Frequency Dependence of Dynamic Soil Properties

Glenn J. Rix
Georgia Institute of Technology
School of Civil and Environmental Engineering
Atlanta, Georgia 30332-0355
glenn.rix@ce.gatech.edu

Most site response analyses are performed with frequency-independent soil properties. For clean, course-grained soils, previous studies have shown that the influence of loading frequency on dynamic soil properties is small (Bolton and Wilson, 1989; Kim 1991). However, in fine-grained soils and coarse-grained soils with significant fines content, the influence of frequency can be important (Kim 1991; Shibuya et al., 1995; Stokoe et al., 1999; d'Onofrio et al., 1999; Matešić and Vucetic, 2003). A detailed understanding of this influence has been hindered by the availability of only sparse (with respect to frequency) measurements and various measurement errors.

Non-Resonance Test Method

A non-resonance (NR) test has been developed and implemented that allows the frequency dependence of dynamic soil properties to be studied (Rix and Meng, 2004). The NR method is based on measurements of the frequency response function between the applied harmonic torque and resulting rotation of the specimen. The test apparatus is identical to that used for resonant column and torsional shear tests. A harmonic excitation $T_0 e^{i\omega t}$ is applied through an electromagnetic drive motor. A voltage-to-current converter is used to minimize the source-load mismatch and virtually eliminate equipment-generated damping due to the interaction between solenoids and magnets within the motor (Meng and Rix, 2003; 2004). The angular displacement response $\phi(0,t)$ of the specimen is detected by a pair of proximity probes.

It may be shown that the frequency response function between the excitation and response of a specimen is (Rix and Meng, 2003):

$$\frac{T_0 e^{i\omega t}}{\phi(0,t)} = \frac{\pi R^4}{2} \rho \omega^2 h \frac{\cot \Omega^*}{\Omega^*} - J_0 \omega^2 \tag{1}$$

where R, h and ρ are the radius, height and mass density of the specimen, respectively; J_0 is the mass polar moment of inertia of the drive system, ω is the circular frequency; and $i^2 = -1$. The term

$$\Omega^*(\omega) = \sqrt{\frac{\rho \omega^2 h^2}{G^*(\omega)}} \tag{2}$$

is a function of the complex-valued shear modulus $G^*(\omega) = G_1(\omega) + i \cdot G_2(\omega)$. The experimental procedure consists of measuring the frequency response function and solving Equation 1 for

$G^*(\omega)$. The magnitude of the complex shear modulus and damping ratio $D(\omega)$ are then calculated using:

$$\left| G^*(\omega) \right| = \sqrt{G_1^2(\omega) + G_2^2(\omega)} \tag{3a}$$

$$D(\omega) = \frac{1}{2} \cdot \frac{G_2(\omega)}{G_1(\omega)} \tag{3b}$$

The NR method allows for rapid measurement of the shear modulus and damping ratio of soils over a broad frequency range from 0.01 to 30 Hz. As such, the test method is well suited for studying frequency-dependent phenomenon in soils.

Kramers - Kronig Relationships

Even at very small strains, soils subjected to dynamic loads have both the ability to store strain energy (elastic behavior), and to dissipate strain energy over a finite period of time (viscous behavior). Thus their mechanical behavior can be described from a phenomenological perspective by the theory of linear viscoelasticity. In linear viscoelastic materials, the real and imaginary components of the complex-valued shear modulus, $G^*(\omega)$, are not independent. They are related via the Kramers-Kronig relationships, which arise from the fundamental properties of linearity and causality. An approximate form of the Kramers-Kronig relationships suitable for this study is:

$$D(\omega) = \frac{\pi}{4} \left(\frac{d \ln G_d(\varpi)}{d \ln(\varpi)} \right)_{\varpi = \omega} \tag{4}$$

and is based on the assumption that $\frac{dG_1(\varpi)}{d \ln \varpi}$ is small, which is true for $D(\omega)$ < 10%.

Test Results

Non-resonance tests have been performed on several remolded and undisturbed soil specimens to demonstrate the influence of frequency on their dynamic properties. Test results for remolded kaolin, undisturbed clayey sand, undisturbed high-plasticity clay, and undisturbed low-plasticity silt are shown in Figure 1. The tests were performed at shear strain amplitudes less than the linear threshold strain of the soils. Although there is experimental scatter, particularly in the damping ratio measurements, the trends from 0.01 to 30 Hz are well defined. The magnitude of the complex-valued shear modulus increases monotonically with frequency as expected. The damping ratio spectra exhibit minima centered at about 0.1 to 1 Hz.

The dense measurements with respect to frequency and the relatively broad frequency range associated with the NR test allows one to check the consistency of the data with the approximate form of the Kramers-Kronig relationships. Equation 4 was used to calculate the damping ratio spectra with the results shown as solid lines in Figure 1. The agreement between measured and calculated damping ratio spectra is excellent.

Two aspects of the test results are noteworthy. First, it is commonly assumed that the shear modulus of many fine-grained soils increases linearly with frequency. According to Equation 4, a linear increase in shear modulus corresponds to constant damping ratio. Clearly this is not the case for the soil specimens in Figure 1. Second, although the measured and calculated damping ratio spectra are similar in shape to the conceptual diagram proposed by Shibuya et al. (1995), there is no range of frequencies over which the damping ratio is constant (i.e., hysteretic damping).

Conclusions

Non-resonance measurements and the approximate Kramers-Kronig relationship are valuable tools to study the frequency (or strain rate) dependence of soils. The NR method provides for denser measurements with respect to frequency that reveal important details in material behavior including inflection points in shear modulus spectra and minima in damping ratio spectra. The NR method is also capable of spanning a broad frequency range because the inertia of the drive system is properly taken into account. The Kramers-Kronig relationship is used to help confirm the validity of the measurements by comparing measured and calculated damping ratio spectra.

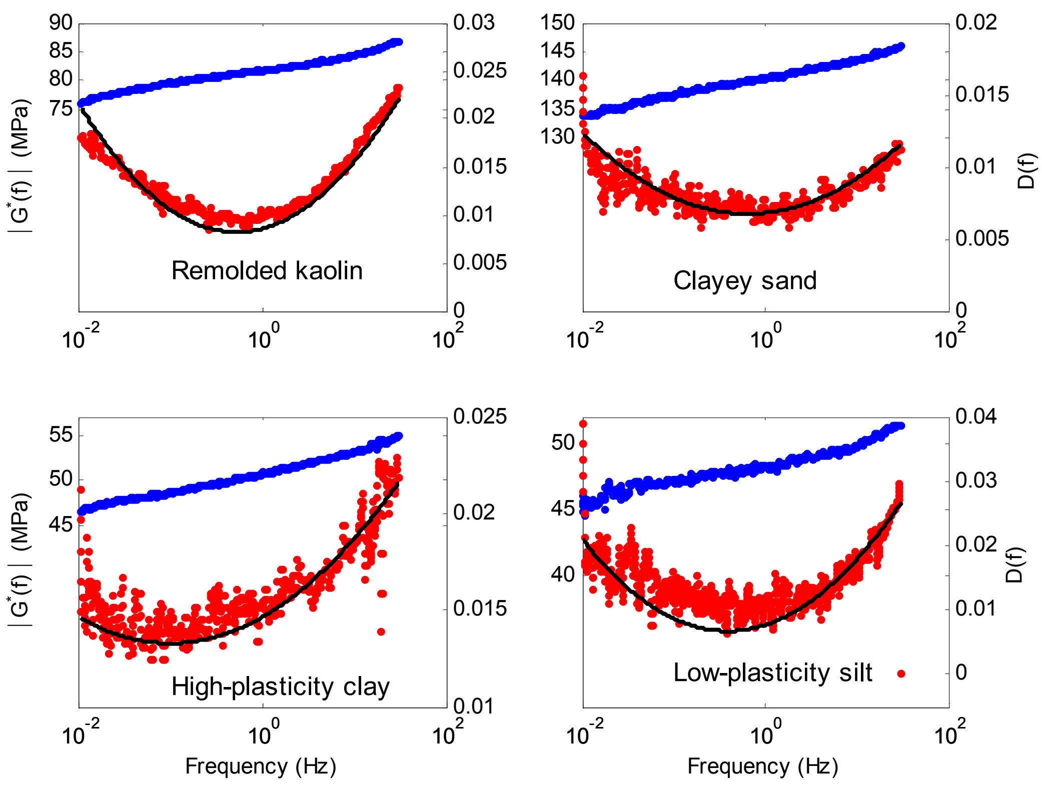

Figure 1 Modulus and damping ratio spectra of remolded and undisturbed specimens