Development of a New Family of Normalized Modulus Reduction and Material Damping Curves

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Dynamic Soil Properties

Shear Modulus, \( G \)

Material Damping Ratio, \( D \)

\[ G_{\text{max}} \]

\[ D_{\text{min}} \]
Examples of Empirical Relationships Based on Laboratory Studies

- Seed et al., 1970
- Hardin and Drnevich, 1972
- Kokusho, 1980
- Seed et al., 1986
- Sun et al., 1988
- Idriss, 1990
- Vucetic and Dobry, 1991
- Ishibashi and Zhang, 1993
Nonlinear Behavior of Sandy and Gravelly Soils

Range for Sand Seed and Idriss (1970)
Range for Gravel Seed et al. (1986)
Effect of Soil Plasticity on Nonlinear Behavior (Vucetic and Dobry, 1991)

Normalized Shear Modulus, $G/G_{\text{max}}$

Shearing Strain, $\gamma$, %

- **Non-Plastic**
- PI = 15 %
- PI = 30 %
- PI = 50 %
- PI = 100 %
- PI = 200 %
Objective

To generate a new family of empirical $G/G_{\text{max}} \sim \log \gamma$ and $D \sim \log \gamma$ curves such that the observed effects of various parameters on $G/G_{\text{max}}$ and $D$ are represented more accurately:

- Shearing Strain Amplitude, $\gamma$
- Soil Type (expressed by PI, $C_u$, $D_{50}$)
- Effective Confinement, $\sigma_o'$
- Number of Cycles, $N$
- Loading Frequency, $f$
- Overconsolidation Ratio, OCR

much like Hardin and Drnevich, 1972
Proposed 5-Parameter Model (Modified Hyperbolic Model)

\[
\frac{G}{G_{\text{max}}} = \frac{1}{1 + \left(\frac{\gamma}{\gamma_r}\right)^a}
\]

\[
D = D_{\text{min}} + D_{\text{Masing}} \times \left(b \times \left(\frac{G}{G_{\text{max}}}\right)^c\right)
\]
Proposed Model: $\frac{G}{G_{\text{max}}} = \frac{1}{1 + \left(\frac{\gamma}{\gamma_r}\right)^a}$

Normalized Shear Modulus, $\frac{G}{G_{\text{max}}}$

Shearing Strain, $\gamma$, %
Effect of Reference Strain, $\gamma_r$, on:

1. $\frac{G}{G_{\text{max}}} - \log \gamma$

2. $\tau - \log \gamma$
   
   (a = 1)
Effect of Coefficient “a” on $G/G_{\text{max}} - \log \gamma$ Curves
Masing (1926) Behavior

Shear Stress, $\tau$, MPa

Shearing Strain, $\gamma$, %
Masing Behavior: \( D - \log \gamma \)

\[
D_{\text{Masing}}(\gamma) = \frac{8 \left( \int \tau \, d\gamma - \frac{\tau \gamma}{2} \right)}{4 \pi \frac{\tau \gamma}{2}}
\]

Material Damping Ratio, \( D, \% \)

Shearing Strain, \( \gamma, \% \)
Proposed Model: $D \sim \log \gamma$

$$D = D_{\text{Masing}} \times \left[ b \times \left( \frac{G}{G_{\text{max}}} \right)^c \right] + D_{\text{min}}$$
$G_{\text{max}}$ Degradation with $\gamma$: “c” Parameter

Single-Amplitude Shearing Strain at which High-Amplitude Cycling Occurred, %
Relationships Between Five Parameters and Soil Type and Loading Conditions: Plastic Soils

- $\gamma_r = f(\ PI, \ OCR, \ \sigma_0')$
- $a = \text{constant} = 0.92$
- $D_{min} = f(\ PI, \ OCR, \ \sigma_0', \ f)$
- $b = f(\ N)$
- $c = \text{constant} = 0.10$
Bayesian Approach

Bayesian Approach is a systematic way of combining information based on experience (or intuition) with observational data.

• The problem is structured analytically.
• Unknown parameters are modeled as random variables.
• Expected values based on experience and confidence intervals associated with these estimates are determined.
• These values are updated such that the likelihood of occurrence of the observational data is maximized.
Recommended Values: Plastic Soils

\[ \gamma_r = (\phi_1 + \phi_2 \times \text{PI} \times \text{OCR}^{\phi_3}) \times \sigma'_o^{\phi_4} \]
\[ a = \phi_5 \]

where: \( \sigma'_o \) = mean effective confining pressure (atm),

\( \text{PI} \) = soil plasticity (%),

\( \text{OCR} \) = overconsolidation ratio,

and

\( \phi_1 = 0.0352, \quad \phi_2 = 0.0010, \quad \phi_3 = 0.3246, \quad \phi_4 = 0.3483, \quad \phi_5 = 0.9190 \)
Results: $\gamma_r$ for PI = 60%, OCR = 4 and $\sigma_o' = 4$ atm
Recommended Values: Plastic Soils

\[
D_{\text{min}} = (\phi_6 + \phi_7 \times \text{PI} \times \text{OCR}^{\phi_8}) \times \sigma_0^{\phi_9} \times [1 + \phi_{10} \times \ln(f)]
\]

\[
b = \phi_{11} + \phi_{12} \times \ln(N)
\]

where: \( \sigma'_o \) = mean effective confining pressure (atm),

\( \text{PI} \) = soil plasticity (%),

\( \text{OCR} \) = overconsolidation ratio,

\( f \) = loading frequency,

\( N \) = number of loading cycles,

and \( \phi_6 = 0.8005, \phi_7 = 0.0129, \phi_8 = -0.1069, \phi_9 = -0.2889, \phi_{10} = 0.2919, \phi_{11} = 0.6329, \phi_{12} = -0.0057 \).
Results: $D_{\text{min}}$ for PI = 60%, OCR = 4 and $\sigma_o' = 4$ atm
Effect of $\sigma'_o$ on the Nonlinear Behavior of Sands

Seed and Idriss (1970)

Average for Sands

Range

Shearing Strain, $\gamma$, %

Normalized Shear Modulus, $G/G_{\text{max}}$

Silty Sand (SM)
PI = 0 %

- 0.25 atm
- 1 atm
- 4 atm
- 16 atm

Seed and Idriss (1970)

Average for Sands

Range
Effect of $\sigma'_o$ on the Nonlinear Behavior of Sands

Seed and Idriss (1970)
- Avg. for Sands
- Range
- Silty Sand (SM)
  - PI = 0 %
  - $0.25 \text{ atm}$
  - $1 \text{ atm}$
  - $4 \text{ atm}$
  - $16 \text{ atm}$

Material Damping Ratio, $D$, %

Shearing Strain, $\gamma$, %

$N = 10 \text{ cycles}$
$f = 1 \text{ Hz}$
Effect of Plasticity on Nonlinear Soil Behavior

Normalized Shear Modulus, $G/G_{\text{max}}$

This Study (1 atm)
- Non-Plastic
- $\text{PI} = 15\%$
- $\text{PI} = 30\%$
- $\text{PI} = 50\%$
- $\text{PI} = 100\%$

Vucetic and Dobry (1991)

Shearing Strain, $\gamma$, %
Effect of Plasticity on Nonlinear Soil Behavior

Vucetic and Dobry (1991)

This Study (1 atm)

Material Damping Ratio, \( D, \% \)

Shearing Strain, \( \gamma, \% \)

- Non-Plastic
- PI = 15 %
- PI = 30 %
- PI = 50 %
- PI = 100 %

N = 10 cycles
f = 1 Hz
Standard Deviations for $G/G_{\text{max}} \log \gamma$

\[ \sigma_{NG} = \exp(\phi_{13}) + \sqrt{\frac{0.25}{\exp(\phi_{14})} - \left(\frac{G}{G_{\text{max}}} - 0.5\right)^2} \frac{\exp(\phi_{14})}{\exp(\phi_{14})} \]

where:
- $\sigma_{NG}$ = standard deviation for normalized modulus reduction curve
- $G/G_{\text{max}}$ = estimated normalized shear modulus, and
- $\phi_{13}$ and $\phi_{14}$ = parameters that relate standard deviation to mean estimate of normalized shear modulus
Uncertainty Associated with the Predicted $G/G_{\text{max}} - \log \gamma$ Curves

Normalized Shear Modulus, $G/G_{\text{max}}$

Silty Sand (SM)
- PI = 0 %
- OCR = 1
- $f = 1$ Hz
- $N = 10$
- $\sigma_{o}' = 1$ atm
Standard Deviations for $D - \log \gamma$

$$\sigma_D = \exp(\phi_{15}) + \exp(\phi_{16}) \cdot \sqrt{D}$$

where:

$\sigma_D$ = standard deviation for material damping curve,

$D$ = estimated material damping ratio, and

$\phi_{15}$ and $\phi_{16}$ = parameters that relate standard deviation to the mean estimate of material damping ratio
Uncertainty Associated with the Predicted D – log γ Curves

Silty Sand (SM)
- PL = 0 %
- OCR = 1
- f = 1 Hz
- N = 10
- \( \sigma_{o'} = 1 \text{ atm} \)

Material Damping Ratio, D, %

Shearing Strain, \( \gamma \), %
Accomplishments

• An empirical formulation to estimate $G/G_{\text{max}} - \log \gamma$ and $D - \log \gamma$ curves for different soils under various loading conditions was generated.
• This formulation was calibrated using data collected at UT over the past decade (with significant input from ROSRINE/PEER).
• $G/G_{\text{max}} - \log \gamma$ and $D - \log \gamma$ curves predicted using the formulation were observed to be consistent with the general trends reported in the literature and observed during the course of this study.
• The uncertainties associated with the predicted curves were also evaluated within the formulation.
Comparison Between SP, GP, and GW

C = Well Graded Gravel (GW)

G / G_{\text{max}}

Particle Size, mm

Percent Finer

Shearing Strain, $\gamma$, %

Particle Size, mm

Inc. $C_u$

Shearing Strain, $\gamma$, %

Percent Finer

Inc. $C_u$

Shearing Strain, $\gamma$, %

Comparison Between SP, GP, and GW

ABC

Inc. Cu

C = Well Graded Gravel (GW)
Problem with Using the Wrong $G_{\text{max}}$
Effect of Sampling on $V_S$ Values

Modulus Ratio, $G_{\text{max}, \text{lab}} / G_{\text{max}, \text{field}}$

Range from ROSRINE Study

General Trend
Thank You

- Don Anderson, CH2M Hill
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- Robert Pyke, Consultant
- Cliff Roblee, Caltrans
- John Schneider, EPRI
- Walt Silva, Pacific Engineering