90E-03	-1.07E-01	-1.68E-02	3.89E-03
7.5	-4.41E-01	1.73E-01	6.26E-02	7.73E-02	-2.58E-02	-1.39E-02	3.84E-02	-1.44E-02	-5.92E-03	-9.36E-02	-1.63E-02	2.20E-03
10	-3.95E-01	1.23E-01	7.79E-02	7.10E-02	-2.12E-02	-1.43E-02	2.13E-02	-4.42E-03	-6.15E-03	-8.17E-02	-1.53E-02	2.16E-03

* Standard error in modeling σ according to Equation (12).

Vertical Component: Median *DSF*



Vertical Component: Standard Deviation



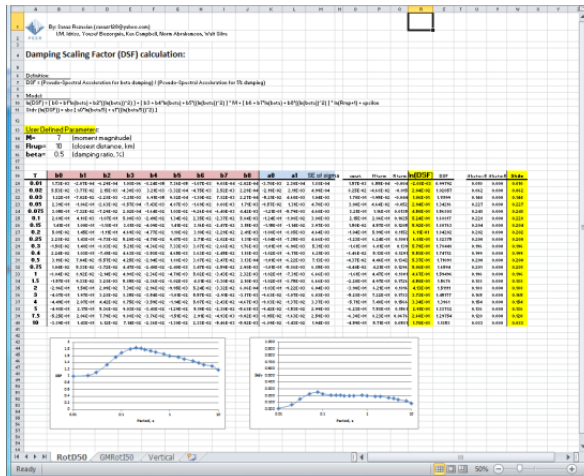
Deliverables / Products

- **PEER Report** : available for free download



http://peer.berkeley.edu/publications/peer_reports/reports_2012/webPEER-2012-01-REZAEIAN.pdf

- **Conference Paper**: 15WCEE Paper Number 0421
- **Excel File and Matlab Codes** : available upon request





Thank You

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