APPLICATIONS OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

EARTHQUAKE WORD PROBLEMS:

As with any word problem, the trick is convert a narrative statement or question to a mathematical statement.

Before we start, let's talk about earthquakes and how we measure their intensity. In 1935 Charles Richter defined the magnitude of an earthquake to be

$$M = \log \frac{I}{S}$$

where I is the intensity of the earthquake (measured by the amplitude of a seismograph reading taken 100 km from the epicenter of the earthquake) and S is the intensity of a "standard earthquake" (whose amplitude is 1 micron $=10^{-4}$ cm).
The magnitude of a standard earthquake is

$$M = \log \frac{S}{S} = \log 1 = 0$$

Richter studied many earthquakes that occurred between 1900 and 1950. The largest had magnitude of 8.9 on the Richter scale, and the smallest had magnitude 0. This corresponds to a ratio of intensities of 800,000,000, so the Richter scale provides more manageable numbers to work with.

Each number increase on the Richter scale indicates an intensity ten times stronger. For example, an earthquake of magnitude 6 is ten times stronger than an earthquake of magnitude 5. An earthquake of magnitude 7 is 10 x 10 = 100 times strong than an earthquake of magnitude 5. An earthquake of magnitude 8 is 10 x 10 x 10 = 1000 times stronger than an earthquake of magnitude 5.

Example 1: Early in the century the earthquake in San Francisco registered 8.3 on the Richter scale. In the same year, another earthquake was recorded in South America that was four time stronger. What was the magnitude of the earthquake in South American?

Solution: Convert the first sentence to an equivalent mathematical sentence or equation.

$$M_{SF} = \log \frac{I_{SF}}{S} = 8.3$$

$$8.3 = \log \frac{I_{SF}}{S}$$

Convert the second sentence to an equivalent mathematical sentence or equation.

Developed into worksheet from following the source:
http://www.sosmath.com/algebra/logs/log5/log56/log5611/log5611.html

$$
\begin{aligned}
M_{SA} &= \log \frac{I_{SA}}{S} \\
I_{SA} &= 4I_{SF} \\
M_{SA} &= \log \frac{4I_{SF}}{S}
\end{aligned}
$$

Solve for M_{SA}.

$$
\begin{aligned}
M_{SA} &= \log \frac{4I_{SF}}{S} \\
&= \log 4I_{SF} - \log S \\
&= \log 4 + \log I_{SF} - \log S \\
&= \log 4 + (\log I_{SF} - \log S) \\
&= \log 4 + \log \frac{I_{SF}}{S} \\
&= \log 4 + 8.3 \\
&= 0.602059991328 + 8.3 \\
&= 8.90205999133 \\
M_{SA} &= 8.9
\end{aligned}
$$

The intensity of the earthquake in South America was 8.9 on the Richter scale.

Example 2: A recent earthquake in San Francisco measured 7.1 on the Richter scale. How many times more intense was the San Francisco earthquake described in Example 1?

Solution: The intensity (I) of each earthquake was different. Let I_1 represent the intensity the early earthquake and I_2represent the latest earthquake.

$$First \ : \ 8.3 = \log \frac{I_1}{S}$$

$$Second \ : \ 7.1 = \log \frac{I_2}{S}$$

What you are looking for is the ratio of the intensities: $\frac{I_1}{I_2}$.

So our task is to isolate this ratio from the above given information using the rules of logarithms.

$$\begin{aligned} \log \frac{I_1}{S} - \log \frac{I_2}{S} &= 8.3 - 7.1 \\ \left(\log \frac{I_1}{S}\right) - \left(\log \frac{I_2}{S}\right) &= 8.3 - 7.1 \\ (\log I_1 - \log S) - (\log I_2 - \log S) &= 1.2 \\ \log I_1 - \log S - \log I_2 + \log S &= 1.2 \\ \log I_1 - \log I_2 &= 1.2 \\ \log \frac{I_1}{I_2} &= 1.2 \end{aligned}$$

Convert the logarithmic equation to an exponential equation.

$$\begin{aligned} \log \frac{I_1}{I_2} &= 1.2 \\ 10^{1.2} &= \frac{I_1}{I_2} \\ \frac{I_1}{I_2} &= 15.8489319246 \\ \frac{I_1}{I_2} &\approx 16 \end{aligned}$$

The early earthquake was 16 times as intense as the later earthquake.

EARTHQUAKE PROBLEMS:

Problem 1: Early in the century an earthquake measured 8.0 on the Richter scale. In the same year, another earthquake was recorded that measured six time stronger on the Richter scale. What was the magnitude of the earthquake of the stronger earthquake?

Problem 2: A recent earthquake measured 6.8 on the Richter scale. How many times more intense was this earthquake than an earthquake that measured 4.3 on the Richter scale?

Problem 3: If one earthquake is 31 times as intense as another, how much larger is its magnitude on the Richter scale?

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