# International Workshop on Uncertainties in Nonlinear Soil Properties and their Impact on Modeling Dynamic Soil Response

Sponsored by the National Science Foundation and PEER-Lifelines Program

# PEER Headquarters, UC Berkeley, March 18-19, 2004

## Plenary Session 2: Status of Soil Testing and Material Models

# Emerging Trends in Dynamic Simple Shear Testing

by

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Geotechnical test results are the most fundamental component of practically any geotechnical analysis. Their validity and accuracy define the validity and accuracy of the analysis. Soil material properties determined in the laboratory or in the field represent the foundation of geotechnical analyses and design procedures. The laboratory and field test data also form the basis for studies of spatial variability of soil properties and various probabilistic approaches to asses the uncertainties in determining soil parameters. Accordingly, if there are fundamental aspects of the cyclic stress-strain behavior of soil that have not been tested, or have not been tested adequately or accurately enough, their testing will impact and improve the modeling of dynamic soil response in a truly fundamental manner.

A domain of cyclic soil behavior that needs to be fully explored is the small-strain cyclic soil behavior. In spite of numerous studies in last 15 years (e.g., Jamiolkowski et al., 1999; Shibuya et al., 1994) there are still fundamental small-strain behavior issues that remain to be resolved. In the last 15 years standard triaxial, torsional and simple shear devices have been continuously improved and modified for cyclic small-strain testing purposes. Many of these modifications are very complex and have taken advantage of most current technologies of parameter measurements and data processing. At the same time, the resonant column device, which has been used for the testing of shear moduli and damping at very small strains since its introduction decades ago, has been also used extensively (e.g., Kim et al. 1991; Stokoe et al., 1995, 1999). The resonant column device has been used either alone or in combination with torsional shear device.

Another domain of cyclic soil behavior that has not been adequately investigated is the cyclic stress-strain behavior at very large cyclic strains. Although this kind of testing does not require complicated modifications of testing devices, the data available in the literature are surprisingly scarce.



Fig. 1 NGI-type direct simple shear device at the UCLA Soil Dynamics Laboratory

To investigate the unresolved fundamental aspects of the cyclic stress-strain behavior at very small and very large cyclic shear strain amplitudes,  $\gamma_c$ , a long-term testing effort using simple shear devices has been undertaken at UCLA. This effort encompasses many different soils. The NGI-type concept of direct simple shear (DSS) device with circular specimen confined in a wire-reinforced rubber membrane has been used (Bjerum and Landva, 1966). Either constant-volume equivalent-undrained, or cyclic settlement NGI-type DSS tests were conducted.

Because small-strain properties cannot be accurately tested in the standard, commercially available NGI-type simple shear device shown above in Fig. 1, a new simple shear device for small-strain cyclic testing was designed and built at UCLA in 1993 (Doroudian and Vucetic, 1995). The new testing apparatus is shown in Fig 2. It is called the dual-specimen direct simple shear (DSDSS) device. The most unique feature of the DSDSS device is that two specimens of the same soil are tested simultaneously; instead of just one as usually is the case in geotechnical laboratory practice. This special configuration, in conjunction with very stiff components of the device and non-contact displacement transducers, enables almost complete elimination of problems associated with undesirable false deformations, system compliance and friction. Furthermore, the cyclic horizontal displacement is applied manually (in a "quiet" manner) on the middle cap between the specimens, thus avoiding undesirable vibrations that would be introduced by hydraulic, pneumatic or electrical motor. As a result, very small strains and stresses could be applied and measured accurately in a controlled manner. As shown in Fig. 3, cyclic loops can be recorded clearly at  $\gamma_c = 0.001\%$ . Other DSDSS results confirm that cyclic loops can be recorded comfortably at  $\gamma_c$  as small as 0.0003% (Doroudian and Vucetic, 1998; Matesic and Vucetic, 2003; Vucetic and Tabata, 2003).



Fig. 2 UCLA Dual-specimen direct simple shear device – DSDSS device



Fig. 3 Typical results of variable-frequency cyclic DSDSS test on clay (Matesic and Vucetic, 2003)

The large-strain cyclic simple shear tests have been conducted in both the standard DSS device shown in Fig. 1 and the DSDSS devices. An example of typical results obtained in the standard DSS device is shown below in Fig. 4.



Fig. 4 Typical large-strain cyclic behavior of low-plasticity clay in the standard DSS device; cyclic shear strain amplitude,  $\gamma_c$ = 15.00% (Hsu and Vucetic, 2002)

The long-term goals of the cyclic DSS and DSDSS testing at UCLA are illustrated in Fig. 5. They include the refinement and extension of the data on secant shear modulus,  $G_s$ , and equivalent viscous damping ratio,  $\lambda$ , into the domains of very small shear strains well below the volumetric cyclic threshold shear strain,  $\gamma_{tv}$ , and very large cyclic shear strains between  $\gamma_c \approx 1.0\%$  and 15.0%. In the context of these long term goals, extensive studies on the magnitude of  $\gamma_{tv}$  for different soils and its correlation to the soil's plasticity index, PI, have also been conducted. More specifically, the values of  $\gamma_{tv}$  for cyclic settlement, cyclic stiffness degradation and equivalent pore water pressure have been investigated separately with the help of DSS and DSDSS devices. The entire long-term investigation at UCLA has been going on for more that 10 years and it has encompassed cyclic testing in different research projects supported by different institutions, such as NSF, University of California Office of the President, Lawrence Livermore National Laboratory, Caltrans, PEER, Southern California Earthquake Center (SCEC), and the Okinawa International Exchange and Human Resources Development Foundation in Japan.

Over 160 cyclic tests were conducted on more than 60 soils. Given such a wide scope of the long-term study, and the fact that in the small-strain DSDSS tests many cycles of loading can be applied repeatedly under different conditions without altering the specimen microstructure, very large amounts of cyclic stress-strain data have been generated. To analyze and compare these data systematically, a conveniently structured database with a cyclic loop as its elementary unit has been developed (Hsu and Vucetic, 2002). In the database every cyclic loop in each test is characterized by a series of parameters such as, plasticity index, PI, void ratio, e, degree of saturation, S, vertical stress,  $\sigma_v$ , cyclic shear strain amplitude,  $\gamma_c$ , secant shear modulus, G<sub>s</sub>, damping ratio,  $\lambda$ , frequency, f, shape of applied cyclic shear strain, etc. The main power of the database is that very large number of cyclic loops and their characteristics can be compared and manipulated between different tests in various ways, thus enabling the creation and instant graphical presentation of many useful behavioral trends and comparisons. The data points from the most comprehensive study are presented below in Fig. 6. The modulus reduction and damping curves derived from this data points are presented in Fig. 7. It can be seen that both the

 $G_s$  and  $\lambda$  data cover the range of  $\gamma_c$  from 0.0003% to 15% in large quantities, while as shown in Fig. 6 quite a few data were recorded even below 0.0002%. From such a wealth of data several truly fundamental aspects of the cyclic stress-strain behavior have been analyzed. Many findings have been published while some are in the process of being published; some available data are still being analyzed, while some aspects of the cyclic behavior are just being tested.

The list of investigated research subjects, corresponding publications, and current research needs are summarized in Table 1. Specific research subjects that need to be studied are identified. The subjects for which significant raw data are available and need to be systematically processed and thoroughly analyzed are also identified.



Fig. 5 Average secant shear modulus reduction ( $G_s/G_{max}$  versus  $\gamma_c$ ) and damping ( $\lambda$  versus  $\gamma_c$ ) curves suggested by Vucetic and Dobry (1991) with domains that have been investigated at UCLA (Hsu and Vucetic, 2002)



Fig. 6 Cyclic-loop data points of  $G_s$  and  $\lambda$  from the most comprehensive UCLA study (Hsu and Vucetic, 2002)

In the following paragraphs and figures some selected but limited research results are presented. Although these results are relevant for the modeling of seismic soil response, they may not be the most important, because one of the reasons for showing them is to illustrate the capabilities of the DSS and DSDSS testing.



Fig. 7 Modulus reduction and damping curves derived from the data points presented in Fig. 6 (Hsu and Vucetic, 2002)

GENERAL RESERCH TOPICS	SPECIFIC RESEARC SUBJECTS	MAJOR PUBLICATIONS AND THE STATUS OF UNPUBLISHED DATA AND RESEARCH NEEDS
SECAN SHEAR MODULUS, G <sub>s</sub>	Degree of nonlinearity and precise shape of $G_s$ - log $\gamma_c$ and $G_s/G_{max}$ -log $\gamma_c$ curves at small strains	Hsu and Vucetic (2002) Lanzo and Vucetic (1999a) Lanzo, Vucetic and Doroudian (1997) Matesic and Vucetic (2003) - Existing data need analysis
	Effect of vertical stress, $\sigma_v$ , and overconsolidation ratio, <i>OCR</i> , on the shape of G <sub>s</sub> -log $\gamma_c$ and G <sub>s</sub> /G <sub>max</sub> -log $\gamma_c$ curves at small strains	Hsu and Vucetic (2002) Lanzo, Vucetic and Doroudian (1997)
	Effect of frequency of cyclic loading, $f$ , and average strain rate, $d\gamma/dt$ , on the shape of G <sub>s</sub> - log $\gamma_c$ and G <sub>s</sub> /G <sub>max</sub> -log $\gamma_c$ curves at small strains	Matesic and Vucetic (2003) Vucetic, and Tabata (2003) Vucetic, Tabata and Matesic (2003)
	Effect of the frequency of cyclic loading, $f$ , and average strain rate, $d\gamma/dt$ , on $G_{max}$ at small strains	Hsu and Vucetic (2002) Tabata (2004) - More research needed
	Extension of $G_s$ -log $\gamma_c$ and $G_s/G_{max}$ -log $\gamma_c$ curves to very large strains	Hsu and Vucetic (2002) - More research needed
DAMPING RATIO, λ	Effect of plasticity index, <i>PI</i> , on damping ratio curve, $\lambda$ -log $\gamma_c$ , at very small $\gamma_c$	Hsu and Vucetic (2002) Lanzo and Vucetic (1999a,b) Vucetic, Lanzo and Doroudian (1998a)
	Effect of vertical stress, $\sigma_v$ , and overconsolidation ratio, <i>OCR</i> , on damping ratio curve, $\lambda$ -log $\gamma_c$ , at very small $\gamma_c$	Hsu and Vucetic (2002) Vucetic, Lanzo and Doroudian (1998a) - Existing data need analysis
	Effect of the shape of applied cyclic shear straining (triangular versus sinusoidal versus trapezoidal, stc.) on damping ratio, $\lambda$ , and damping curve, $\lambda$ -log $\gamma_c$ , at small $\gamma_c$	Hsu and Vucetic (2002) Lanzo and Vucetic (1999b) Vcetic, Lanzo and Doroudian (1998a, b) - Existing data need analysis
	Effect of the frequency of cyclic loading, <i>f</i> , and average strain rate, $d\gamma/dt$ , on damping ratio, $\lambda$ , and damping curve, $\lambda$ -log $\gamma_c$ , at small $\gamma_c$	Lanzo and Vucetic (1999a) Lanzo, Doroudian and Vucetic (1999) Tabata (2004) - More research needed
	Damping ratio, $\lambda$ , and damping curve, $\lambda$ -log $\gamma_c$ , at very large cyclic strain amplitudes, $\gamma_c$	Hsu and Vucetic (2002) - More research needed

Table 1. Research subjects investigated at UCLA, corresponding publications and research needs

RELATIONSHIP BETWEEN MONOTONIC AND CYCLIC STRESS- STRAIN BEHAVIOR	Comparison between the cyclic and monotonic stress-strain behavior: (a) validity of Masing rules, and (b) comparison between the $G_s/G_{max}$ -log $\gamma_c$ curves obtained from cyclic and monotonic tests	Hsu and Vucetic (2002) Tabata (2004) - More research needed
	Effects of the magnitude and variation of strain rate, $d\gamma/dt$ , on the monotonic stress-strain behavior and its relation to cyclic small-strain behavior	Tabata (2004) - More research needed
VOLUMETRIC CYCLIC THRESHOLD SHEAR STRAIN, $\gamma_{\rm fv}$	Volumetric cyclic threshold shear strain, $\gamma_{tv}$ , for cyclic settlement, including the effects of <i>PI</i> and $\sigma_v$	Hsu and Vucetic (2004)
	Volumetric cyclic threshold shear strain, $\gamma_{tv}$ , for cyclic stiffness degradation, including the effects of <i>PI</i> and $\sigma_v$	- Submitted for publication
	Volumetric cyclic threshold shear strain, $\gamma_{tv}$ , for equivalent cyclic pore water pressure in NGI- type DSS cyclic test, including the effects of <i>PI</i> and $\sigma_v$	Hsu and Vucetic (2002) - Submitted for publication
SOIL TESTING ISSUES	Effect of sample disturbance on Gs, $G_{max}$ and $\lambda$ , and the $G_s/G_{max}$ -log $\gamma_c$ and $\lambda$ -log $\gamma_c$ curves	- Comprehensive research needed
	Effect of the lateral stress mobilization in NGI- type DSS test on cyclic stress-strain properties	- Comprehensive research needed

Effect of average strain rate  $\dot{\gamma} = d\gamma/dt$  on  $G_s$  and  $G_s$  -log  $\gamma_c$  trend

The effects of the frequency of cyclic loading and corresponding strain rate have been extensively investigated. The effect of the higher frequency, f, and corresponding average strain rate,  $\dot{\gamma} = d\gamma/dt = 4$  f  $\gamma_c$ , on  $G_s$  is commonly quantified by the strain-rate shear modulus parameter:

$$\alpha_{G} = \frac{\Delta G_{s}}{\Delta \log \dot{\gamma}} = \frac{\left(G_{s}\right)_{\text{high-}f} - \left(G_{s}\right)_{\text{low-}f}}{\log \dot{\gamma}_{\text{high-}f} - \log \dot{\gamma}_{\text{low-}f}} = \frac{\left(G_{s}\right)_{\text{high-}f} - \left(G_{s}\right)_{\text{low-}f}}{\log \frac{\dot{\gamma}_{\text{high-}f}}{\dot{\gamma}_{\text{low-}f}}}.$$

Parameter  $\alpha_G$  describes the increase of  $G_s$  for the tenfold (one order of magnitude) increase of  $\dot{\gamma}$ . If  $\alpha_G$  is normalized by some convenient reference  $G_s$ , the strain-rate shear modulus factor,  $N_{\dot{\gamma}-G}$ , is obtained:

$$N_{\dot{\gamma}-G} = \frac{\left(G_{s}\right)_{\text{high}-f} - \left(G_{s}\right)_{\text{low}-f}}{\log \frac{\dot{\gamma}_{\text{high}-f}}{\dot{\gamma}_{\text{low}-f}}} \frac{1}{\left(G_{s}\right)_{\text{ref}}} = \frac{\alpha_{G}}{\left(G_{s}\right)_{\text{ref}}}.$$

Factor  $N_{\dot{\gamma}-G}$  was introduced by Isenhower and Stokoe (1981) and it describes the relative increase of  $G_s$  for the tenfold increases of  $\dot{\gamma}$  with respect to a reference  $G_s$ . In this respect,  $N_{\dot{\gamma}-G}$ is more convenient for quantifying the rate effects than  $\alpha_G$ . Figure 8a below shows how the  $G_{s}$ - $\gamma_c$  data points are scattered vertically at certain levels of  $\gamma_c$  for a clay if  $\dot{\gamma}$  varies significantly. Figure 8b-e shows further how  $G_s$  increases consistently with  $\dot{\gamma}$  at the same levels of  $\gamma_c$ . Finally, Fig. 9 presents the trends of  $\alpha_G$  and  $N_{\dot{\gamma}-G}$  with  $\gamma_c$  for different soils. For example, it can be seen in Fig. 9b that  $G_s$  in clays tested at  $\gamma_c = 0.001\%$  increases by 4-8% for a tenfold increase of  $\dot{\gamma}$ . At smaller  $\gamma_c = 0.0005\%$  this increase is even larger, between 7 and 11%.



Fig. 8 Influence of  $\dot{\gamma}$  on  $G_s$  for a clay (Matesic and Vucetic, 2003)

Such a strong effect of  $\dot{\gamma}$  on  $G_s$  indicates that the shape of the  $G_s/G_{max}$ -log  $\gamma_c$  curve at small strains depends significantly on the values of the applied  $\dot{\gamma}$  and corresponding frequencies, f. This effect can produce different curvatures of the  $G_s/G_{max}$ -log  $\gamma_c$  curves at small strains, and in this way it can affect drastically the outcome of a site response study. For example, under

otherwise regular testing conditions, a  $G_s/G_{max} = 1.0$  curve representing a perfectly linear soil can be obtained easily at very small strains if at all levels of  $\gamma_c$  the tests are conducted at the same frequency, f. This and other effects of  $\dot{\gamma}$  on  $G_s$  and their ramifications are described in detail in Matesic and Vucetic (2003) and Vucetic and Tabata (2003).



Fig. 9 Trends of  $\alpha_G$  and  $N_{\dot{\gamma}-G}$  with  $\gamma_c$  for several sands and clays (Matesic and Vucetic, 2003)

## Effect of frequency, f, on damping ratio $\lambda$

The data obtained in the DSDSS device reveal that the effect of frequency, f, on damping ratio  $\lambda$  and  $\lambda$ -log  $\gamma_c$  curve is more intricate than previously concluded by others on the basis of limited testing. In Fig. 10 below, this effect is shown for one high plasticity clay. It can be seen that below f = 0. 1 Hz damping ratio  $\lambda$  decrease with f, while above f = 0. 1 Hz it increases. For other soils somewhat different trend has been obtained. This trend also depends on the shape of cyclic straining. Finding out the precise effect of frequency, f, and corresponding average strain rate,  $\dot{\gamma} = d\gamma/dt$ , on damping ratio  $\lambda$  can substantially improve the modeling of soil response, because it is well known that the site response depends strongly on the magnitude of  $\lambda$ .



Fig. 10 Effect of frequency, f, on damping ratio  $\lambda$  (Lanzo and Vucetic, 1999a; Lanzo, Doroudian and Vucetic 1999)

## Volumetric cyclic threshold shear strain, $\gamma_{ty}$ , for cyclic settlement

The trend of the volumetric cyclic threshold shear strain for cyclic settlement with the plasticity index, PI, is shown below in Fig 11. While considerable data on  $\gamma_{tv}$ , for cyclic settlement of sands were published before by others, the data on clayey soil have been scarce. A literature review has revealed only three  $\gamma_{tv}$  values for cyclic settlement of clay, most likely because of the complexity and high cost of such experimental efforts. Besides that, these three results show no trend with PI. Figure 12 shows furthermore that  $\gamma_{tv}$  levels correspond to surprisingly large G<sub>s</sub>

reduction of 55 to 80%. Construction of Fig. 12 was made possible by testing both  $\gamma_{tv}$  and  $G_s$  on the same soils, which seems to be the only such combined experimental study to date.



Fig. 11 Trend of volumetric cyclic threshold shear strain,  $\gamma_{tv}$ , for cyclic settlement with plasticity index, PI (Hsu and Vucetic, 2004)



Fig. 12 Location of the  $\gamma_{tv}$  points on the G<sub>s</sub>/G<sub>max</sub>-log  $\gamma_c$  curves (Hsu and Vucetic, 2004)

### **Conclusions**

The main conclusion is that some fundamental aspects of the cyclic stress-strain behavior of soils are still not known or understood to a satisfactory extent, in spite of the fact that the outcome of dynamic site response analysis depends primarily on the soil properties. Accordingly, the investigations of certain fundamental aspects of the cyclic stress-strain behavior need to be conducted. These investigations can be conducted in the range of larger cyclic shear strains with the help of standard cyclic simple shear device, and in the range of small cyclic shear strains with the help of UCLA dual-specimen direct simple shear device.

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