Substructuring, Integration Methods and Simulation Errors

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Outline

- Introduction
- Substructuring
- Integration Methods
- Simulation Errors
- Geographically distributed Hybrid Simulation
- Real-time Hybrid Simulation
Introduction

Laboratory Testing in Structural & Earthquake Engineering

- Shaking Table Testing
- Quasi-Static Testing
- Hybrid Simulation
Introduction

Realism of Dynamic Response Evaluation

Test cost

Quasi-Static

HS

Shaking table
Nature of the problem requires substructuring

Presence of experimental substructures require the use of special integration methods

Presence of a transfer system introduce simulation errors

Rate dependent materials require real-time hybrid simulation (RTHS)

Making use of multiple labs extend the method to geographically distributed testing
Analytical substructures are generally those that can be modeled with confidence.

Experimental substructures are those that are difficult to model due to lack of prior data, complicated geometry, material inelastic behavior, boundary conditions, etc.
Substructuring

- Analytical substructures can vary between no analytical substructure (only mass and damping modeling) to analytical substructures with many degrees of freedom

- Substructuring and the boundary conditions between analytical and experimental substructures define the experimental degrees of freedom and how the experimental degrees of freedom will be controlled
CASE 1: CANTILEVER COLUMN with MASS [No MASS MOMENT of INERTIA or ANALYTICAL SUBSTRUCTURE]

Red : Experimental
Blue: Analytical
Substructuring Cases

CASE 1: CANTILEVER COLUMN with MASS [No MASS MOMENT of INERTIA or ANALYTICAL SUBSTRUCTURE]

Red : Experimental
Blue: Analytical
Substructuring Cases

CASE 2: CANTILEVER COLUMN with MASS and MASS MOMENT of INERTIA [No ANALYTICAL SUBSTRUCTURE]

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 2: CANTILEVER COLUMN with MASS and MASS MOMENT of INERTIA [No ANALYTICAL SUBSTRUCTURE]

Red : Experimental
Blue: Analytical
Substructuring Cases

CASE 3: TWO COLUMNS without ANALYTICAL SUBSTRUCTURE

Red: Experimental
Blue: Analytical

\[ u_1 \]

\[ u_2 \]

\[ m_1 \]

\[ m_2 \]
Substructuring Cases

CASE 3: TWO COLUMNS without ANALYTICAL SUBSTRUCTURE

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 4: TWO COLUMNS with an EXPERIMENTAL and an ANALYTICAL SUBSTRUCTURE

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 4: TWO COLUMNS with an EXPERIMENTAL and an ANALYTICAL SUBSTRUCTURE

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 4-1: TWO COLUMNS with an EXPERIMENTAL and an ANALYTICAL SUBSTRUCTURE

Red: Experimental
Blue: Analytical

\[ u_1 \quad m_1 \quad u_3 \quad m_2 \quad u_2 \]
CASE 4-1: TWO COLUMNS with an EXPERIMENTAL and an ANALYTICAL SUBSTRUCTURE

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 4-2: TWO COLUMNS with an EXPERIMENTAL and an ANALYTICAL SUBSTRUCTURE

- Red: Experimental
- Blue: Analytical

Spring with a lateral force-deformation relation
Substructuring Cases

CASE 4-2: TWO COLUMNS with an EXPERIMENTAL and an ANALYTICAL SUBSTRUCTURE

Red: Experimental
Blue: Analytical

Spring with a lateral force-deformation relation

\[ m_1 \quad \text{and} \quad m_2 \]

\[ u_2 - u_1 \]
CASE 5: PORTAL FRAME with ONE OF THE COLUMNS AND BEAM AS ANALYTICAL SUBSTRUCTURE

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 5: PORTAL FRAME with ONE OF THE COLUMNS AND BEAM AS ANALYTICAL SUBSTRUCTURE

Red : Experimental
Blue: Analytical
Substructuring Cases

CASE 6: MULTI-BAY MULTI-STORY FRAME with ANALYTICAL SUBSTRUCTURING

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 6: MULTI-BAY MULTI-Story FRAME with ANALYTICAL SUBSTRUCTURING

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 6-1: MULTI-BAY MULTI-STORY FRAME with ANALYTICAL SUBSTRUCTURING

Red: Experimental
Blue: Analytical
Substructuring Cases

CASE 6-1: MULTI-BAY MULTI-STORY FRAME with ANALYTICAL SUBSTRUCTURING

Red: Experimental

Blue: Analytical
Integration Methods

Analytical Simulation + Experimental Simulation = Hybrid Simulation

All the integration methods developed for analytical simulations are not suitable for hybrid simulation.

Example: The most common and standard integration method for analytical simulation, Implicit Newmark Integration.
Integration Methods

Implicit Newmark Integration

\[
p_{i+1} - m\ddot{u}_{i+1} - c\dot{u}_{i+1} - p_r(u_{i+1}) = 0
\]

\[
u_{i+1} = \nu_i + \Delta t \dot{\nu}_i + \frac{(\Delta t)^2}{2} \left[ (1 - 2\beta)\ddot{\nu}_i + 2\beta \ddot{\nu}_{i+1} \right]
\]

\[
\dot{\nu}_{i+1} = \dot{\nu}_i + \Delta t \left[ (1 - \gamma)\ddot{\nu}_i + \gamma \ddot{\nu}_{i+1} \right]
\]

\(\gamma\) and \(\beta\) define the variation of accelerations over a time step:

- \(\gamma = 1/2\) and \(\beta = 1/4\) : constant average acceleration
- \(\gamma = 1/2\) and \(\beta = 1/6\) : linear acceleration

Equilibrium and difference equations represent a nonlinear system of equations,

\[
f(u_{i+1}) = p_{i+1} - m\ddot{u}_{i+1} - c\dot{u}_{i+1} - p_r(u_{i+1}) = 0
\]

which can be solved using iterative methods such as Newton-Raphson method

\[
f'(u^k_{i+1})\Delta u^k_{i+1} = -f(u^k_{i+1})
\]
Iterations of Implicit Newmark are not suitable for hybrid simulation:

- Iterations may not converge
- Iterations result in artificial loading and unloading
- Nonuniform displacement increments: velocity and acceleration oscillations within the step
Integration Methods

HS compatible integrators

- Explicit Newmark Integration
- Operator Splitting Method
- Implicit Newmark Integration with Fixed Number of Iterations

Do not require iterations
Explicit Newmark

Algorithm

1) Compute the displacements

\[ u_{i+1} = u_i + \Delta t \dot{u}_i + \frac{1}{2} \Delta t^2 \ddot{u}_i \]

2) Impose the displacements \( u_{i+1} \) and compute/measure forces \( p_r(u_{i+1}) \)

3) Compute the accelerations

\[ \left[ \mathbf{m} + \Delta t \gamma \mathbf{c} \right] \ddot{u}_{i+1} = p_{i+1} - p_r(u_{i+1}) - c\left[ \ddot{u}_i + \Delta t (1 - \gamma) \dddot{u}_i \right] \]

\[ m_{\text{eff}} \ddot{u}_{i+1} = p_{\text{eff}} \]

4) Compute the velocities

\[ \dot{u}_{i+1} = \dot{u}_i + \Delta t \left[(1 - \gamma) \dddot{u}_i + \gamma \dddot{u}_{i+1}\right] \]
Explicit Newmark

- Becomes explicit by setting $\beta = 0$
- Initial and tangent stiffness matrices are not required
  \[
  [m + \Delta t \gamma c]\ddot{u}_{i+1} = p_{i+1} - p_r(u_{i+1}) - c[\dot{u}_i + \Delta t(1 - \gamma)\ddot{u}_i]
  \]
- Iterations are not required
- Computationally very efficient
  \[
  [m + \Delta t \gamma c]\ddot{u}_{i+1} = p_{i+1} - p_r(u_{i+1}) - c[\dot{u}_i + \Delta t(1 - \gamma)\ddot{u}_i]
  \]
  \[
  m_{\text{eff}}\ddot{u}_{i+1} = p_{\text{eff}}
  \]
  - $m_{\text{eff}}$ is constant during the solution
  - $m_{\text{eff}}$ needs to be factored only once
- No numerical damping is present to suppress higher-mode participation.
Explicit Newmark

- Conditionally stable: $\frac{dt}{T_n} < \Pi$

- Not applicable to structures with massless DOF

- If stability condition is satisfied, most suitable method for both slow and real-time hybrid simulation
Alpha – Operator Splitting (OS) Method

- Iterations are not required (one predictor, one corrector)
- Eliminates the iterations by ignoring the $\ddot{u}_{i+1}$ term in the displacement difference equation
- Tangent stiffness matrix is not required

$$m_{\text{eff}}\ddot{u}_{i+1} = p_{\text{eff}}$$

$$m_{\text{eff}} = m + (1 - \alpha)\Delta t \gamma c + \Delta t^2 \beta(1 + \alpha)K^I$$

$$p_{\text{eff}} = (1 - \alpha)p_{i+1} + \alpha p_i - (1 - \alpha)p_r(\ddot{u}_{i+1}) - \alpha p_r(\ddot{u}_i) - \left[(1 - \alpha) c \Delta t (1 - \gamma) + \alpha \Delta t^2 \beta K^I\right] \ddot{u}_i - c \ddot{u}_i$$

- Computationally efficient
  - $m_{\text{eff}}$ is constant during the solution
  - $m_{\text{eff}}$ needs to be factored only once
- Unconditionally stable for softening response
- Numerical damping is present
- Quite suitable for slow and real-time hybrid simulation
Implicit Newmark Integration with Fixed Number of Iterations

- Uniform displacement increments between iterations
- Number of iterations constant → No convergence problems
- Number of iterations should be determined with prior analyses
- Very suitable for slow hybrid simulation
- Restricted use in real-time hybrid simulation

- Integration time step = 0.01 sec
- # of iterations = 10

Each iteration needs to be physically completed in 1 msec
Not always possible
Simulation Errors

ERROR SOURCES

- Errors due to Structural Modeling
- Errors due to Numerical Methods
- Experimental Errors
Errors due to Structural Modeling

- Structural Idealization
  - Discrete parameter system
  - Lumped mass vs Consistent Mass Matrices

- Modeling Damping
  - Viscous Damping is represented analytically
  - Spurious moments may be introduced when initial stiffness proportional damping is used
Errors due to Numerical Methods

- Use of initial stiffness matrix instead of tangent stiffness matrix for experimental substructures may lead to errors.

- Experimental errors increase with the number of iterations.

- Higher modes are more susceptible to error propagation than the lower modes. Therefore numerical damping is desirable for suppressing higher mode errors.

- However, numerical damping may lead to errors if it reduces lower mode participation.
Experimental Errors

1. Apply $u_{1,i+1}$ to the test specimen
2. Measure the corresponding resisting force $f_{e,i+1}$

- Reliability of a hybrid simulation depends on the accuracy of $f_c$
- All the errors that occur during stages 1 and 2 are experimental errors and affect hybrid simulation
Experimental Errors

Random errors:

- They have no distinguishable pattern and generally no specific physical effects can be anticipated.

- **Examples:**
  1. Random electrical noise in wires and electronic systems
  2. Random rounding-off or truncation in the A/D conversion of electrical signals

- They do not introduce significant errors to hybrid simulation.
Experimental Errors

Systematic errors:

- They may lead to error propagation and numerical instability
- **Examples:**
  1. Measurement errors
  2. Hybrid simulation technique (ramp and hold, continuous, real-time)
  3. Servo-hydraulic closed control loop

1. Measurement errors

  - Errors in load cells & displacement transducers of actuators due to:
    a. Calibration
    b. Friction or slop in the attachments
    c. A/D and D/A conversions
Experimental Errors

Hybrid Simulation Technique

Ramp and Hold Method

- Force relaxation during hold phase

- Force

Actuator Disp.

Computation duration

Ramp

Hold

Ramp

Computation duration

Time

Disp
Experimental Errors

Hybrid Simulation Technique

Continuous Testing (Predictor-Corrector Algorithms)
Control-loop errors

- Overshooting
- Undershooting or Delay
- Negative Damping & Instability

Experimental Errors

Displacement 

Displacement 

Displacement 

Command 

Feedback 

Time delay

Increased Damping
Simulation Errors

Methods to Reduce the Effects of Errors

- Tuning
- Advanced Control Methods
- Integration Methods with Numerical Damping
- Error Compensation Methods
Geographically Distributed HS

- Lab 1 in The Americas
- Lab 2 in Asia
- Lab 3 in Europe
- Lab 4 in Australia
Real-time Hybrid Simulation (RTHS)

- **Requirement for real time:**
  
  Loading rate = Computed velocity

- **Slow HS:** Sufficient for most cases when rate effects are not important.

- **RTHS:** Essential for rate-dependent materials and devices, e.g. viscous dampers, friction pendulum isolators or polymer insulators.
Thank you!
Geographically Distributed HS

Geographically distributed HS test between nees@berkeley and UNIKA, Germany

**Experimental substructure:**
Friction device and a fixed tuned-mass-damper @UNIKA

**Analytical substructure:**
SDOF mass with viscous damping @Berkeley

**OpenFresco:** The Open-source Framework for Experimental Setup and Control
http://openfresco.berkeley.edu/