Seismic Performance Assessment Analysis and its Uses Allin Cornell Stanford University



Objectives

- What can SPAA do for us? Or rather what do we need?
- Use SPAA to get a limit state annual probability result
- Use it to help motivate the basis of the Framing Equation
- Show its relationship to the needs



The Framing Equation

$$\lambda_{DV}(z) = \iint_{y x} G_{DV|DM}(z | y) | dG_{DM|IM}(y | x) | | d\lambda_{IM}(x) |$$

 $\lambda_{DV}(z)$ = Mean Annual Frequency of $DV \ge z$

- **DV** Decision Variable(s) (costs, lives lost, collapse limit states, ...)
- **DM** Damage Measure(s) (displacements, fractures, ...)
- IM Ground Motion Intensity Measure(s) (PGA, S_a, ...)



PBEE Assessment Needs: Can SPA-Analysis Help?

- Limit State (e.g., collapse) Probabilities
- Familiar format(s) for checking that limit state probabilities meet objectives.

 $\phi \cdot \mathbf{C} = \gamma \cdot \mathbf{D}$

• Economic Loss Assessment



Elements of the Assessment





Ground Motion Hazard





Dynamic Response vs. Ground Motion Intensity





Dynamic Global Instability Capacity





0.06

0.03

Limit State Probability

$$\lambda_{\rm f} = \lambda P[D > C]$$

$$\lambda_{f} = \lambda \int P[D > C | s] f_{S_{a}}(s) ds$$

$$\lambda f_{S_a}(s)ds = \left|\frac{d\lambda_{S_a}(s)}{ds}\right| ds = \left|d\lambda_{S_a}(s)\right|$$



Limit State Probability (cont'd)

$$\lambda_{f} = \int P[D > C | s] | d\lambda_{S_{a}}(s) |$$

$$\lambda_{f} = \iint F_{C}(x) f_{D|S_{a}}(x \mid s) ds \mid d\lambda_{S_{a}}(s) \mid$$



Limit State Probability (cont'd)

$$\lambda_{f} = \iint F_{C}(x) f_{D|S_{a}}(x \mid s) ds \mid d\lambda_{S_{a}}(s) \mid$$
$$= \iint F_{C}(x) \mid dG_{D|S_{a}}(x \mid s) \mid d\lambda_{S_{a}}(s) \mid$$



Lim. State Prob.& Framing Equation

$$\lambda_{f} = \iint F_{C}(x) | dG_{D|S_{a}}(x | s) | | d\lambda_{S_{a}}(s) |$$

$$\lambda_{DV}(z) = \iint_{y x} G_{DV|DM}(z | y) | dG_{DM|IM}(y | x) | | d\lambda_{IM}(x) |$$



Lim. State Prob. & Framing Equation

$$\lambda_{f} = \iint F_{C}(x) \left\| dG_{D|S_{a}}(x \mid s) \right\| d\lambda_{S_{a}}(s) \right\|$$

$$\lambda_{\mathrm{DV}}(z) = \iint_{y \, x} G_{\mathrm{DV}|\mathrm{DM}}(z \mid y) \left| dG_{\mathrm{DM}|\mathrm{IM}}(y \mid x) \right| \left| d\lambda_{\mathrm{IM}}(x) \right|$$



Lim. State Prob. & Framing Equation

$$\lambda_{f} = \iint \mathbf{F}_{C}(\mathbf{x}) | dG_{D|S_{a}}(\mathbf{x} | \mathbf{s}) | d\lambda_{S_{a}}(\mathbf{s}) |$$

$$\lambda_{DV}(z) = \iint_{y x} G_{DV|DM}(z | y) | dG_{DM|IM}(y | x) | | d\lambda_{IM}(x) |$$



Lim. State Prob.: Closed Form

$$\lambda_{f} = \iint F_{C}(x) | dG_{D|S_{a}}(x | s) || d\lambda_{S_{a}}(s) |$$

After fitting specific forms to these functions:

$$\lambda_{f} = \lambda_{S_{a}}(s_{a}^{\hat{C}}) \exp[\frac{1}{2} \frac{k^{2}}{b^{2}} (\beta_{C}^{2} + \beta_{DM|S_{a}}^{2})]$$



Lim. State Prob.: Estimation





Lim. State Prob. >LRFD Format

$$\lambda_{f} = \lambda_{S_{a}}(s_{a}^{\hat{C}}) \exp[\frac{1}{2} \frac{k^{2}}{b^{2}} (\beta_{C}^{2} + \beta_{DM|S_{a}}^{2})]$$

Re-arranges to:

$$e^{-1/2(k/b)\beta_{C}^{2}} \cdot \hat{C} = e^{1/2(k/b)\beta_{D|Sa}^{2}} \hat{D}_{S_{a}^{\lambda_{o}}}$$



Lim. State Prob.: LRFD Check





Displacement-Based "LRFD" Format

$$e^{-1/2(k/b)\beta_{C}^{2}} \cdot \hat{C} = e^{1/2(k/b)\beta_{D|Sa}^{2}} \hat{D}_{Sa}^{\lambda_{o}}$$

$$\mathbf{\phi} \cdot \mathbf{C} = \mathbf{\gamma} \cdot \mathbf{D}$$



Economic Assessment Analysis

$$\lambda_{f} = \iint \mathbf{F}_{C}(\mathbf{x}) | dG_{D|S_{a}}(\mathbf{x} | \mathbf{s}) | d\lambda_{S_{a}}(\mathbf{s}) |$$

$$\lambda_{\text{Dollars}}(z) = \iint G_{\text{Dlls}|\text{D}}(z \mid x) \mid dG_{\text{D}|\text{S}_{a}}(x \mid s) \mid d\lambda_{\text{S}_{a}}(s) \mid$$



Seismic Performance Assessment Analysis

- Provides guidance for integration of Ground Motion Hazard, Nonlinear Dynamic Response and Capacity
- Produces annual frequency of limit state "exceedance"
- In integral form, solution by simulation, numerical integration and approximate closed form approximations

Seismic Performance Assessment Analysis

- Closed from solutions lead also to familiar LRFD-like formats for the assessment
- Economic loss analysis falls into similar integral form.
- Both limit state and economic analysis can be represented by a common generic form

