## F. Earthquakes



Level 2 The Epicenter of an Southwestern Earthquake
KM\#6

Brief Description: This study examines the speed of $P$ and $S$ waves emitted from an earthquake, Equations relating distance traveled and speed are written, then the equations are used to find the epicenter as the intersection of three circles, each centered at a seismic station.
Mathematical Content: Basic algebra; equation of a line; radius of a circle. Kéyah Math Student Worksheets The link above right will provide the module questions in a format for students without computer for Math Tools.

## Objectives:

$\bullet$
The Kéyah Math

This is a study illustrating how earthquakes are located using data received at seismic stations. In particular, a 3.1 magnitude earthquake is used in this study. You will learn how geologists found out where it occurred.

## Warm-up Steps

Use questions for thought as a preliminary discussion about the topic.

1. Do you know of any recent earthquakes near where you live?
2. If so, was any damage done?
3. Where have some earthquakes that you know about occurred?
4. How do you think that scientist know where an earthquake happened?
5. Why is it important to know where an earthquake occurred?

## Introduction

An earthquake occurs when rocks in the crust move. This movement releases energy that is transmitted outward as seismic waves. Seismic stations have instruments that can sense and record seismic waves, even those from earthquakes that occur thousands of miles away from the station. Earthquakes generate several different types of seismic waves, and these waves each travel at different speeds through rock. The fastest waves (traveling at about $6 \mathrm{~km} / \mathrm{sec}$ through most crustal rocks) are the first to arrive at seismic stations and are called primary ( $\mathbf{P}$ ) waves. The next fastest waves travel at about $3.5 \mathrm{~km} / \mathrm{sec}$, so they arrive at seismic stations later and have been termed secondary (S) waves. The farther these waves travel away from the source of the earthquake, the more the $P$ waves will outpace the $S$ waves.

The epicenter of an earthquake is a virtual point on the surface that is located directly above the source of the earthquake. The farther a given seismic station is from an epicenter, the longer the time interval between the arrival of the $P$ waves and the arrival of the $S$ waves. This time interval can be expressed mathematically as a
function of the distance from the epicenter and the speeds of the $P$ and $S$ waves through rock. If we know what those two speeds are, and measure the time interval between the $P$ and $S$ arrivals at the seismic station, we can easily calculate how far the epicenter was from that seismic station.

The sequence of problems below will show you how this is done.

## Using math to find the epicenter of an earthquake

## Notation

The variables are:
$t_{p}=$ number of seconds a P wave travels after the instant of the earthquake;
$t_{s}=$ number of seconds a $S$ wave travels after the instant of the earthquake;
$\mathrm{d}_{\mathrm{p}}=$ distance (in km) P waves have traveled in $t p$ seconds; and
$\mathrm{d}_{\mathrm{s}}=$ distance (in km ) S waves have traveled in ts seconds

Step 1: Write an equation that shows how far each type of wave traveled after $t$ seconds.
Hint: Use the well-known formula, distance $=$ rate $\times$ time, to write the two equations that express distance traveled by the waves in terms of lapsed time. Your answer should be written in the form
$d_{p}=m_{p} t_{p}$, and $d_{s}=m_{s} t_{s}$, where $m_{p}$ and $m_{s}$ are the slopes of the lines.
Answer: $d_{p}=6 t_{p} ; d_{s}=3.5 t_{s}$

## Step 2:

A) Solve each of the two equations from Step 1 for times, $t_{s}$ and $t_{p}$. Since the distance is the same for each wave, denote this by $\mathbf{D}\left(=d_{p}=d_{s}\right)$.

Answer: $t_{p}=d_{p} / 6 ; t s=d_{s} / 3.5$
B) Take the difference of the results from Part A; be sure your answer is positive. Denote this difference by the variable $\mathbf{U}$.

Answer: $\boldsymbol{V}=\frac{\mathbf{2 5}}{\mathbf{2 1}} \boldsymbol{E}$
C) What does your answer to Part B represent?

Answer: Difference in arrival times for P - and S-waves
D) Solve the equation from Part $B$ for distance $D$.

Answer: $\boldsymbol{D}=\frac{\mathbf{2 1}}{\mathbf{2 5}} \boldsymbol{U}$

The table below shows four seismic stations near New Mexico (represented by 3- and 4-letter codes) with latitude and longitude coordinates given in columns 2 and 3 . Columns 4 and 5 indicate the times that $P$ and $S$ waves were received at the stations. The last three blank columns are for arrival time of P - and S -waves differences $\mathbf{U}$, distances $\mathbf{D}$ from the stations to the epicenter of the earthquake, and scaled down distances for the map shown below.

| Station | Arrival Time <br> P-waves | Arrival Time <br> S-waves | Time Diff <br> U | Dist to <br> Epicenter <br> D | Scaled <br> Distance for <br> Map |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1. TUC | $\mathbf{3 : 3 1 : 2 5}$ | $\mathbf{3 : 3 1 : 5 4}$ | 30 | 252 | 2.45 |
| 2. ANMO | $\mathbf{3 : 3 1 : 2 8}$ | $\mathbf{3 : 3 2 : 0 0}$ | 32 | 269 | 2.62 |
| 3. GDL2 | $\mathbf{3 : 3 1 : 4 5}$ | $\mathbf{3 : 3 2 : 2 9}$ | 44 | 370 | 3.6 |
| 4. LTX | $\mathbf{3 : 3 2 : 2 2}$ | $\mathbf{3 : 3 3 : 3 2}$ | 70 | 588 | 5.72 |

## Step 3:

A) From the data given in the table above, compute the time differences in seconds that $P$ and $S$ waves were received at the stations. Complete the column for $\mathbf{U}$ in the table.
B) Use your answers to Part A and the equation from Step 2B to find the distances from the stations to the epicenter of the earthquake to fill in the column for $\mathbf{D}$.

The distances you found in Step 3 can be used to find the epicenter of an earthquake. You know how far away the epicenter is from any one station but you don't know what direction. For example, if the epicenter is 1000 km away from the seismic station, you know it must be somewhere on the circle centered at the seismic station with radius 1000 km .

The map on this page is to be used to locate the epicenter. First, print this map.


## Step 4.

This a Google Earth map. The four markers locate the four stations listed in the table. From the latitude and longitude coordinates in the table, locate and identify each station.

## Answer: See Map Shown

## Step 5.

Next, you will need to figure out the scale on the map.
A) Measure, in inches, the east-west distance on the map along the southern border of New Mexico from Texas to Arizona.

The real distance between these locations is 565.56 kilometers (Google Earth).
B) Now use this information to figure the scale for this map in km/inch.

Answer: A and B. On the Google map printed from my computer, the distance across the southern border of New Mexico is 5.5 inches. So the scale is $\mathbf{1 0 2 . 8 k s} /$ inch . Note: this may vary with printing from different computers.

Step 6.
This is the final step!
A) Convert the distances $\mathbf{D}$ to fit the scale on the map; fill in the last column, Scaled Distance for Map, in the table with these data.
B) Use the distances you computed in the table for $\mathbf{U}$ and the scale for the map to draw a circle with center at each station and radius the scaled distance from that station to the epicenter.
C) Look at these circles and locate the epicenter.
D) What are the coordinates of the epicenter? What city or town is it near?

Answer: B, C, D) Draw Circles with centers at Stations and radii shown in the table. Epicenter at 33.00N, 108.23W - just north of Silver City (See map)

Note: P- and S-wave travel is actually non-linear. For simplicity, we use a linear approximation in this study.

## Follow-up Steps

1. Review the geology concepts used in this study.
2. Review the math that you used to find the epicenter.
3. Why is three the minimum number of stations that would be necessary to locate the earthquake?
