

REAL-TIME HYBRID SIMULATION WITH ADAPTIVE ACTUATOR CONTROL FOR STRUCTURAL ENGINEERING RESEARCH

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ABSTRACT

Real-time hybrid simulation combines experimental testing and numerical simulation, and thus provides a viable experiment technique for structural engineering research. Servo-hydraulic actuators are typically utilized to facilitate large- or full-scale real-time hybrid simulation. Time delay introduced in an actuator's response due to servo-hydraulic dynamics can destabilize the simulation if not compensated properly. This paper discusses an adaptive compensation method for real-time hybrid simulation to minimize actuator delay. An evolutionary variable is introduced to adapt the compensation parameter based on an actuator tracking indicator. Laboratory tests involving large-scale real-time hybrid simulations of a two-story-four-bay moment resisting frame with Magneto-Rheological (MR) dampers are conducted to experimentally demonstrate the effectiveness of the proposed adaptive compensation method. The steel moment resisting frame is modelled analytically with the aid of a finite element program and the MR dampers are taken as the experimental substructures to be physically tested. The actuator tracking capability is shown to be greatly improved and reliable experimental results are achieved, even when a good estimate of actuator delay is not used and variable actuator delay occurs during the simulation.

INTRODUCTION

Experimental testing is an essential tool to evaluate the performance of structures under extreme events such as earthquakes. It enables researchers to study the response of structures to earthquakes, and provides data for developing numerical models and assessing design methodologies for new types of structural systems. Various testing techniques have been developed for structural engineering research, and include quasi-static cyclic testing, shake-table testing, pseudodynamic testing, and hybrid simulation. Although shake-table testing provides the most realistic means of simulating seismic effects in the laboratory, the test structure is usually a scaled-down version of the prototype structure to accommodate the capacity of the shake table. Due to the scaling effect, shake-table tests of reduced-scale models may not accurately replicate the behaviour of some structural components of the prototype structure such as control devices.

Numerous rate-dependent devices have been recently developed for seismic hazard mitigation [Soong *et al.* 2002]. The behaviour of these devices varies under different loading frequencies and evaluation of their performance requires that the tests be conducted using a real-time scale so that the true response of the test structure can be obtained during the experiment. Conventional experiment techniques, such as quasi-static cyclic testing and pseudodynamic testing, are not suitable when rate-dependent devices exist in the test structure since these methods of testing are conducted using an expanded time scale. Two alternatives have been suggested to overcome this challenge to facilitate experimental research involving rate-dependent devices. The first involves

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the use of a large capacity shake tables that enables the seismic response of large-scale structures to be experimentally [NIED]. Large shake table tests however often lead to a great amount of cost associated with test structure construction and table operation. Another alternative is to extend the conventional pseudodynamic testing and hybrid simulation methods from an expanded time scale to a real-time scale, and is known as real-time pseudodynamic testing and real-time hybrid simulation [Nakashima *et al.* 1999; Blakeborough *et al.* 2001].

In a real-time hybrid simulation the structural system is divided into analytical and experimental substructures, where the analytical substructures are numerically modelled and the experimental substructures (i.e., test structure) are physically tested in the laboratory. An integration algorithm is usually used to solve for the structural response in a real-time hybrid simulation. By numerically modelling parts of the structural system using the finite element method, real-time hybrid simulation provides a viable alternative to experimentally evaluate the performance of large- or full-scale structural systems, especially systems with rate-dependent seismic devices. The interaction between the integration algorithm and the substructures ensures that the dynamic performance of the entire structural system is considered throughout the simulation.

Unlike conventional experimental techniques, such as quasi-static testing and pseudodynamic testing, real-time hybrid simulation requires that the servo-hydraulic actuator(s) impose command displacement(s) accurately onto the experimental substructure(s) in a real-time manner. Due to inherent servo-hydraulic dynamics the actuator has an inevitable time delay in response to the displacement command. This time delay is usually referred to as actuator delay and will result in a desynchronization between the measured restoring force(s) from the experimental substructure(s) and the integration algorithm in a real-time hybrid simulation. Studies on the effect of actuator delay [Wallace *et al.* 2005; Chen and Ricles 2008b] show that actuator delay is equivalent to creating negative damping which can destabilize a real-time hybrid simulation if not compensated properly.

Actuator delay compensation is often used to minimize actuator delay and achieve accurate actuator control for real-time hybrid simulation. Horiuchi *et al.* [1999] and Horiuchi and Konno [2001] proposed two compensation schemes for actuator delay that are based on polynomial extrapolation and a linear acceleration assumption. Methods originating from control engineering theory have also been used in real-time testing, and include derivative feedforward [Jung and Shing 2006]. The above compensation methods are often utilized for a predefined constant actuator delay. Chen and Ricles [2009] showed that the performance of these compensation methods can be analyzed through a frequency response analysis of the equivalent discrete transfer function.

An accurate estimate of actuator delay is often difficult to acquire before a real-time test. Moreover, the actuator delay might be variable during a real-time hybrid simulation due to the nonlinearities in the servo-hydraulic system and the experimental substructure(s). Compensation methods for variable actuator delay have also been developed. Darby *et al.* [2002] proposed an online procedure to estimate and compensate for actuator delay during a real-time hybrid test using a proportional feedback system. Bonnet *et al.* [2007] applied model reference adaptive minimal control synthesis (MCS) to real-time testing. Carrion and Spencer [2007] used a feedforward-feedback controller in conjunction with inverse modelling to compensate for variable actuator delay when conducting real-time hybrid simulation of substructures such as semi-active MR dampers. Chen and Ricles [2008a] developed an adaptive inverse compensation

method based on a simplified discrete model of a servo-hydraulic system. In this paper, the application of real-time hybrid simulation with adaptive inverse compensation method is demonstrated by experimentally evaluating the seismic performance of a steel moment resisting frame (MRF) with magneto-rheological (MR) dampers.

ADAPTIVE INVERSE COMPENSATION METHOD FOR ACTUATOR DELAY COMPENSATION

The modelling of the servo-hydraulic system can be complicated due to nonlinearities in the system. To incorporate the actuator delay into a stability analysis of a real-time hybrid simulation Chen and Ricles [2008b] proposed a simplified first-order discrete model to model the servo-hydraulics. The inverse of the simplified model was subsequently applied for actuator delay compensation in real-time hybrid simulation of a single-degree-of-freedom structure with an elastomeric damper by Chen *et al.* [2009]. Good performance was observed for actuator tracking with the resulting inverse compensation. The inverse compensation model is expressed as

$$G_c(z) = \frac{X^p(z)}{X^c(z)} = \frac{\alpha_{es} \cdot z - (\alpha_{es} - 1)}{z} \quad (1a)$$

In Eq. (1a) α_{es} is the estimated actuator delay constant; z is the complex variable in the discrete z-domain; $X^c(z)$ is the discrete z-transform of the displacement command d_{i+1}^c , which is usually computed by an integration algorithm in a real-time hybrid simulation; $X^p(z)$ is the discrete z-transform of the predicted displacement d_{i+1}^p from the compensation method to be sent to the servo-hydraulic actuator to compensate for actuator delay; and $G_c(z)$ is the discrete transfer function for the inverse compensation method. Applying the inverse discrete z-transform to Eq. (1a) leads to the following equivalent difference equation for the inverse compensation method:

$$d_{i+1}^p = \alpha_{es} \cdot d_{i+1}^c - (\alpha_{es} - 1) \cdot d_i^c \quad (1b)$$

In Eq. (1b) d_{i+1}^p and d_{i+1}^c are the predicted displacement and command displacement for the $(i+1)^{\text{th}}$ time step, respectively, and d_i^c the command displacement for the i^{th} time step. Eq. (1b) indicates that the inverse compensation method can be interpreted as an extrapolation using the command displacements of the previous time steps. It can also be observed from Eqs. (1a) and (1b) that the performance of the inverse compensation method is dependent on the accuracy of the delay constant α_{es} , which is often estimated from previous experience. To minimize the effect of an inaccurately estimated or variable actuator delay, Chen and Ricles [2008a] developed an adaptive inverse compensation method based on the inverse compensation method, which is formulated as

$$G_c(z) = \frac{X^p(z)}{X^c(z)} = \frac{(\alpha_{es} + \Delta\alpha) \cdot z - (\alpha_{es} + \Delta\alpha - 1)}{z} \quad (2)$$

In Eq. (2), $\Delta\alpha$ is an evolutionary variable with an initial value of zero, and is determined using the following adaptive control law

$$\Delta\alpha(t) = k_p \cdot TI(t) + k_i \cdot \int_0^t TI(\tau) d\tau \quad (3)$$

In Eq. (3) k_p and k_i are proportional and integrative gains for the adaptive control law, respectively; and TI is the tracking indicator based on the enclosed area of the hysteresis in the synchronized subspace plot, as shown in Fig. 1, where the actuator command displacement d^c is

plotted against the actuator measured response d^m . The calculation of TI for each time step can be formulated as [Mercan 2007]

$$TI_{i+1} = 0.5(A_{i+1} - TA_{i+1}) \quad (4a)$$

$$A_{i+1} = A_i + dA_{i+1} = A_i + 0.5(d_{i+1}^c + d_i^c)(d_{i+1}^m - d_i^m) \quad (4b)$$

$$TA_{i+1} = TA_i + dTA_i = TA_i + 0.5(d_{i+1}^m + d_i^m)(d_{i+1}^c - d_i^c) \quad (4c)$$

where d_{i+1}^m is the measured displacement of the servo-hydraulic actuator at the $(i+1)^{\text{th}}$ time step.

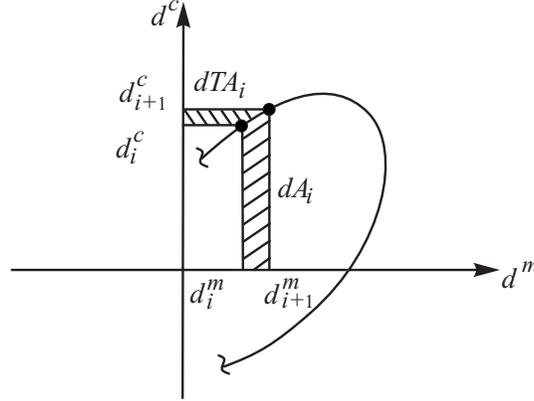


Fig. 1. Definition of tracking indicator

It can be observed that the adaptive inverse compensation method reduces to the inverse compensation method when both the gain k_p and k_i are set equal to zero. When α_{es} , k_p , and k_i are set equal to zero, no actuator delay compensation occurs during the real-time hybrid simulation. Generally, a larger value of k_p results in a faster response and a larger oscillation in the evolutionary variable, while increasing the integrative gain k_i reduces the oscillation and leads to a smaller steady state error. In this paper, the integrative gain k_i is selected to be one tenth of the proportional gain k_p based on studies involving numerical simulations, where $k_i = 0.1k_p$ was found to produce good adaptation with a small error.

EXPERIMENTAL EVALUATION OF THE SEISMIC PERFORMANCE OF A STEEL MRF WITH MR DAMPERS USING REAL-TIME HYBRID SIMULATION

Real-time hybrid simulation is conducted on a steel MRF with MR dampers in passive mode to demonstrate the performance of the adaptive inverse compensation method. MR dampers are control devices proposed for vibration control of engineering structures. Experimental studies involving predefined displacements have been reported that led to the development of phenomenological models for MR dampers [Dyke *et al.* 1996; Yang *et al.* 2002; Bass and Christenson 2007]. Shake table tests of structures with small-scale MR dampers also have also been conducted by researchers [Jansen and Dyke 2008]. These experiments provide valuable insight into the behaviour of MR dampers. However, the real-time tests with predefined displacements do not account for the structure-damper interaction during the earthquake, and the scaled shake table tests may not realistically represent the nonlinear behaviour of large-scale MR dampers that exist in the prototype structure. Large-scale real-time hybrid simulation provides a viable technique to investigate the performance of large-scale MR dampers for seismic hazard mitigation with structure-damper interaction considered.

Prototype Steel MRF Structure

A 2-story, 6-bay by 6-bay prototype office building is selected as the prototype structure for the experimental study presented in this paper. The building is assumed to be located on a stiff soil site near Los Angeles and has four identical perimeter steel MRFs to resist lateral forces. Fig. 2 shows the plan view and the perimeter frame of the prototype structure. The experimental study presented in this paper focuses on one typical perimeter MRF, which is designed with MR dampers as shown in Fig. 2(b). The yield strength of the material for the MRF is assumed to be equal to 345 MPa. The gravity loads described in the International Building Code [ICC 2006] are considered in the design. A smooth design response spectrum with parameters $S_{DS}=1.0$, $S_{D1}=0.6$, $T_0=0.12$ sec. and $T_s=0.6$ sec. represents the design basis earthquake (DBE) [BSSC 2003]. The computer program SAP2000 [2008] is utilized for selecting the frame member sizes in accordance with the AISC-LRFD provisions [AISC 2005]. The same column cross section and various beam cross-sections are used for the two-story MRF.

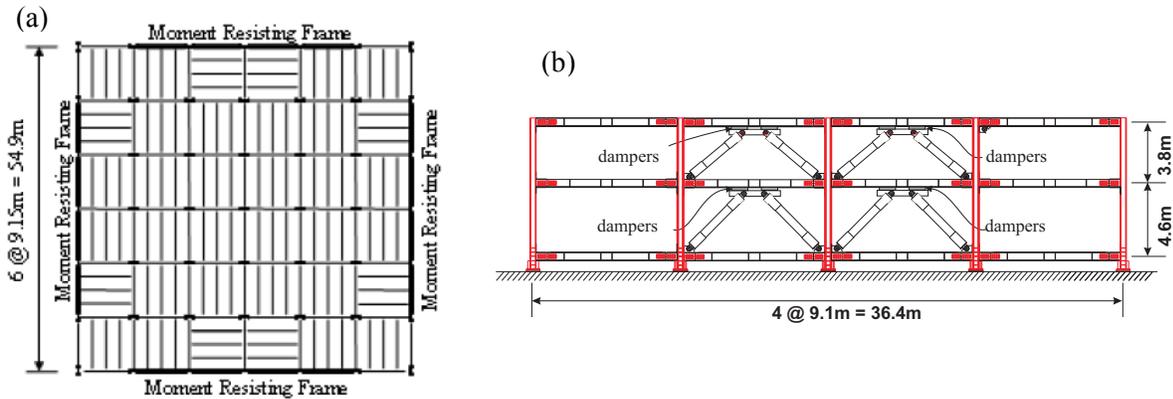


Fig. 2. Prototype building (a) plan view; (b) perimeter MRF with dampers and braces

A simplified design procedure developed by Lee *et al.* [2007] is utilized to design the MRF with MR dampers, where the properties of the resulting MRF are tabulated in Table 1, including column and beam cross-sections, fundamental period of vibration and story stiffness. The MR dampers are assumed to be in passive mode with the maximum current input of 2.5 Amps. A total of six and four MR dampers are determined by the simplified design procedure for the first and second stories, respectively. To utilize the MR dampers, the perimeter MRF is re-designed with a design base shear equal to 50% of that of the conventional SMRF.

Table 1. Properties of MRF with MR Dampers

Column	Beams		Fundamental period (sec)	Story stiffness (kN/m)	
	1 st story	2 nd story		1 st story	2 nd story
W14x120	W24x55	W18x40	1.21	36007	23894

Experimental Setup

The real-time hybrid simulation was performed using the NEES Real-Time Multi-Directional (RTMD) Facility at Lehigh University. Fig. 3 shows the experimental setup for the real-time hybrid simulation, which consists of two experimental substructures (two MR dampers), two servo-hydraulic actuators with supports and roller bearings; reaction frames, and beams securing the MR dampers to the strong floor. The large-scale MR dampers used in this study are

manufactured by Lord Corporation and have a nominal capacity of 200 kN at the maximum current input of 2.5 Amps [Bass and Christenson 2007]. The MR fluid damper has an available stroke of 584 mm and an accumulator to accommodate a temperature change in the fluid up to 44.4 °C. An Advanced Motion Control PWM servo-amplifier is utilized to provide an electrical current command signal that controls the electromagnetic field for the damper. Since the dampers at a story level of the prototype structure are placed in parallel in the MRF, they are assumed to be subjected to the same velocity and displacement, and hence each of the MR dampers test setups in the laboratory represents all of the dampers in one story. The measured restoring force from each MR damper setup is multiplied by the number of dampers to obtain the total restoring force of all the dampers at a story level in the MRF. The MRF is analytically modeled using a nonlinear finite element program with a total 122 degrees of freedom and 71 elements [Karavasilis *et al.* 2009]. The highest natural frequency of the MRF is around 20k Hz.

The two actuators each have a 500 mm stroke and different maximum force capacity of 1700 kN and 2300 kN, respectively. Two servo-valves, each with a flow capacity of 2500 liters/min, are mounted on each actuator to enable them to achieve a maximum velocity of 760 mm/sec and 560 mm/sec, respectively. The servo-controller for the actuator used in the real-time tests consisted of a digital PID controller with the proportional gain of 20, integral time constant of 5.0 resulting in an integral gain of 4.0, differential gain of 0 and a roll-off frequency of 39.8 Hz [Lehigh RTMD 2009].

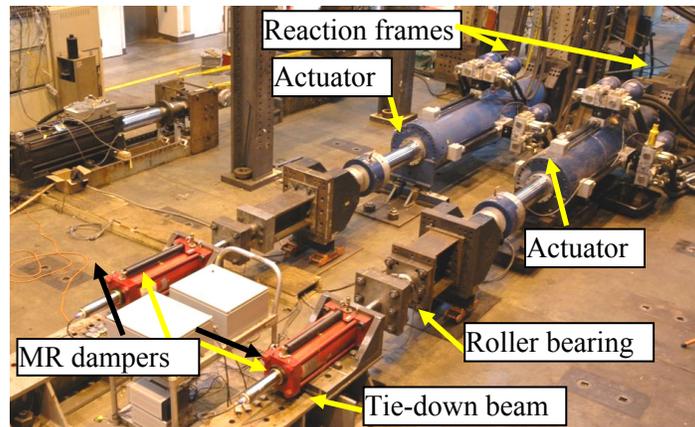


Fig. 3. Experimental setup for real-time hybrid simulation

Real-Time Hybrid Simulation Results

The unconditionally stable explicit CR integration algorithm [Chen and Ricles 2008c] is used for the real-time hybrid simulation. The structural response calculated by the CR algorithm is translated into the displacement for the DOFs of the experimental substructures, which is then imposed to the MR dampers by the servo-hydraulic actuators. The integration time step Δt used for the present study is equal to 10/1024 sec. To ensure a smooth and continuous actuator response, a ramp generator is used to interpolate the command displacement at the servo controller sampling rate, which is 1/1024 sec.

The TCU105-E component of the 1999 Chi-Chi earthquake recorded at the TCU105 station was used as the ground motion for the real-time hybrid simulation. The ground motion is scaled to the design basis earthquake (DBE) by employing the scaling procedure of Somerville [1997].

The estimates of the actuator delay constant for the adaptive inverse compensation are $\alpha_{1es}=30$ and $\alpha_{2es}=45$ based on previous experiments. The adaptive gains are set to be $k_p=0.4$ and $k_i=0.04$ for both actuators.

Real-time hybrid simulation results are presented in Figs. 4 and 5. The floor displacement and damper deformation time histories are presented in Fig. 4. The MRF is observed to have a maximum lateral displacement magnitude of 35.2 mm and 62.6 mm for the first and second floors, respectively. The resulting maximum story drifts are about 0.8% and 0.7% story drift and much less than the design drift of 1.6%. The experimental substructures, i.e., the MR dampers, developed a maximum deformation magnitude of 35.8 mm and 29.3 mm for the first and second floors, respectively.

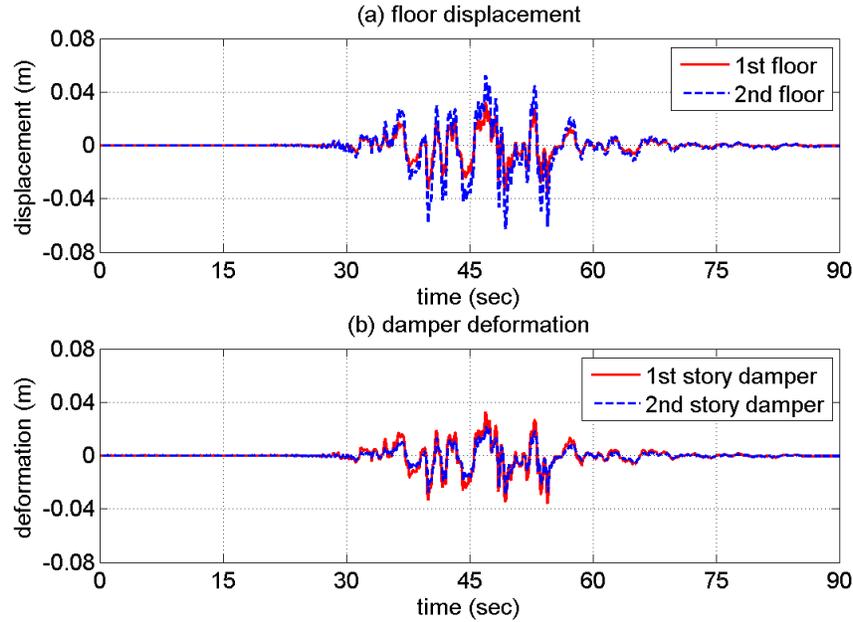


Fig. 4. Real-time hybrid simulation results of MRF subjected to DBE earthquake: (a) floor displacements; (b) damper deformations

The actuator tracking for the real-time hybrid simulation is shown in Fig. 5. The comparisons between the command and measured displacements are presented in Fig. 5(a) and 5(b) for the actuators attached to the first and second story dampers, respectively. Good agreement can be observed in the comparison, indicating that good actuators control is achieved. The time history of the TI is presented in Fig. 6(a) and 6(b) for the two actuators, where small values of the TI can be observed and indicates good actuator control during the real-time hybrid simulation.

The time histories for the evolutionary variable $\Delta\alpha$ are presented in Fig. 7(a) and 7(b) for the actuators. Spikes of small amplitude can be observed for the evolutionary variable $\Delta\alpha$ for both actuators, where the adaptive compensation tried to accommodate a sudden increase in the actuator delay due to the increased deformation in the MR dampers and the associated larger forces and velocities developed by the actuators. It can also be observed that the evolutionary variable $\Delta\alpha$ for the two actuators have different trends, where the actuator attached to the first story damper is observed to have a oscillation around zero while the actuator attached to the second story damper has a negative value of $\Delta\alpha$ between zero and -10. This means that the accuracy of the estimated actuator delay for the two actuators is different. The actuator attached to the second story damper appears to have a slight over-compensation induced by the value used

for the initial estimate α_{es} . The different trends in $\Delta\alpha$ can be attributed to the different power curve capacities of the two actuators resulting in different delay when applying a similar force and velocity, where the smaller 1700 kN actuator used for the 1st story damper has a faster maximum velocity than the 2300 kN actuator used for the 2nd story, as described previously.

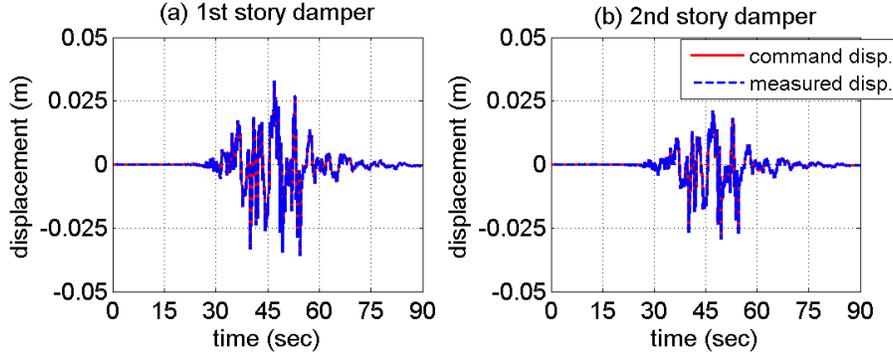


Fig. 5. Comparison of command and measured actuator displacements

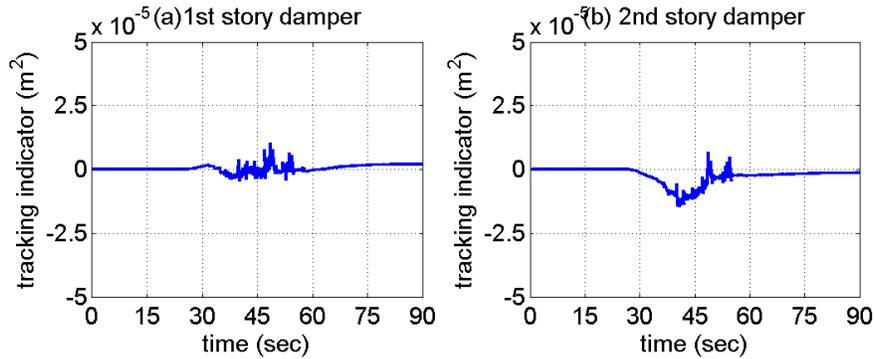


Fig. 6. Time history for tracking indicator

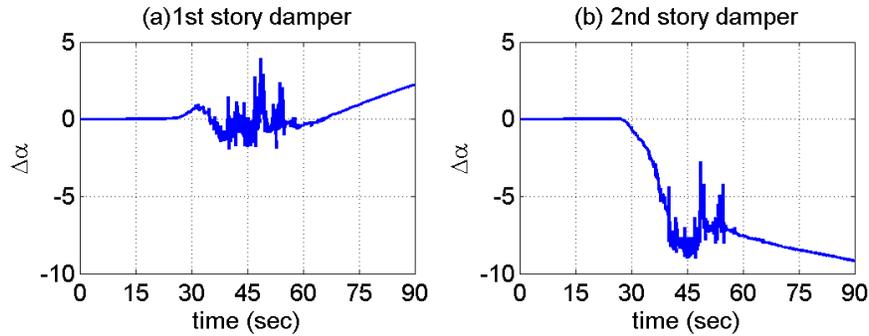


Fig. 7. Time history for evolutionary variable $\Delta\alpha$

SUMMARY AND CONCLUSIONS

An adaptive inverse compensation method is discussed in this paper to minimize actuator delay in order to achieve reliable and accurate real-time hybrid simulation results. The adaptive compensation method employs an initial estimate for the actuator delay and utilizes an evolutionary variable to accommodate inaccurately estimated and variable delay during a real-time hybrid simulation. The adaptive control law for the evolutionary variable is based on a tracking indicator. The adaptive inverse compensation is experimentally demonstrated through a

real-time hybrid simulation of a steel MRF with MR dampers in passive mode. The MRF with MR dampers is designed using a simplified design procedure to achieve a prescribed performance objective under earthquake loading. One ground motion record is selected and scaled to the DBE level for the simulation. Accurate actuator control is shown to be achieved through the adaptive inverse compensation method in terms of actuator control error as well as tracking indicator, thus enabling a successful real-time hybrid simulation to be performed. The experimental results also show that the MRF designed by the simplified procedure employed in this paper satisfies the performance objective. The results from this paper demonstrate the potential of real-time hybrid simulation with adaptive inverse compensation for assessing the performance of structures with rate-dependent devices.

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