Evolution of Soil Models Since the 1970s

by Robert Pyke[1]

Introduction

In Pyke (1979) the author wrote "there is currently a trend towards increasing use of fully nonlinear stress-strain relationships" for analyses of soil response under cyclic loading, such as those caused by earthquakes. That statement has turned out to be less than accurate over the intervening 25 years. There are several reasons for this. They include an unjustified preference for using half-baked semi-empirical procedures in geotechnical engineering but also include the lack of consensus on what nonlinear material models to use. As stated by Chandrakant Desai: "Soil models are like religion. Everyone believes in his own but not in anyone-else's". It should also be said that modeling soil behavior under complex and cyclic loadings is not an easy task but the task has been made harder by the lack of data from truly two or three-dimensional element tests in the laboratory and even the lack of measured stress-strain data under cyclic simple shear conditions for a wide range of loading conditions. This paper is limited to a discussion of models for representing soils under cyclic simple shear loadings and consists of two parts. In the first part the evolution of simple models for this loading condition is summarized. In the second part the effect of some differences in modeling assumptions are illustrated using a one-dimensional site response problem taken from actual practice.

Early History

The earliest nonlinear site response analyses used models that were based on either a hyperbolic shape or the Ramberg-Osgood relationship and cycled in accordance with what are commonly referred to as the Masing rules. A basic hyperbola can be defined by two parameters, the shear modulus at small strains, G_{max}, and the shear strength, τ_{max}. Since it is well known that soil stress-strain relationships do not necessarily follow a plain hyperbola exactly, one or more additional parameters may be introduced to modify the shape of the hyperbola. The Ramberg-Osgood relationship is a three-parameter model that affords a better fit to actual data than a plain hyperbola but has the oddity that it expresses strain in terms of stress. This fitted nicely with the method of characteristics, which was the solution scheme favored by some early workers, but is inconvenient for most solution schemes since they require stress in terms of strain.

The Masing rules flow from the hypotheses of Masing (1926). Essentially they state that: (1) on reversals the stress-strain relationship goes back to a slope of $G_{max:}$ and (2) after the first quarter cycle of loading, the scale of the stress-strain relationship changes by a factor of two. In Pyke (1979) the author discussed the limitations of the second Masing hypothesis and suggested an alternate that he called the Cundall-Pyke

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hypothesis. Later in this paper it will be demonstrated that the first Masing hypothesis is also invalid. Through no fault of Masing, adoption of his hypotheses has been one of the obstacles to better modeling of the nonlinear behavior of soils. Because it was written in German, it seems that few modern workers have ever read his paper. In it he describes his hypothesis, a simple experiment to test it, and concludes that the hypothesis is not supported by the experiment!

Limitations of Early Models

Early nonlinear models for site response analyses had up to four limitations.

1. They tended to observe the Masing rules which not only makes fitting of real soil behavior more difficult in general but specifically prevents the development of permanent strains in cases where there is an initial shear stress in the direction of the subsequent cyclic loading. This limitation is not critical in one-dimensional analyses whose sole purpose is to compute the site response, but it is critical in one, two or three-dimensional analyses that are intended to compute permanent deformations, which is one of the goals of nonlinear analyses. In a nonlinear analysis in which the material model complies with the second Masing hypothesis, the computed deformations will result solely from the irregularities in the input motion rather than from the material behavior.
2. They tended not to follow the degradation (flattening of the secant modulus) and degeneration (change of shape) that occurs in the stress-strain relationships of most soils under cyclic loading. Again, this may not be critical in analyses whose sole purpose is to compute site response as indicated by response spectra but it may be critical to the computation of displacement profiles and displacement histories, which are increasingly required as the input to more sophisticated soil-structure interaction analyses.
3. They did not produce the small strain damping that is observed in element tests and implied by field measurements. This may not be significant in analyses of site response for strong ground motions but may be for analyses of weak motions.
4. They almost universally produced more damping at moderate to large strains than is observed in element tests. However, it is not clear whether this is a always a significant shortcoming in nonlinear analyses since some nonlinear models that generate higher damping than observed in element tests have performed well in back-calculations of the motions recorded in vertical arrays, as shown for example in EPRI (1993).

Relative Merits of One-Dimensional and Three-Dimensional Models

Space and time limitations prevent a full discussion of the many three-dimensional nonlinear models that have been developed in the last 25 years, thus this paper and presentation are limited to a discussion of simple models for representing soils under cyclic simple shear loadings. Nonetheless, three-dimensional models should, as a minimum, match the performance of a more complete simple or one-dimensional model, as discussed in subsequent sections of this paper, when applied in a one-dimensional analysis. Three-dimensional models in general have one big advantage over simple one-dimensional models and one big disadvantage. The big advantage is that if, and this is a big if, degeneration and degradation are properly modeled, the effects of the second horizontal component of motion and the vertical component of motion can automatically be taken into account in a "one-dimensional" site response analysis that is driven by two or three components of motion. The big disadvantage is that they are necessarily more complicated to describe and to communicate to parties other than the original developer.

A Comment on the Term "Reference Strain"

The quantity defined τ_{max} divided by G_{max}, defined by Hardin and Drnevich (1972) as the reference strain turns out to be useful both for expressing the stress-strain relationship in mathematical form and as a measure of the relative values of G_{max} and τ_{max}. Soils with larger values of reference strain have greater shear strengths relative to their small strain moduli and show more elastic, less nonlinear, stress-strain behavior than soils with smaller values of reference strain. Thus plastic clays exhibit high values of reference strain and gravelly soils exhibit low values. However, some confusion arises because if the shear stress - shear strain relationship is a plain hyperbola, the reference strain is also the strain at which the shear modulus is reduced to half its initial value of $G_{max,}$ and it is convenient therefore to read off the reference strain from a modulus reduction curve as the strain that corresponds to a value of G/G_{max} of 0.5. This is the sense in which the term reference strain is used by Darendeli (2001). However, in the author's work the reference strain always means the initial reference strain, or the ratio of the initial values of τ_{max} and G_{max}, which values are invariably modified as the stress-strain relationship is cycled so that the strain at which G/G_{max} equals 0.5 may vary significantly from this initial ratio.

Illustration of the Evolution of a Simple Model

The form of the shear stress - shear strain loops generated by a simple model employing a plain hyperbola and the Masing rules and the modulus reduction and damping curves generated by such a model are shown in Figure 1. The example is for a reference strain of 0.1 percent and for comparison the upper bound of the range of modulus reduction curves for sand published by Seed and Idriss (1970) and the lower bound of their curves for damping are also shown. Note that the model appears to generate excessive damping. If such a model is used to construct modulus reduction and damping curves for use in equivalent linear analyses the damping ratios need to be reduced by a factor in the order of 0.5 in order to obtain curves comparable to laboratory data.

If the basic hyperbola is modified by use of the parameter "a" defined by Darendeli (2001) (called alpha in the author’s model to distinguish it from Hardin and Drnevich’s "a") the shape of the stress-strain loops can be changed and the modulus reduction and damping curves modified as shown in Figure 2. However, this does not solve the problem of excessive damping as the better fits to laboratory data that are shown in Figures 2 through 5 result from the use of a damping reduction factor of 0.5 (the same as Darendeli's parameter "b").

Figure 3 shows the effect of using the Cundall-Pyke hypothesis and the effect of introducing low strain damping using the scheme described in EPRI (1993) and Pyke (1993).

The simple hyperbolic model described in EPRI (1993), sometimes referred to as the HDCP soil model, also includes two schemes for introducing degradation and degeneration of the stress-strain loops - a scheme that is suitable for saturated sands based on the work of Seed, Martin and Lysmer (1976), and a scheme that is more suitable for clayey soils based on the work of Idriss, Dobry and Singh (1978). An example stress-strain loop and modulus reduction and damping curves for clays are shown in Figure 4. Figure 5 shows an example stress-strain loop for saturated sand and a comparison of modulus reduction and damping curves for both dry and saturated sands. Clearly such models have some utility for generating families of curves that may be used in equivalent linear analyses in addition to their use in nonlinear analyses.

While the introduction of degradation and degeneration reduces the damping ratios obtained at high strains, as shown in Figures 4 and 5, it has proven to be difficult to match laboratory measured values of damping over a wide range of strains unless G_{max} is allowed to degrade, that is, unless the first Masing hypothesis is also abandoned. While Hardin and Drnevich (1972) suggested that there was experimental support for G_{max} remaining constant, Stokoe (personal communication, 2000) advises that this is not so. Lacking more detailed experimental data, the author has implemented a scheme whereby the user specifies the decrease in G_{max} per log cycle and the range of strains over which this reduction occurs. The modulus reduction and damping curves obtained using this procedure and parameters selected to provide a good fit to the Seed and Idriss curves are shown in Figure 6. A second example of the use of the new procedure is given in Figure 7 in which modulus reduction and damping curves for an initial reference strain of 0.3 percent using the old procedure, which are known to provide a good fit to experimental data for young Bay Mud, are compared with similar curves obtained using the new procedure.

Effects on Site Response Analyses

The effects of the evolution of the simple hyperbolic model on site response analyses is illustrated by an example taken from actual practice, namely calculation of the site response for possible new offshore runways at San Francisco International Airport. At the time that these studies were suspended, two alternatives for the runway platform were being considered: a hydraulically placed fill with consolidation of the underlying young Bay Mud enhanced by the use of "wick" drains; and a concrete deck supported by large diameter piles. The soil profile under a portion of the proposed runways consists of 70 feet of young Bay Mud, underlain by 60 feet of interbedded sands and clays, 100 feet of Old Bay Clay, and Franciscan bedrock. The majority of the results shown are for this basic profile, which represents the free-field condition for the pile-supported platform, but limited results are also shown for the fill alternative in which the Young Bay Mud layer is shortened and stiffened as a result of consolidation under the weight of the fill.

For the natural soil profile and 1000-year return period ground motions the response of the young Bay Mud is very nonlinear with cyclic shear strains at the base of this stratum as large as 10 percent. Equivalent linear analyses do not even converge under these conditions. Multiple-level displacement histories that were generated by nonlinear site response analyses were used as input to advanced soil-pile-structure interaction analyses in order to study the behavior of various pile types and sizes. The results, and indeed the feasibility of the pile-supported solution, were sensitive not only to the details of the profile assumed for analysis but also to the details of the soil model.

The site response analyses were conducted using the computer program TESS (TAGAsoft, 2004), which uses an explicit finite difference solution for the one-dimensional wave propagation problem. The properties of the young Bay Mud layer were defined by the parameters used to produce the shear modulus and damping curves that are shown in Figure 7. The results are best viewed as animated displacement profiles using a post-processor but for the purposes of this paper they are summarized by the shear stress - shear strain loops at the base of the young Bay Mud and by the acceleration and displacement response spectra at the ground surface or mudline.

As a starting point, the results obtained for both the natural profile and the profile with fill, using the older soil model that has excessive damping, are shown in Figure 8. The stress-strain loops are shown only for the natural profile. Note that a permanent offset develops solely as a result of the irregularities of the particular acceleration history that has been used as the input motion. Note also that the displacement response spectra in the period range from 2 to 3 seconds provide perhaps the best indication of the severity of the loading applied to the structure.

In order to show the effect of a gently sloping ground surface the results obtained for the natural profile assuming the mudline has a 5 percent slope are shown in Figure 9. Only one stress-strain loop is shown as an example but the acceleration and displacement response spectra are shown for two cases, one for each of the possible directions in which the input motion can be applied. The gently sloping surface does not make much difference to the response spectra but the computed final displacements at the mudline are -4.4 feet for the horizontal ground surface and 6.8 and 13.5 feet for the two cases with a sloping ground surface.

The previous examples have used the Cundall-Pyke hypothesis but the effect of substituting the Masing hypothesis is shown in Figure 10. Not only is the appearance of the stress-strain loops different from that seen in Figure 8 but also there are noticeable differences in the mudline acceleration and displacement response spectra.

The effect of introducing degradation of G_{max} and obtaining damping values consistent with experimental data is shown in Figure 11. The stress-strain loops in this figure should be compared with those in Figure 8. Note that while both the acceleration and displacement response spectra are higher, consistent with lower damping, the stress-strain loops are much tighter. This can also be seen in Figure 12, which shows one of the stress-strain loops and the acceleration and displacement response spectra for the two possible cases with a mudline slope of 5 percent. The computed final displacements at the mudline are now -1.1 feet for the horizontal ground surface and -2.1 and 1.4 feet for the two cases with a sloping ground surface even though the maximum displacement at the mudline has increased from 5.3 to 5.6 feet. Thus, as might have been expected, the permanent offsets computed using the earlier "fat" stress-strain relationship appear to have been in error. Only by closely following the stress-strain relationship of the soil can one hope to make reasonable estimates of permanent deformations.

Conclusions

1. Neither of the Masing hypotheses is valid.
2. While some progress has been made in the last 25 years, in order to make further progress we need more element test data that fully describe complex, cyclic loading in one, two and three-dimensions.
3. What we really need is more robust nonlinear analysis tools and well-documented case histories rather than equivalent linear analyses and semi-empirical procedures based on poorly documented case histories or centrifuge tests!

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student Mehmet Darendeli, and from discussions over a period of many years with Peter Cundall, and the late Bill Joyner.

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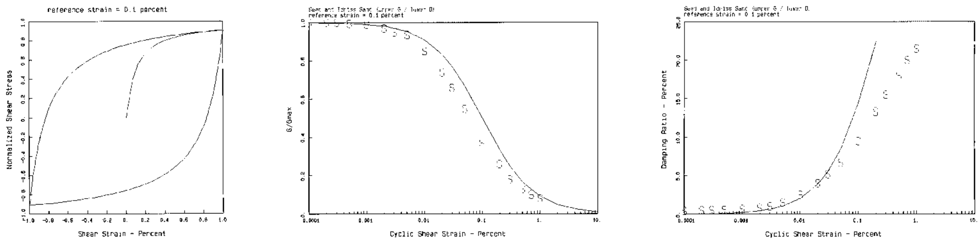

Fig.1 Simple Hyperbolic Model

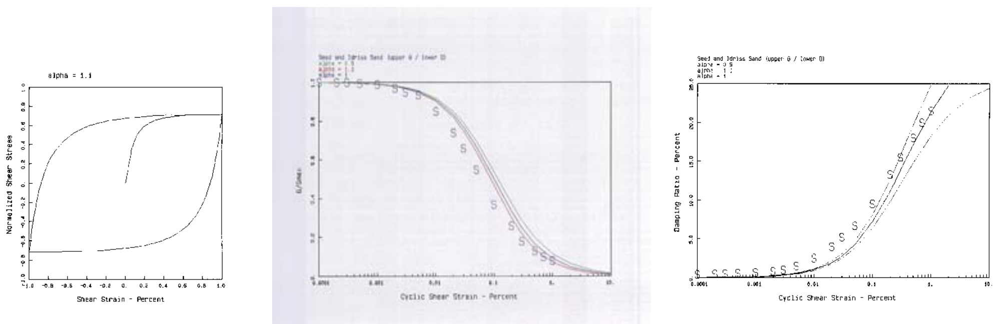

Fig.2 Effect of Alpha Parameter

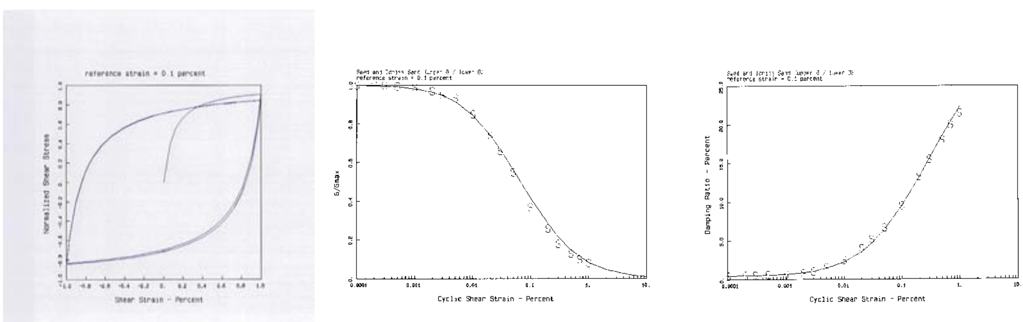

Fig.3 Effect of Cundall-Pyke Hypothesis and Low Strain Damping

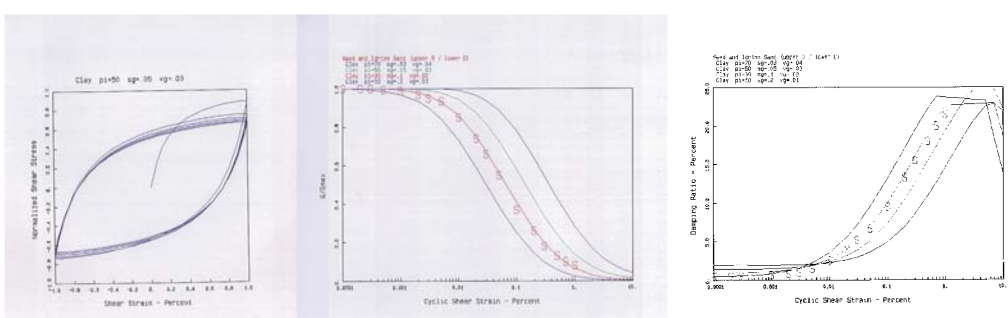

Fig.4 Example Curves for Clay

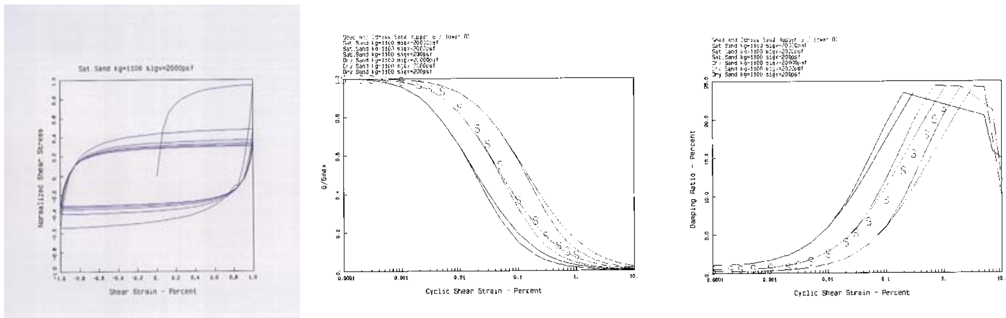

Fig.5 Example Curves for Sand

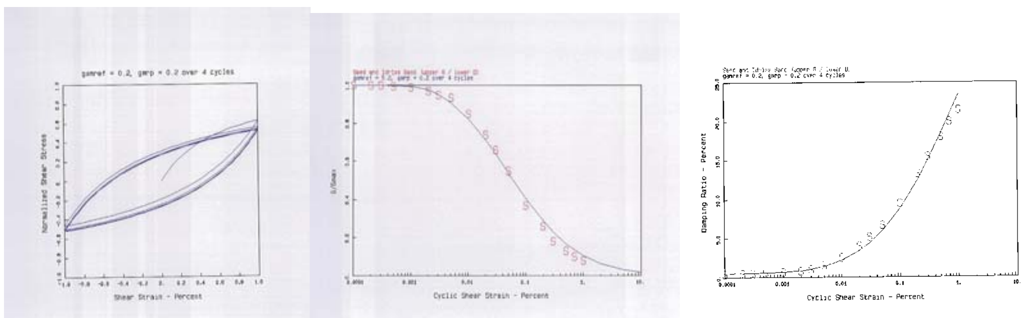

Fig.6 New Model with Degradation of G_{max}

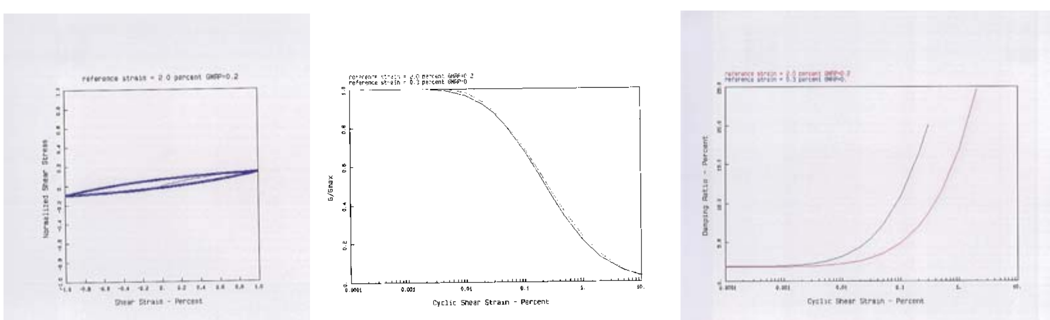

Fig.7 Comparison of Models for Young Bay Mud

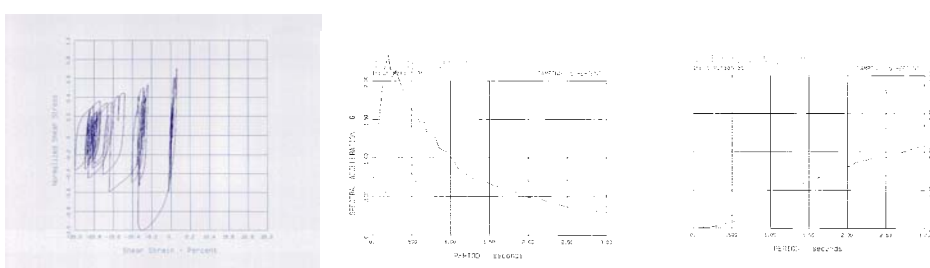

Fig.8 Example Site Response Using Old Model

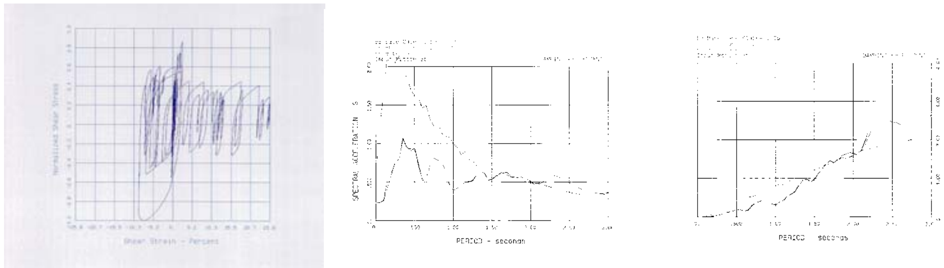

Fig.9 Effect of Sloping Ground Surface – Old Model

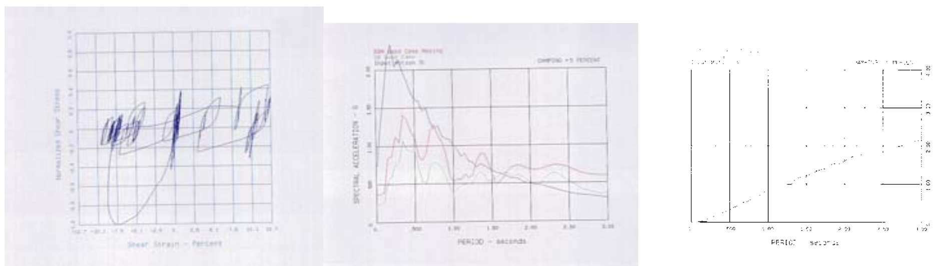

Fig.10 Effect of Masing Hypothesis

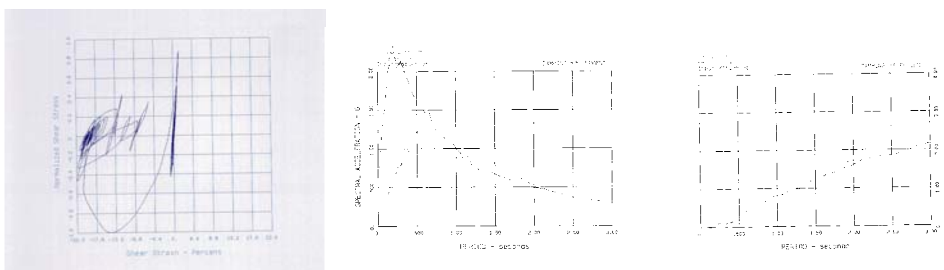

Fig.11 Site Response Using New Model

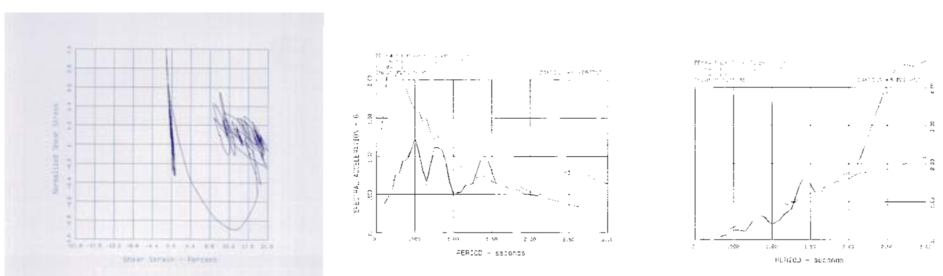

Fig.12 Effect of Sloping Ground Using New Model