Basics of Hybrid Simulation

Selim Günay,
Research Scientist, PEER
University of California, Berkeley
Outline

1. Hybrid Simulation (HS) Background
2. HS Classification
3. Benefits of HS
4. Review of HS Related Research
HS Background

Analytical Simulation

+ Experimental Simulation

= Hybrid Simulation
**HS Background**

**Analytical Simulation:**

\[
\begin{bmatrix}
  m_1 & 0 \\
  0 & m_2 \\
\end{bmatrix}
\begin{bmatrix}
  \ddot{u}_1^t \\
  \ddot{u}_2^t \\
\end{bmatrix}
+ \begin{bmatrix}
  -f_2 + f_1 \\
  f_2 \\
\end{bmatrix}
= \begin{bmatrix}
  0 \\
  0 \\
\end{bmatrix}
\]

\[
\ddot{u}_1^t = \dddot{u}_1 + \dddot{g} \\
\ddot{u}_2^t = \dddot{u}_2 + \dddot{g}
\]

\[
\begin{bmatrix}
  m_1 & 0 \\
  0 & m_2 \\
\end{bmatrix}
\begin{bmatrix}
  \dddot{u}_1 \\
  \dddot{u}_2 \\
\end{bmatrix}
+ \begin{bmatrix}
  -f_2 + f_1 \\
  f_2 \\
\end{bmatrix}
= -\begin{bmatrix}
  m_1 \\
  m_2 \\
\end{bmatrix}
\dddot{g}
\]

\[
-f_2 = m_2 \dddot{u}_2^t \\
-f_1 + f_2 = m_1 \dddot{u}_1^t
\]
For the linear-elastic case

\[ f_1 = k_1 u_1 \]
\[ f_2 = k_2 (u_2 - u_1) \]
Damping is commonly represented as a linear combination of mass and stiffness matrices.

\[
C = \alpha M + \beta K
\]

\[
\begin{bmatrix}
    m_1 & 0 \\
    0 & m_2
\end{bmatrix}
\begin{bmatrix}
    \ddot{u}_1 \\
    \ddot{u}_2
\end{bmatrix}
+ \begin{bmatrix}
    c_{11} & c_{12} \\
    c_{21} & c_{22}
\end{bmatrix}
\begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix}
+ \begin{bmatrix}
    k_1 + k_2 & -k_2 \\
    -k_2 & k_1
\end{bmatrix}
\begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix}
= -\begin{bmatrix}
    m_1 \\
    m_2
\end{bmatrix}
\ddot{u}_g
\]
HS Background

Analytical Simulation:

Objective of Analytical Simulation: Solve the equation of motion using numerical integration methods
HS Background

A straightforward integration application: Explicit Newmark Integration

1) Compute the displacements \( u_{i+1} = u_i + \Delta t \dot{u}_i + \frac{(\Delta t)^2}{2} \ddot{u}_i \)

2) Compute the restoring forces \( f_{i+1} \) corresponding to \( u_{i+1} \)

3) Compute the accelerations \( [m + \Delta t \gamma c] \ddot{u}_{i+1} = p_{i+1} - f_{i+1} - c[\dot{u}_i + \Delta t(1-\gamma)\ddot{u}_i] \)

4) Compute the velocities \( \dot{u}_{i+1} = \dot{u}_i + \Delta t[(1-\gamma)\ddot{u}_i + \gamma \ddot{u}_{i+1}] \)

5) Increment \( i \)
Bottom spring replaced with a test specimen

\[
\begin{bmatrix}
    m_1 & 0 \\
    0 & m_2
\end{bmatrix}
\begin{bmatrix}
    \ddot{u}_1 \\
    \ddot{u}_2
\end{bmatrix}
+ \begin{bmatrix}
    c_{11} & c_{12} \\
    c_{21} & c_{22}
\end{bmatrix}
\begin{bmatrix}
    \dot{u}_1 \\
    \dot{u}_2
\end{bmatrix}
+ \begin{bmatrix}
    -f_a + f_e \\
    f_a
\end{bmatrix}
= - \begin{bmatrix}
    m_1 \\
    m_2
\end{bmatrix} \ddot{u}_g
\]
A straightforward integration application: Explicit Newmark Integration

1) Compute the displacements
\[ u_{i+1} = u_i + \Delta t \ddot{u}_i + \frac{(\Delta t)^2}{2} \dddot{u}_i \]

2a) Compute the restoring force \( f_{a,i+1} \) corresponding to the displacement \( u_{2,i+1} - u_{1,i+1} \)

2b) Impose \( u_{1,i+1} \) to the test specimen and measure the corresponding force \( f_{e,i+1} \)

3) Compute the accelerations
\[
\begin{bmatrix}
\mathbf{m} + \Delta t \gamma \mathbf{c}
\end{bmatrix} \ddot{u}_{i+1} = \mathbf{p}_{i+1} - \mathbf{f}_{i+1} - \mathbf{c} \left[ \dddot{u}_i + \Delta t (1 - \gamma) \dddot{u}_i \right]
\]
\[
\mathbf{m}_{\text{eff}} \ddot{u}_{i+1} = \mathbf{p}_{\text{eff}}
\]

4) Compute the velocities
\[ \dot{u}_{i+1} = \dot{u}_i + \Delta t \left[ (1 - \gamma) \ddot{u}_i + \gamma \dddot{u}_{i+1} \right] \]

5) Increment i
HS Classification

- Slow Hybrid Simulation
- Real-time Hybrid Simulation
  - Actuator Configuration
  - Shaking Table Configuration
  - Actuator + Shaking Table Configuration
Required Loading < Computed velocity
Duration of Hybrid Simulation > N × Δt
N: number of integration steps
Δt: integration time step
Applicable when rate effects are not important
Experimental substructure is connected to actuator(s)
Physical mass generally doesn’t exist
From the experimental perspective, slow hybrid simulation is equivalent to quasi-static testing.

Predetermined displacement commands are based on a load protocol.
From the experimental perspective, slow hybrid simulation is equivalent to quasi-static testing.
Real-time Hybrid Simulation (Actuator Configuration)

- Rate of loading = Computed velocity
- Duration of hybrid simulation = $N \times \Delta t$
  - $N$: number of integration steps
  - $\Delta t$: integration time step
- Crucial when rate effects are important
- Experimental substructure is connected to actuator(s)
- Physical mass generally doesn’t exist
Same quasi-static test setup can be used for real-time HS as long as proper hardware exists, e.g. dynamic actuators, digital controllers, etc.

\[ c = \alpha m \]

- **Computational platform:** DSP
- **Controller**
- **Test Specimen**
- **DAQ**
- Hybrid simulation: Displacement commands determined during the test
- **Computed Displacement**

[Diagram showing the setup with labeled components: Controller, Test Specimen, Hybrid simulation, Computed Displacement, DAQ, Computational platform: DSP.]
HS Classification

- Real-time Hybrid Simulation (Shaking Table Configuration)

- Experimental substructure is located on a shaking table
- Physical mass generally exists
- Rate of loading = Computed velocity
- Duration of hybrid simulation = $N \times \Delta t$
  
  - $N$: number of integration steps
  - $\Delta t$: integration time step
- Crucial when rate effects are important
From the experimental perspective, RTHS in a shaking table configuration is equivalent to conventional shaking table testing.
From the experimental perspective, RTHS in a shaking table configuration is equivalent to conventional shaking table testing.
HS Classification

- Real-time Hybrid Simulation (Actuator + Shaking Table Configuration)

- Experimental substructure is located on a shaking table and connected to an actuator via a conductor cable.
Benefits of HS

Convenience in mass modeling

Shaking Table

Hybrid Simulation

$u_1, m_1, I_m$
Benefits of HS

Convenience in system level testing
Benefits of HS

Convenience in mass modeling
Benefits of HS

Convenience in full scale testing
Benefits of HS

Time efficiency due to elimination of physical construction

Experimentalsubstructure

Analyticalsubstructure

Details in the
HS
application
lecture
Benefits of HS

Economical Convenience

- Realism of Dynamic Response Evaluation
- Test cost
- Shaking table
- HS
- Quasi-Static
Benefits of HS

- Nature of the problem requires substructuring
- Presence of experimental substructures require the use of special integration methods
- Presence of a transfer system introduce simulation errors
- Rate dependent materials require real-time hybrid simulation (RTHS)
- Making use of multiple labs extend the method to geographically distributed testing

40 years of extensive research on various aspects of Hybrid Simulation
CASE 1: CANTILEVER COLUMN with MASS [No MASS MOMENT of INERTIA or ANALYTICAL SUBSTRUCTURE]

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 1: CANTILEVER COLUMN with MASS [No MASS MOMENT of INERTIA or ANALYTICAL SUBSTRUCTURE]

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 2: CANTILEVER COLUMN with MASS and MASS MOMENT of INERTIA [No ANALYTICAL SUBSTRUCTURE]

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 2: CANTILEVER COLUMN with MASS and MASS MOMENT of INERTIA [No ANALYTICAL SUBSTRUCTURE]

Red : Experimental
Blue: Analytical
Case 3: Two Columns without Analytical Substructure

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 3: TWO COLUMNS without ANALYTICAL SUBSTRUCTURE

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 4: TWO COLUMNS with an EXPERIMENTAL and an ANALYTICAL SUBSTRUCTURE

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 4: TWO COLUMNS with an EXPERIMENTAL and an ANALYTICAL SUBSTRUCTURE

Red: Experimental
Blue: Analytical
CASE 4-1: TWO COLUMNS with an EXPERIMENTAL and an ANALYTICAL SUBSTRUCTURE

Red: Experimental
Blue: Analytical
CASE 4-1: TWO COLUMNS with an EXPERIMENTAL and an ANALYTICAL SUBSTRUCTURE

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 4-2: TWO COLUMNS with an EXPERIMENTAL and an ANALYTICAL SUBSTRUCTURE

Red: Experimental
Blue: Analytical

Spring with a lateral force-deformation relation
CASE 4-2: TWO COLUMNS with an EXPERIMENTAL and an ANALYTICAL SUBSTRUCTURE

Red : Experimental
Blue: Analytical

Spring with a lateral force-deformation relation
Substructuring Cases

CASE 5: PORTAL FRAME with ONE OF THE COLUMNS AND BEAM AS ANALYTICAL SUBSTRUCTURE

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 5: PORTAL FRAME with ONE OF THE COLUMNS AND BEAM AS ANALYTICAL SUBSTRUCTURE

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 6: MULTI-BAY MULTI-STORY FRAME with ANALYTICAL SUBSTRUCTURING

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 6: MULTI-BAY MULTI-STORY FRAME with ANALYTICAL SUBSTRUCTURING

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 6-1: MULTI-BAY MULTI-STORY FRAME with ANALYTICAL SUBSTRUCTURING

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 6-1: MULTI-BAY MULTI-Story FRAME with ANALYTICAL SUBSTRUCTURING

Red: Experimental
Blue: Analytical
Integration Methods

Analytical Simulation + Experimental Simulation = Hybrid Simulation

All the integration methods developed for analytical simulations are not suitable for hybrid simulation.

Example: The most common and standard integration method for analytical simulation, Implicit Newmark Integration.
Equilibrium and difference equations represent a nonlinear system of equations, 
\[ f(u_{i+1}) = p_{i+1} - m \ddot{u}_{i+1} - c \dot{u}_{i+1} - p_r(u_{i+1}) = 0 \]
which can be solved using iterative methods such as Newton-Raphson method
\[ f'(u_{i+1}^k) \Delta u_{i+1}^k = -f(u_{i+1}^k) \]
Integration Methods

Iterations of Implicit Newmark are not suitable for hybrid simulation:

- Iterations may not converge
- Displacement overshoot: artificial unloading
- Nonuniform displacement increments: velocity and acceleration oscillations within the step
Integration Methods

HS compatible alternative integrators

- Explicit Newmark Integration
- Operator Splitting Method
- Implicit Newmark Integration with Fixed Number of Iterations

Do not require iterations
Simulation Errors

\[
\begin{bmatrix}
m_1 & 0 \\
0 & m_2
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_1 \\
\ddot{u}_2
\end{bmatrix}
+ \begin{bmatrix}
c_{11} & c_{12} \\
c_{21} & c_{22}
\end{bmatrix}
\begin{bmatrix}
\dot{u}_1 \\
\dot{u}_2
\end{bmatrix}
+ \begin{bmatrix}
-f_a + f_e \\
-f_a
\end{bmatrix}
= -\begin{bmatrix}
m_1 \\
m_2
\end{bmatrix}
\ddot{u}_g
\]

1. Apply \( u_{1,i+1} \) to the test specimen
2. Measure the corresponding force \( f_{e,i+1} \)

- Reliability of a hybrid simulation depends on the accuracy of \( f_e \)
- All the errors that occur during stages 1 and 2 are experimental errors and affect hybrid simulation
Simulation Errors

Random errors:
- They have no distinguishable pattern and generally no specific physical effects can be anticipated.
- **Examples:**
  1. Random electrical noise in wires and electronic systems
  2. Random rounding-off or truncation in the A/D conversion of electrical signals
- They do not introduce significant errors to hybrid simulation.
Simulation Errors

Experimental systematic errors:

- They may lead to error propagation and numerical instability

- **Examples:**
  1. Measurement errors
  2. Hybrid simulation technique (ramp and hold, continuous, real-time)
  3. Servo-hydraulic closed control loop

1. Measurement errors

  - Errors in load cells & displacement transducers of actuators due to:
    a. Calibration
    b. Friction or slop in the attachments
    c. A/D and D/A conversions
Simulation Errors

Control-loop errors (Errors in displacement tracking): Demonstration of the effect of control-loop errors

- Overshooting
- Undershooting or Delay
- Increased Damping
- Negative Damping & Instability

Graphs showing command and feedback displacement over time, with arrows indicating overshooting and undershooting.
Simulation Errors

Control-loop errors: Demonstration tests

T = 0.5 sec
ζ = 5%
Simulation Errors

Control-loop errors: Demonstration tests

\[ T = 0.5 \text{ sec} \]
\[ \zeta = 5\% \]
Simulation Errors

Control-loop errors: Demonstration tests

No time delay

![Graph showing displacement over time with command and feedback lines, comparing analytical and hybrid simulations.](image)
Simulation Errors

14 msec time delay introduced artificially by adjusting the feed-forward gain.
Simulation Errors

Control loop errors: Error identification using free vibration

**Step 1:** Push the hybrid structure, generally in the first mode, to a displacement within the linear range

**Step 2:** Run the free vibration hybrid simulation test from this displaced configuration
Simulation Errors

Control loop errors: Error identification using free vibration

- No error
- Overshoot
- Undershoot or Lag

Computed displacement [in.]

Pushover Free Vibration

Time [sec]
Simulation Errors

Methods to Reduce the Effects of Errors

- Error Compensation Methods
- Integration Methods with Numerical Damping
- Tuning
- Advanced Control Methods
HS Related Research

Geographically Distributed HS

Substructure A
Computations in Berkeley

Lab 1 in The Americas

Substructure C

Lab 2 in Asia

Substructure B

Lab 3 in Europe

Substructure D

Lab 4 in Australia
Geographically Distributed HS

Geographically distributed HS test between nees@berkeley and UNIKA, Germany

**Experimental substructure:** Friction device and a fixed tuned-mass-damper @UNIKA

**Analytical substructure:** SDOF mass with viscous damping @Berkeley

**OpenFresco:** The Open-source Framework for Experimental Setup and Control
http://openfresco.berkeley.edu/
Real-time Hybrid Simulation (RTHS)

- **Requirement for real time:**
  Loading rate = Computed velocity

- Slow HS: Sufficient for most cases when rate effects are not important.

- RTHS: Essential for rate-dependent materials and devices, e.g. viscous dampers, friction pendulum isolators or polymer insulators.
Use of HS for Testing of Electrical Equipment

- Electrical equipment in substations are typically mounted on support structures to provide sufficient clearance of the ground, and to integrate them into the design of the substation.
- Support structures are generally steel frames with well defined geometry and material properties. Therefore they are suitable to be modeled in the computer as analytical substructure.
- Electrical equipment generally have complex geometry and material properties with larger uncertainty.
- HS provides an effective, efficient and economic testing opportunity by combining the electrical equipment testing with support structure modeling.
Use of HS for Testing of Electrical Equipment

HS provides an effective, efficient and economic testing opportunity by testing of the electrical equipment and modeling of the support structures.

1. Application I: Evaluation of the Effect of Support Structure Stiffness and Damping on Porcelain and Polymer Insulators
2. Application II: Full Disconnect Switch Tests in Open and Closed Configurations
3. Application III: Testing of Interconnected Equipment
Thank you!