# Development of a New Family of Normalized Modulus Reduction and Material Damping Curves

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## ABSTRACT

More than 150 intact soils specimens and 50 reconstituted gravelly/sandy specimens have been tested dynamically in the laboratory. The effects of dynamic test parameters (such as effective confining pressure, shearing strain amplitude, and number of loading cycles) and soil parameters (such as median grain size, uniformity coefficient, void ratio, plasticity index and overconsolidation ratio) on the linear and nonlinear shear modulus and material damping ratio have been studied. A modified hyperbolic model that can be used to predict the linear and nonlinear dynamic responses of different soil types has been developed with the database generated from the laboratory results. The data and model parameters have been statistically analyzed using the first-order, second-moment Bayesian method. The effects of the tests parameters on dynamic soils properties were evaluated and quantified within this FSBM framework. One of the most important aspects of this study is estimating not only the mean values of the empirical curves but also estimating the uncertainty associated with this values. The modified hyperbolic model is discussed. Normalized modulus reduction and material damping curves for different soils under different confinement states are shown, including the uncertainty associated with the curves.

**Keywords:** **empirical model, hyperbolic model, all soil types, linear response, nonlinear response, shear modulus, material damping, uncertainty**

## INTRODUCTION

Over the past twelve years, more than 150 intact soil specimens have been tested in the laboratory using combined resonant column and torsional shear, RCTS, equipment at the University of Texas at Austin [1, 2, 3 and 4]. This equipment has a fixed-free configuration, with the bottom of the specimen placed on a fixed base pedestal. Specimens that have been tested have diameters that ranged from 36 to 76 mm, and heights that were nominally twice the diameters. These intact specimens ranged from poorly graded sands (SP) to high plasticity clays (CH).

Because of this significant effort in testing intact specimens, a significant database with "real" soils has been generated. The presence of this database motivated a re-evaluation of empirical curves employed in the state of practice. The re-evaluation showed the need to develop an improved set of empirical curves in terms of nonlinear normalized modulus and material damping curves. The database was statistically analyzed using the first-order, second-moment, Bayesian method (FSBM). The effects of various parameters (such as confining pressure and soil plasticity) on dynamic soil properties were evaluated and quantified. This nonlinear model is discussed below and is used to compare linear and nonlinear dynamic soil properties.

To enlarge the database to include the dynamic properties of gravelly and sandy soils determined in the laboratory, a new free-free torsional resonant column device was design and constructed [5]. This device allowed specimens with a diameter of 152 mm and a nominal height of 305 mm to be tested. Due to the large size of the specimens, only reconstituted, nonplastic, gravelly and sandy specimens have been tested. However, these tests have permitted the effects of parameters such as median grain size and uniformity coefficient of granular soils to be studied. The results from these tests have also been included in the empirical model discussed below. This empirical model is used to make comparisons between the dynamic responses of different soils. Comparisons are also made with empirical models in the literature.

## MODIFIED HYPERBOLIC MODEL

The new nonlinear model used to fit the dynamic measurements is a modified version of the hyperbolic model originally recommended by Hardin and Drnevich [6] to model the $G/G_{max}$ – log $\gamma$ relationship of soils. The modified hyperbolic model for normalized modulus reduction can be expressed with two parameters as [3]:

$$\frac{G}{G_{max}} = \frac{1}{1+\left(\frac{\gamma}{\gamma_r}\right)^{a}} \tag{1}$$

in which $\gamma$ = any given shearing strain,
$\gamma_r$ = the reference shearing strain, and
a = dimensionless exponent.

The two parameters are reference strains, $\gamma_r$, and the exponent “a”. The reference strain, $\gamma_r$, is simply used for curve fitting purposes and is defined as the value of $\gamma$ at the shearing strain where $G/G_{max}$ equals 0.5. This definition of $\gamma_r$ is different from the one proposed by Hardin and Drnevich [6]. The value of $\gamma_r$ is, however, very convenient because it can be determine directly from the RCTS (or other laboratory) measurements. Initially, the value of “a” was taken as a constant [3], with a value of 0.92. However, additional work has shown the value of “a” to vary with soil type [4 and 5], and there is an on-going effort to incorporate this variation in the model.

In the case of the material damping ratio, D, the equation for the nonlinear material damping curves is based on the modified hyperbolic stress-strain curve and Masing behavior. The estimation of D that is based solely on Masing behavior yields higher damping ratios at higher strains than values measured in this study and reported in the literature [6 through 8]. Also, the variation of Masing damping ratio, $D_{Masing}$, with shearing strain lacks the value of the small-strain material damping ratio, $D_{min}$, because $D_{Masing}$ goes to zero in the linear range. Therefore, the empirical equation for D that takes into account the experimental observations is:

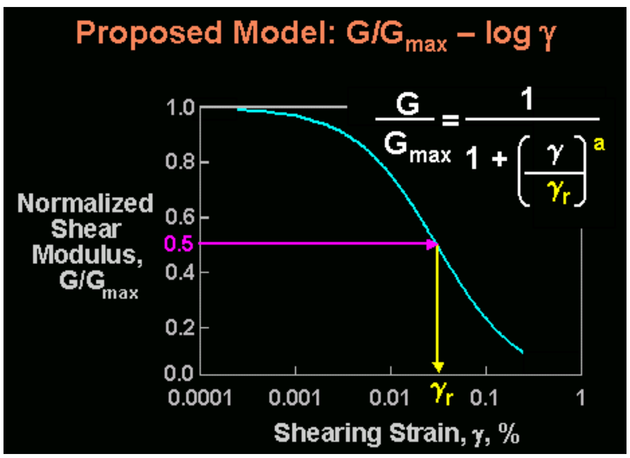
$$D = F^* D_{Masing} + D_{min} \tag{2}$$

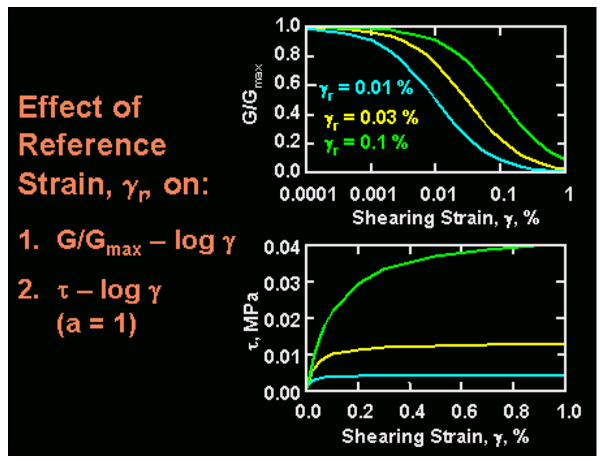
where $$F = b^{*}\left(\frac{G}{G_{max}}\right)^{c} \tag{3}$$

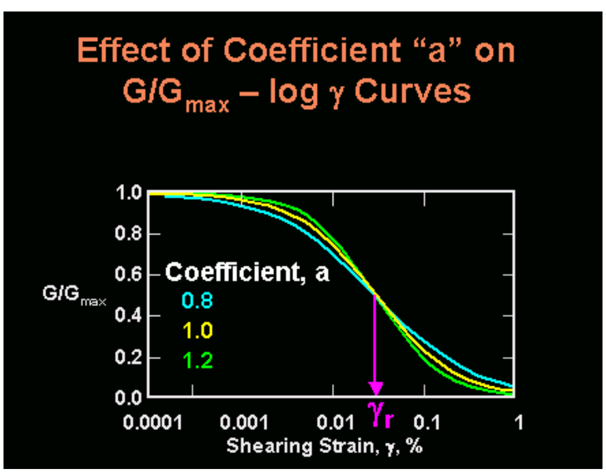
and $$b = \phi_{11} + \phi_{12} * \ln(N) \tag{4}$$

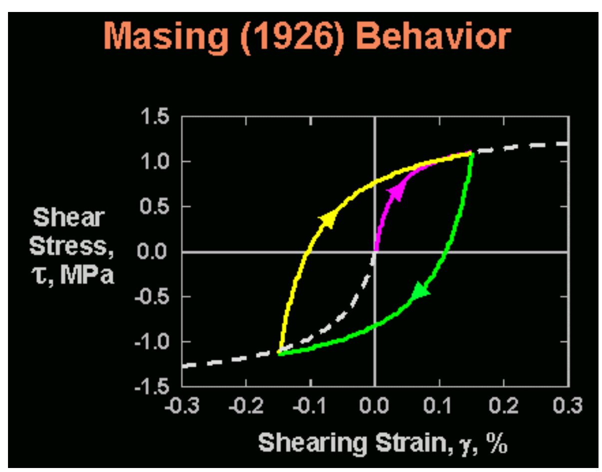
in which N = number of cycles of loading, and $\phi_{11}$ and $\phi_{12}$ are constants. At this time in the model development, "c" is taken as a constant, with a value of 0.1. Work to incorporate variations in this constant is underway.

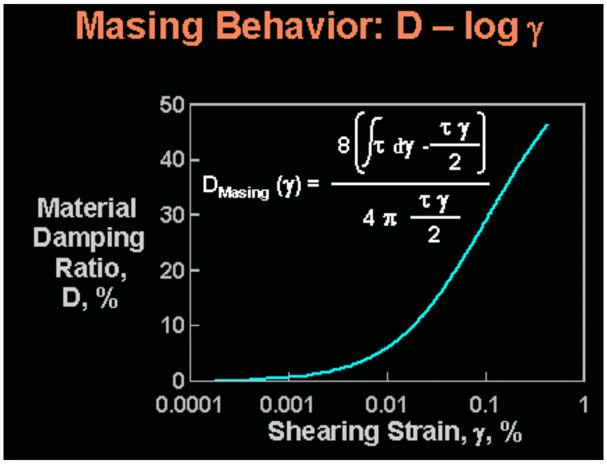
## EFFECTS OF MODEL PARAMETERS

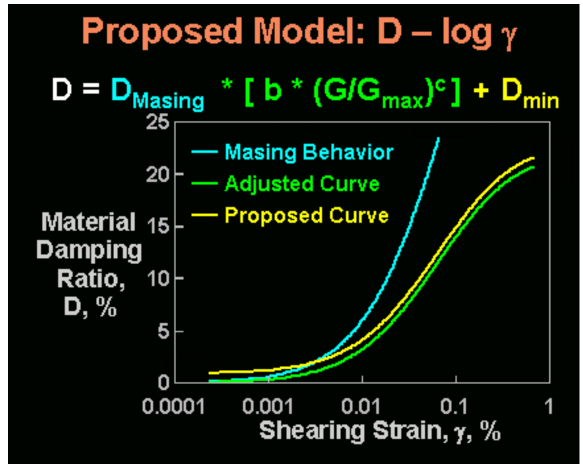


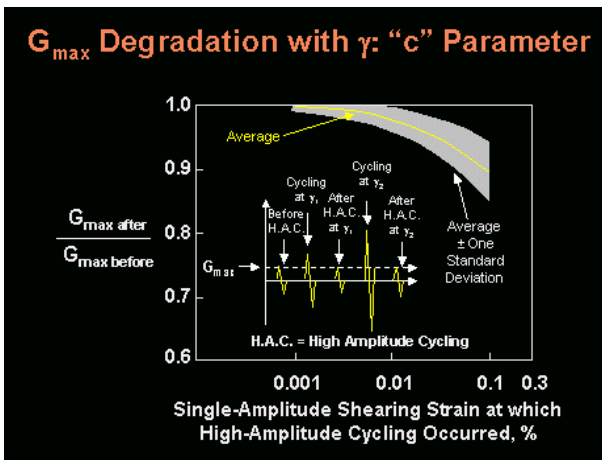






## Relationships Between Five Parameters and Soil Type and Loading Conditions: Plastic Soils

- $\gamma_r$ = f ( PI, OCR, $\sigma_o'$ )
- a = constant = 0.92
- $D_{min}$ = f ( PI, OCR, $\sigma_o'$, f )
- b = f ( N )
- c = constant = 0.10

**EXAMPLE RESULTS**

## Recommended Values: Plastic Soils

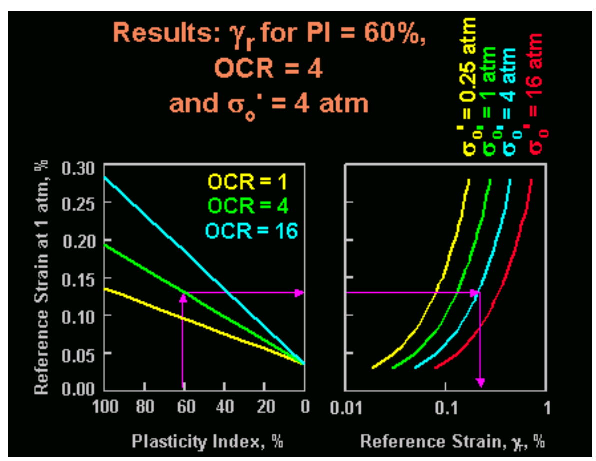
$$\gamma_r = (\phi_1 + \phi_2 * PI * OCR^{\phi_3}) * \sigma_o'^{\,\phi_4}$$

$$a = \phi_5$$

where: $\sigma_o'$ = mean effective confining pressure (atm),
PI = soil plasticity (%),
OCR = overconsolidation ratio,

and

$\phi_1 = 0.0352$, $\phi_2 = 0.0010$, $\phi_3 = 0.3246$,
$\phi_4 = 0.3483$, $\phi_5 = 0.9190$



## Recommended Values: Plastic Soils

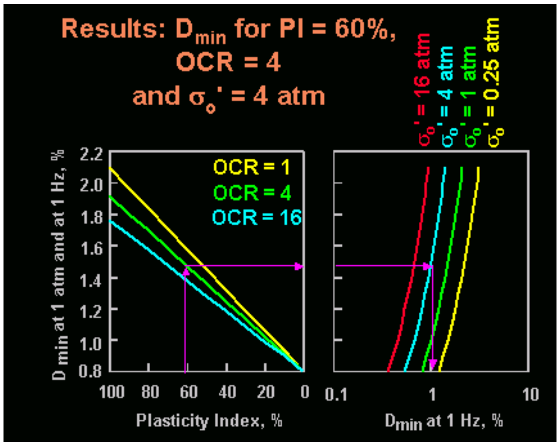
$$D_{min} = (\phi_6 + \phi_7 * PI * OCR^{\phi_8}) * \sigma_o'^{\phi_9} * [1 + \phi_{10} * \ln(f)]$$

$$b = \phi_{11} + \phi_{12} * \ln(N)$$

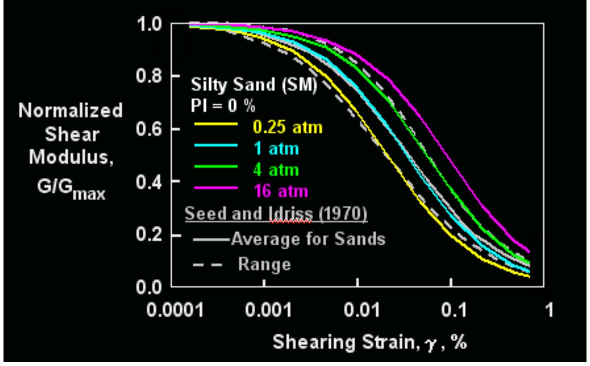
where: $\sigma_o'$ = mean effective confining pressure (atm),
PI = soil plasticity (%),
OCR = overconsolidation ratio,
f = loading frequency,
N = number of loading cycles,

and $\phi_6 = 0.8005$, $\phi_7 = 0.0129$, $\phi_8 = -0.1069$,
$\phi_9 = -0.2889$, $\phi_{10} = 0.2919$, $\phi_{11} = 0.6329$,
$\phi_{12} = -0.0057$

## Results: $D_{min}$ for PI = 60%, OCR = 4 and $\sigma_o'$ = 4 atm




## Effect of $\sigma'_o$ on the Nonlinear Behavior of Sands

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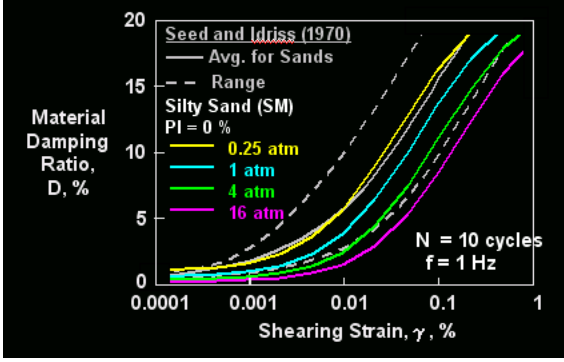
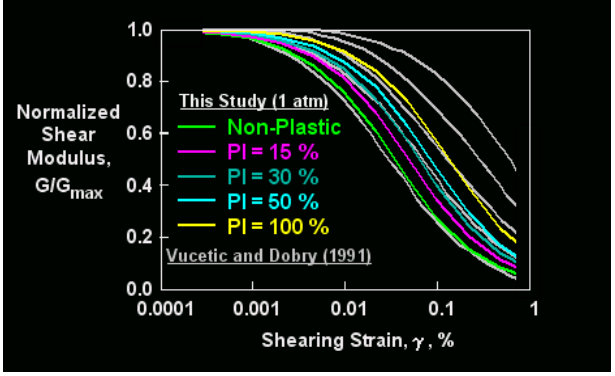
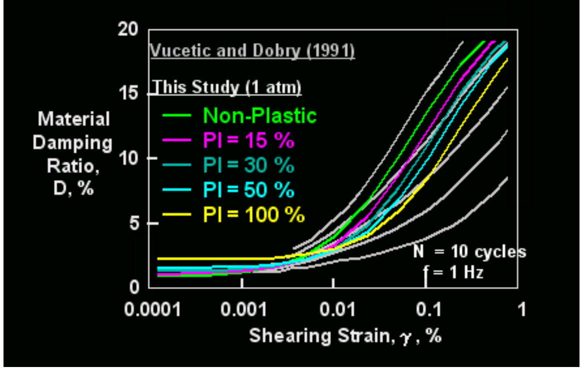
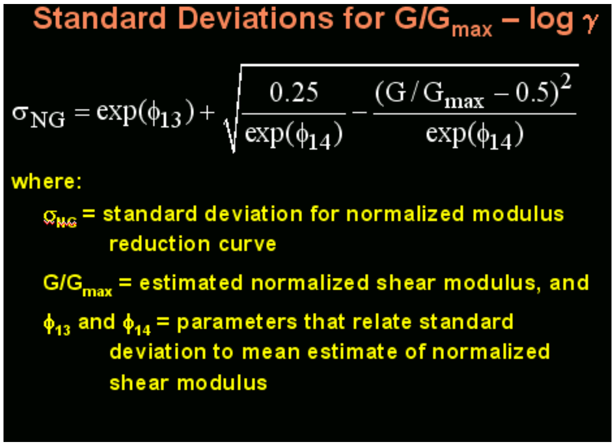


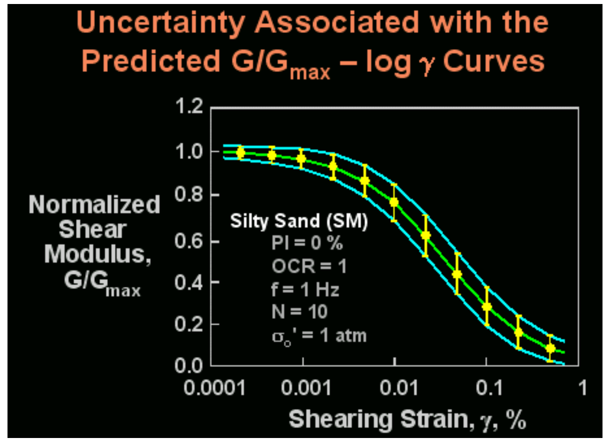

## Effect of Plasticity on Nonlinear Soil Behavior

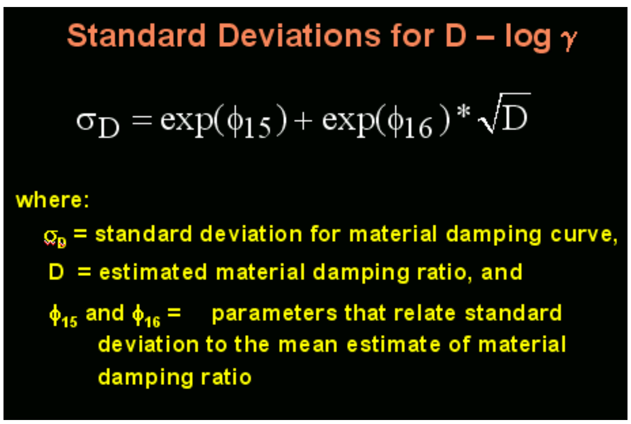


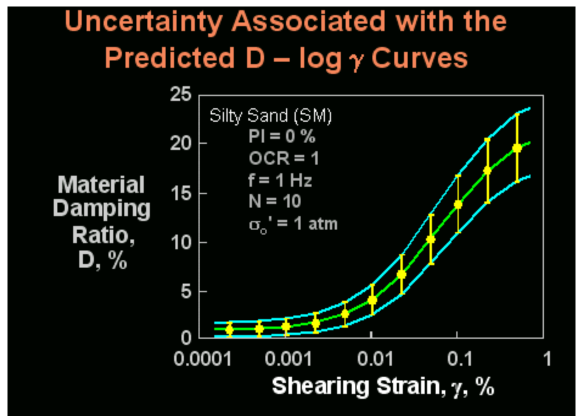

## Effect of Plasticity on Nonlinear Soil Behavior






## DISCUSSION OF GRAVELS, SANDS AND PLASTIC SOILS

Equations 1 through 4, combined with the values of "a", $\gamma_r$, and "b" that have been determined for a wide range in soils, are used to compare the linear and nonlinear dynamic responses of soils ranging from well-graded gravel (GW) to high-plasticity clay (CH). One such comparison is shown in Figure 1 for four different soils. Each soil is confined at an effective isotropic pressure of one atmosphere and is loaded with 10 cycles. The parameters used to characterize the nonplastic granular soils are: (1) median grain size, $D_{50}$, (2) uniformity coefficient, $C_u$, (3) void ratio, e, (4) degree of saturation, $S_r$, and (5) overconsolidated ratio, OCR. The parameters used to characterize the plastic soils (without any sand or gravel particles) are: (1) e, (2) $S_r$, (3) plasticity index, PI and (4) OCR. The values of these parameters are presented in Table 1 along with the values of the parameters used in Equations 1 through 3.

Upon viewing Figure 1, it is clear that there are significant differences in the linear and nonlinear dynamic responses of the four soil types. Some of the differences are explained as follows. First, in Figure 1a, the large value of $D_{50}$ and small value of e result in the well-

Table 1 Parameters Used to Determine the Linear and Nonlinear Responses of GW, SW, CL, and CH Soils Confined at an Effective Isotropic Pressure of 1 atm and Loaded for 10 Cycles. (Response Curves Shown in Figure 1.)

| Soil Type | $\sigma_o'$ (atm) | $D_{50}$ (mm) | $C_u$ | e | $S_r$ (%) | PI (%) | OCR | "a" | $\gamma_r$ (%) | $G_{max}$ (MPa) | "b" | $D_{min}$ (%) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| GW | 1 | 10 | 50 | 0.30[1] | 90 | NP[3] | 1.0 | 0.86 | 0.011 | 306 | 0.62 | 0.50 |
| SW | 1 | 1 | 10 | 0.35[2] | 80 | NP | 1.0 | 0.86 | 0.030 | 183 | 0.62 | 0.80 |
| CL | 1 | NA[4] | NA[4] | 0.64 | 100 | 20 | 1.5 | 0.97 | 0.058 | 81 | 0.62 | 1.05 |
| CH | 1 | NA[4] | NA[4] | 1.12 | 100 | 60 | 1.5 | 1.26 | 0.104 | 58 | 0.62 | 1.54 |

Notes:
1 Void ratio associated with coefficient of uniformity ($C_u$) of 50 and relative density ($D_r$) of about 70 %
2. Void ratio associated with coefficient of uniformity ($C_u$) of 10 and relative density ($D_r$) of about 70 %
3. NP = Nonplastic; 4. NA = Not Applicable

graded gravel (GW) exhibiting the highest value of $G_{max}$. Second, in Figure 1b, the high plasticity clay exhibits the "most linear" response and, hence, the largest value of $\gamma_r$ because of its high PI. On the other hand, the well-graded gravel exhibits the "least linear" response (and lowest value of $\gamma_r$) because it has a large value of $C_u$. Third, in Figure 1c, at small strains, $D_{min}$ of the CH soil is the largest while the GW soil exhibits the smallest value of $D_{min}$. As strain amplitude increases above 0.002%, however, the GW soil exhibits the largest values of D while the CH soil exhibits the smallest values. This order again reverses at shearing strains on the order of 0.2 to 0.5%.

The effect of confining pressure on the G-log $\gamma$, $G/G_{max}$ – log $\gamma$, and D – log $\gamma$ curves is illustrated in Figure 2 for the well-graded sand (SW) and in Figure 3 for the high-plasticity clay (CH). The increase in $G_{max}$ and decrease in $D_{min}$ are clearly shown. The increase in "linearity" with increasing confining pressure is also shown by the shifting of the $G/G_{max}$ – log $\gamma$ curves and D– log $\gamma$ curves to higher strains for the same nonlinear values of $G/G_{max}$ and D. This effect is manifested in increasing value of $\gamma_r$ with increasing effective confining pressure.

Additional effects and comparisons are discussed and shown in the oral presentation, including comparisons with often-used empirical relationships [7 and 8].

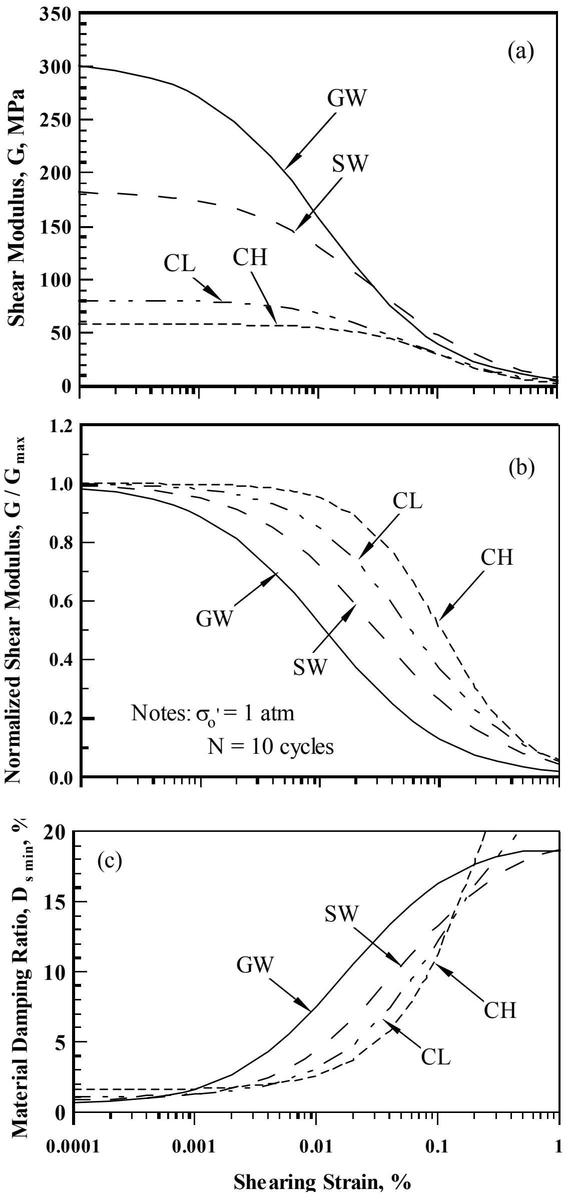



Fig. 1: Comparison of the Linear and Nonlinear Responses of Well-Graded Gravel (GW), Well-Graded Sand (SW), Low-Plasticity Clay (CL), and High-Plasticity Clay (CH) Confined at an Effective Isotropic Pressure of 1.0 atm in Terms of: (a) G – Log $\gamma$, (b) $G/G_{max}$ – Log $\gamma$, and (c) D – Log $\gamma$ Curves.

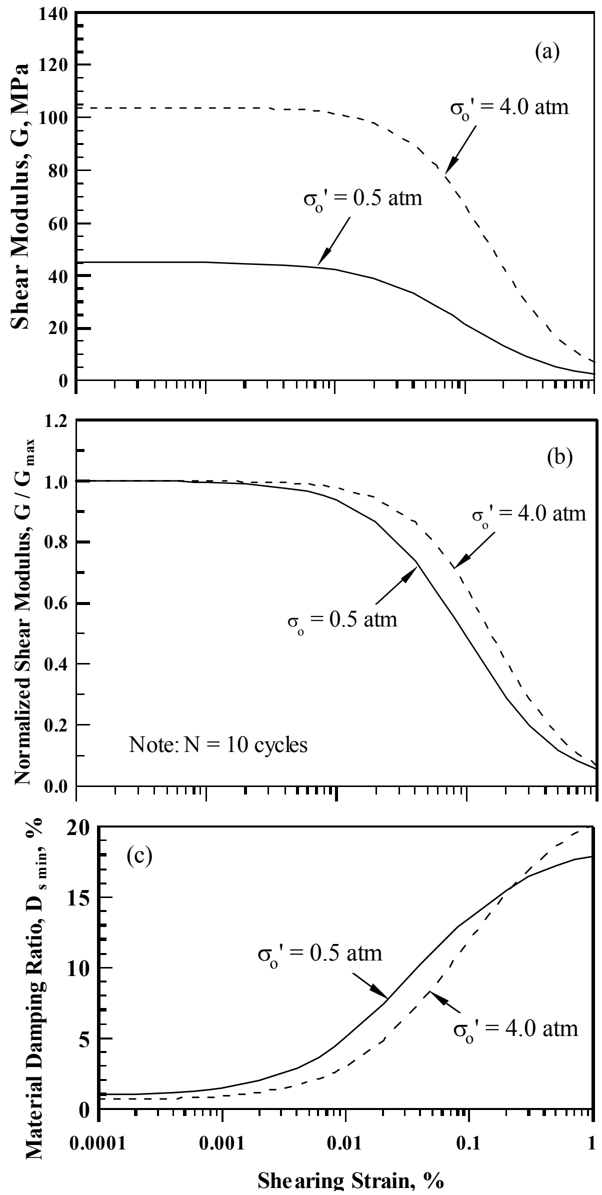



Fig. 2: Comparison of the Linear and Nonlinear Responses of Well-Graded Sand (SW) at Effective Isotropic Confining Pressures of 0.5 and 4.0 atm in Terms of: (a) G – Log $\gamma$, (b) $G/G_{max}$ – Log $\gamma$, and (c) D – Log $\gamma$ Curves.

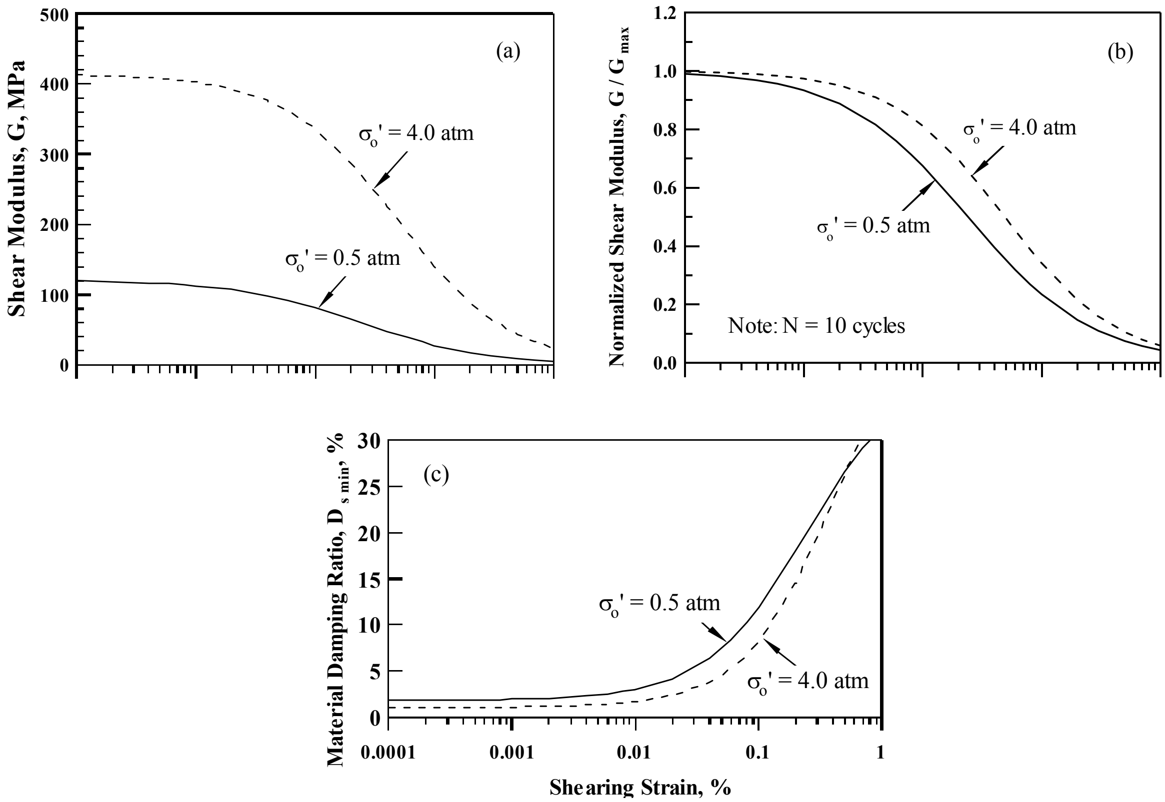
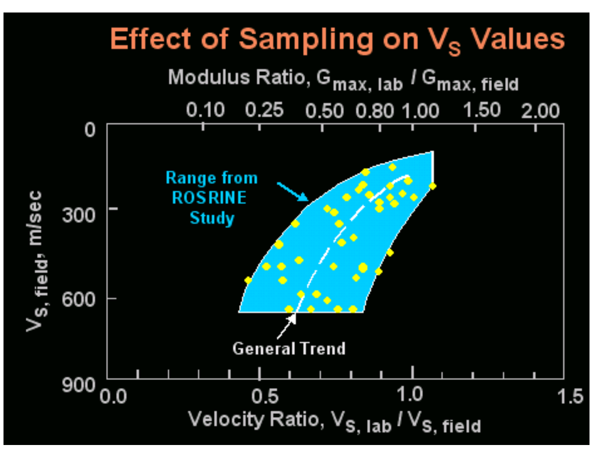
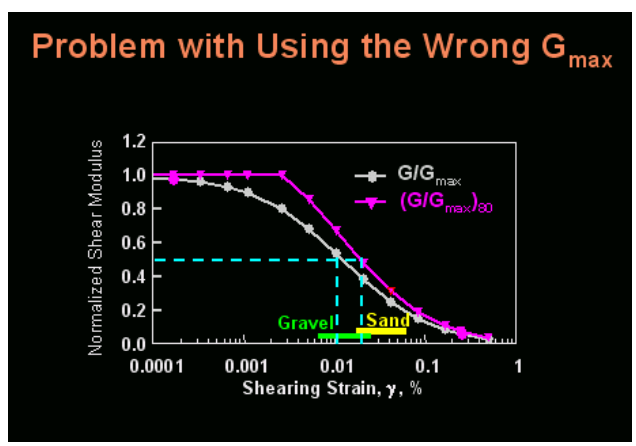



Fig. 3: Comparison of the Linear and Nonlinear Responses of High-Plasticity Clay (CH) at Effective Isotropic Confining Pressures of 0.5 and 4.0 atm in Terms of: (a) G – Log $\gamma$, (b) $G/G_{max}$ – Log $\gamma$, and (c) D – Log $\gamma$ Curves.

## SOME PROBLEM AREAS



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