



Bayesian Model Updating Framework for Simultaneous Parameter, Input, and Noise Estimation

Hamed Ebrahimian, Ph.D., P.E.

Assistant Professor

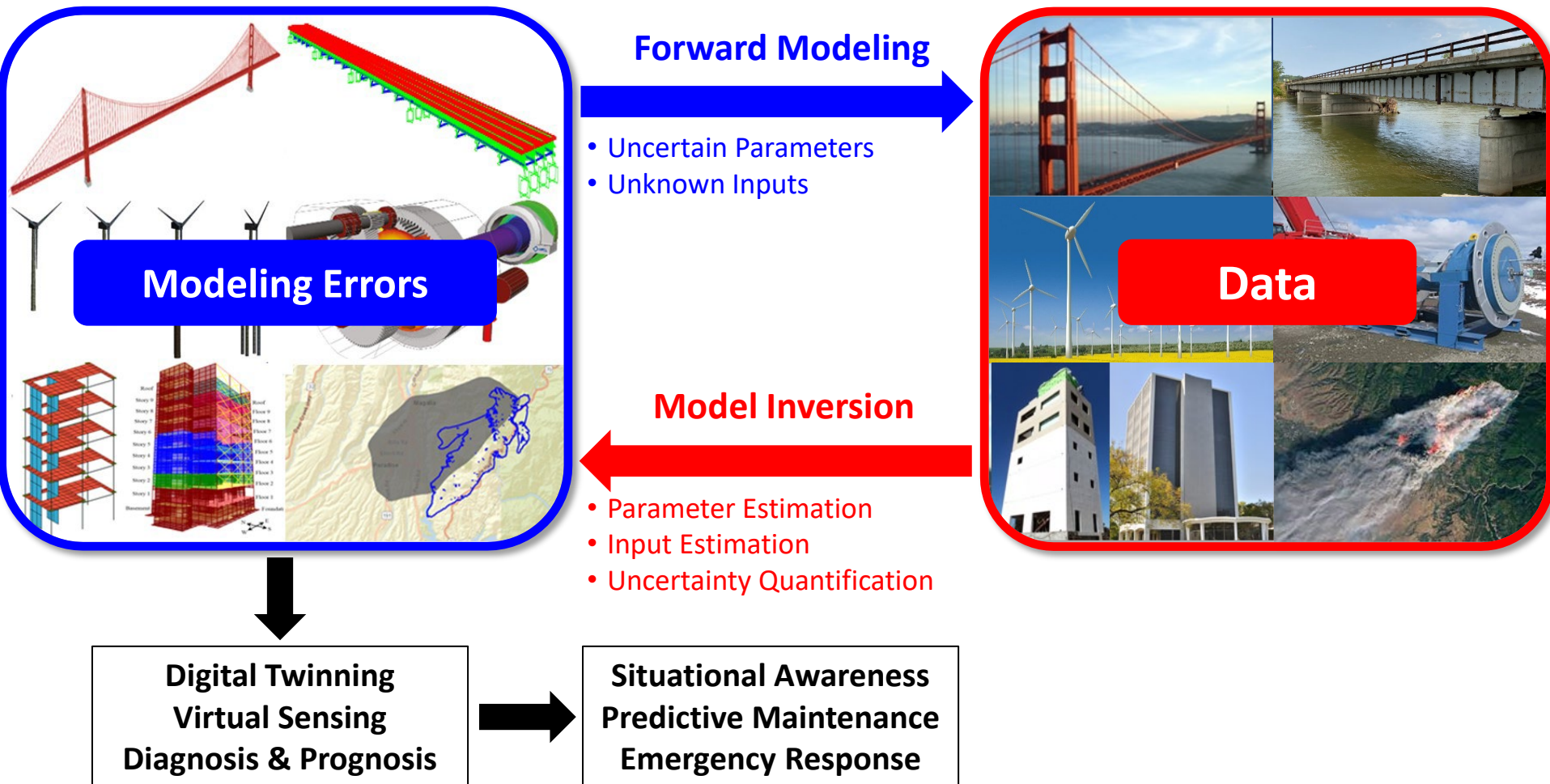
Abdelrahman Taha

Ph.D. Student

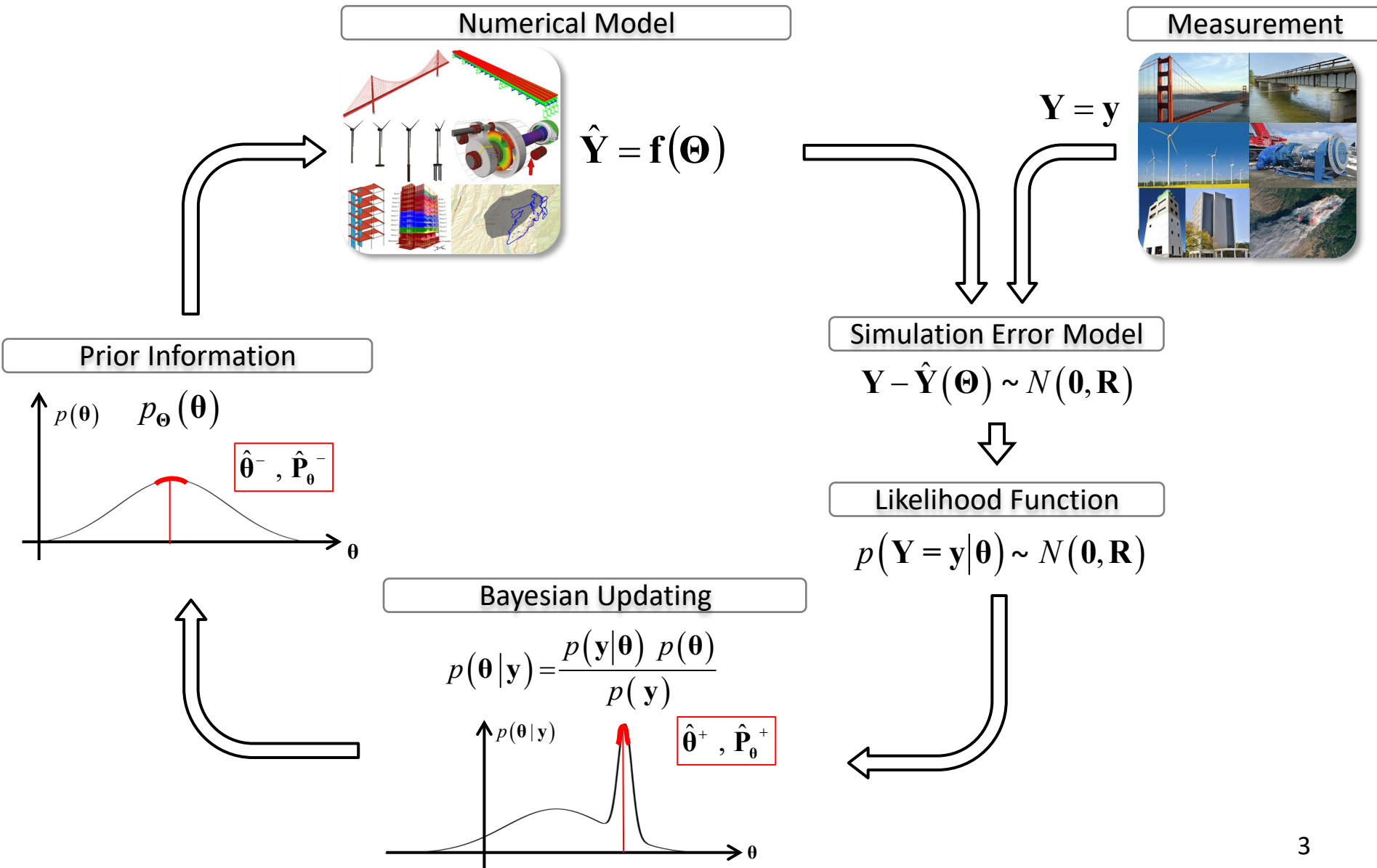
Department of Civil and Environmental Engineering
University of Nevada, Reno



Integration of Physics-Based Models with Data



Bayesian Model Updating



First-Order Approximation: Kalman Filtering

Numerical Model

$$\hat{\mathbf{y}} = \mathbf{f}(\hat{\boldsymbol{\theta}}^-)$$

$$\hat{\mathbf{P}}_{\mathbf{y}} = \underbrace{\left(\frac{\partial \mathbf{f}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \right) \bigg|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^-}}_{\text{FE Response Sensitivity Matrix}} \hat{\mathbf{P}}_{\boldsymbol{\theta}}^- \left(\frac{\partial \mathbf{f}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \bigg|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^-} \right)^T$$

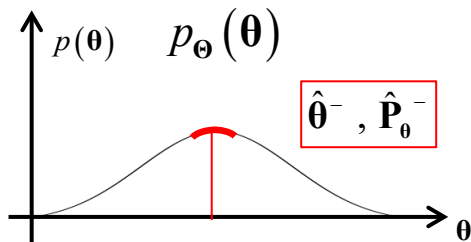
FE Response Sensitivity Matrix

Measurement



$$\mathbf{Y} = \mathbf{y}$$

Prior Information



Simulation Error Model

$$\mathbf{Y} - \hat{\mathbf{Y}}(\boldsymbol{\theta}) \sim N(\mathbf{0}, \mathbf{R})$$

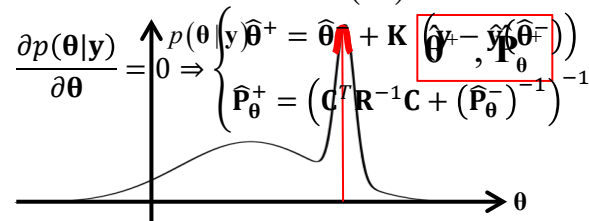


Likelihood Function

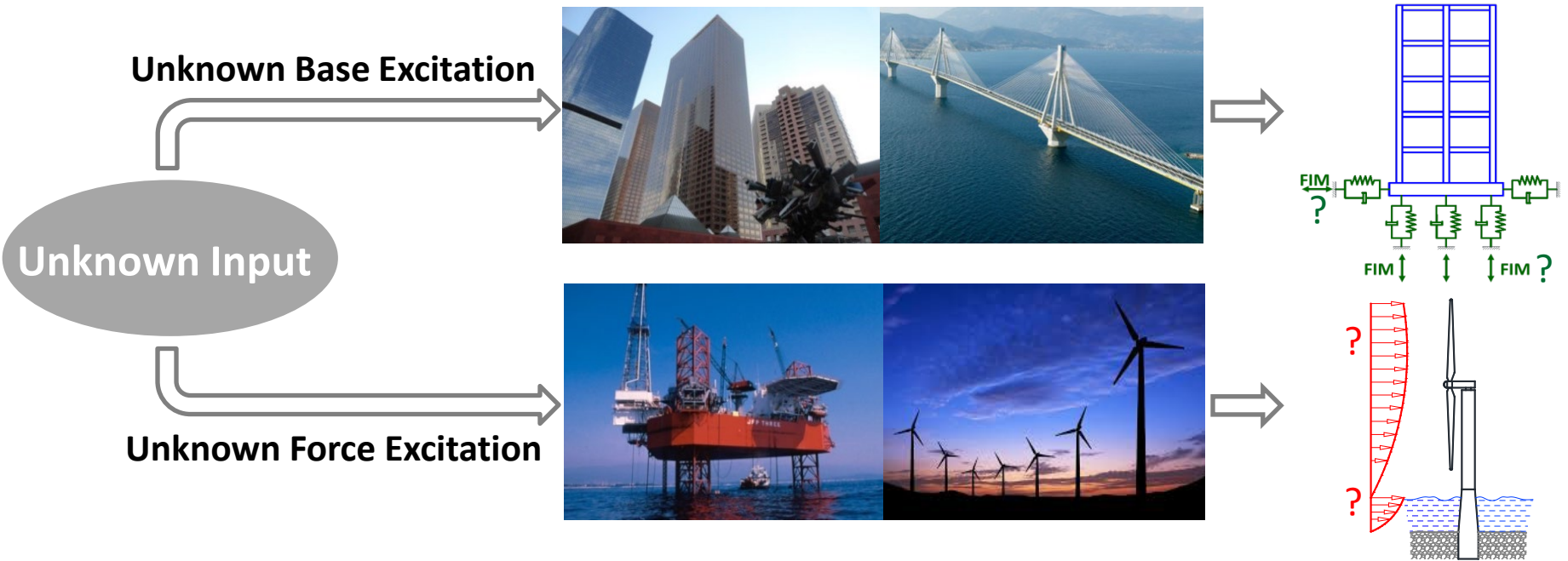
$$p(\mathbf{Y} = \mathbf{y} | \boldsymbol{\theta}) \sim N(\mathbf{0}, \mathbf{R})$$

Bayesian Updating

$$p(\boldsymbol{\theta} | \mathbf{y}) = \frac{p(\mathbf{y} | \boldsymbol{\theta}) p(\boldsymbol{\theta})}{p(\mathbf{y})}$$



Unknown Input Excitation: Output-Only Model Updating



$$\mathbf{M}(\boldsymbol{\theta})\ddot{\mathbf{u}}_k(\boldsymbol{\theta}) + \mathbf{C}(\boldsymbol{\theta})\dot{\mathbf{u}}_k(\boldsymbol{\theta}) + \mathbf{r}_k(\mathbf{u}_{1:k}(\boldsymbol{\theta}), \boldsymbol{\theta}) = \mathbf{L}\mathbf{f}_{1:k}^{unknown}$$

➔ Find $\hat{\boldsymbol{\theta}}, \hat{\mathbf{f}}_{1:k}^{unknown} \mid \left[\hat{\boldsymbol{\theta}}, \hat{\mathbf{f}}_{1:k}^{unknown} \right] = \arg \max_{\boldsymbol{\theta} \in \Theta, \mathbf{f} \in \mathcal{F}} p(\boldsymbol{\theta}, \mathbf{f}_{1:k}^{unknown} \mid \mathbf{y}_{1:k})$

Bayesian Model Updating Applications

Digital Twinning of
Civil Structures

Digital Twinning of
Offshore Wind Systems



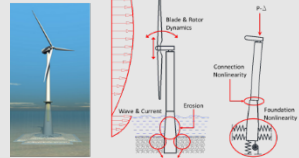
Wildfire
Engineering

Wildfire Simulation and
Data Assimilation

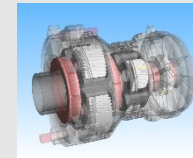


Cross-Disciplinary
Problems

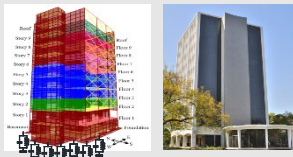
Remote Monitoring of
Offshore Structures



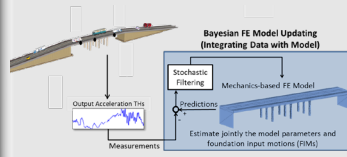
Diagnosis & Prognosis of
Drivetrains



Soil-Structure Model
Updating



Rapid Post-Earthquake
Assessment



Operational Monitoring of
Aging Bridges



Volpe Center



Monitoring and Diagnosis of Aging Bridges

Maintenance Inspection

- Costly
- Periodic
- Subjective
- No system-level insight
- Hidden damage

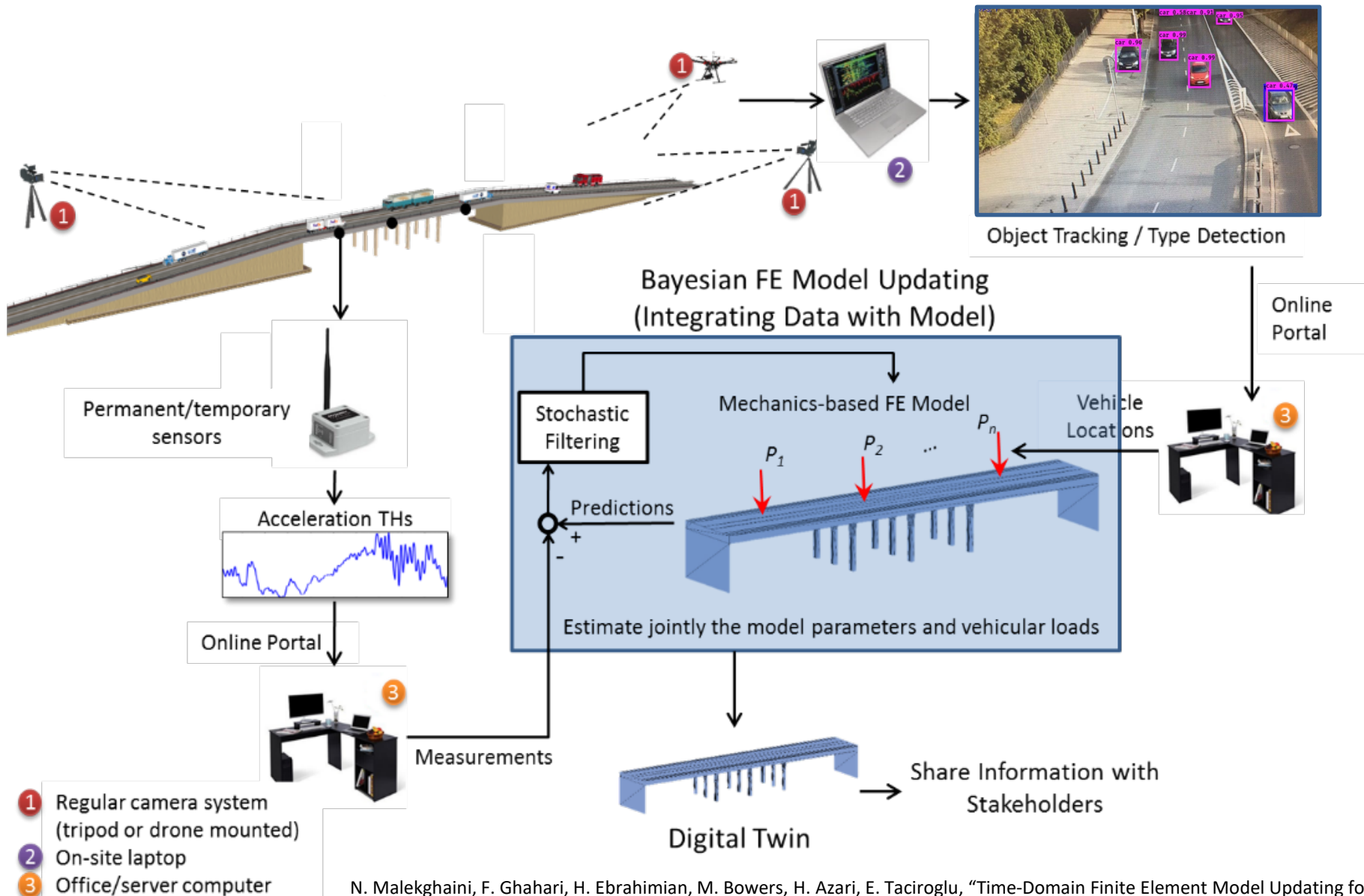


Post-Earthquake Inspection

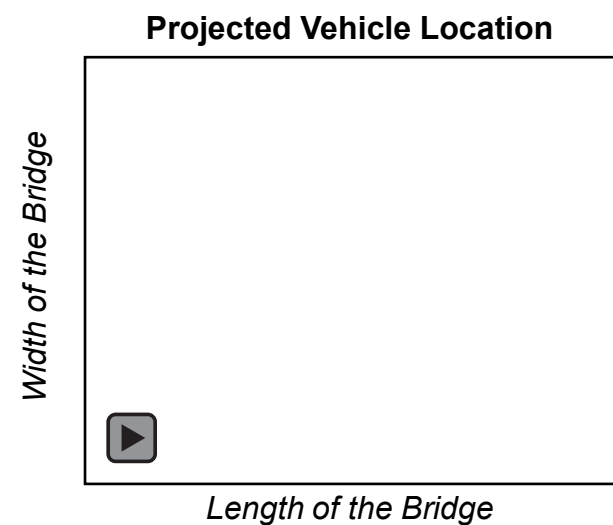
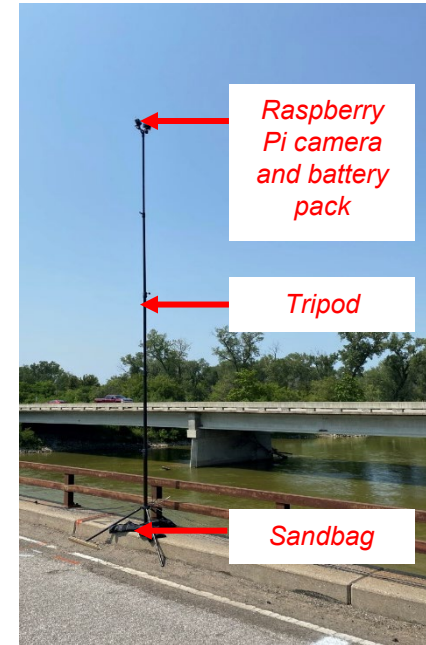
- # of bridges $\times$ inspection time $\times$ chaos = ?
- Intensity-based metrics can be inaccurate!
- Inspection complexity
- Hidden damage
- No system-level insight



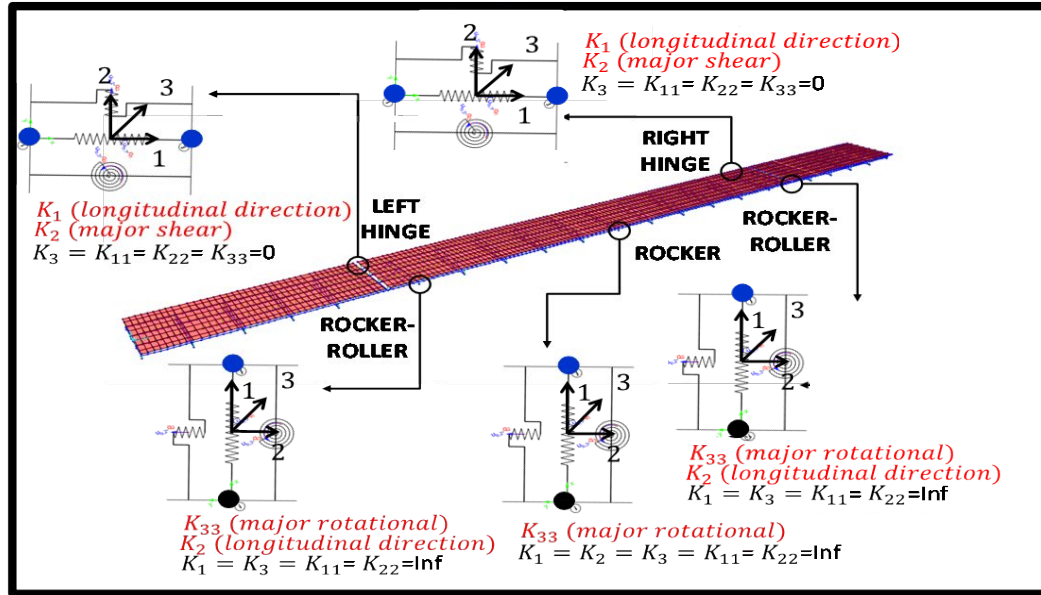
Operational Monitoring Framework



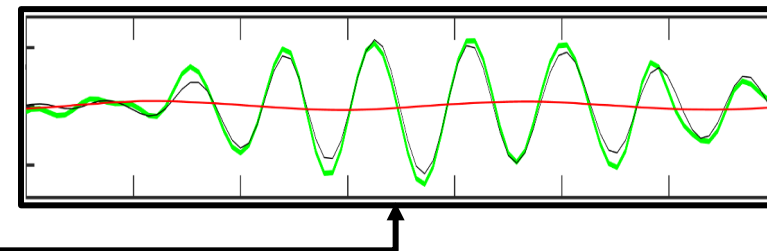
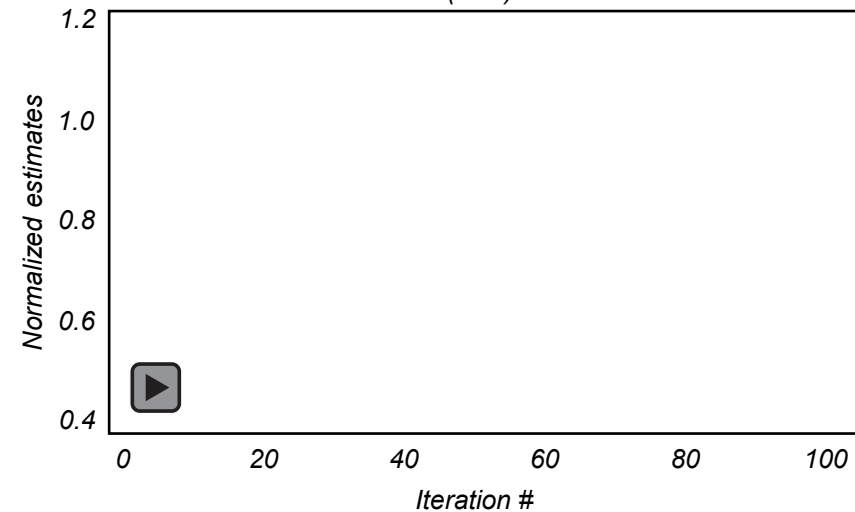
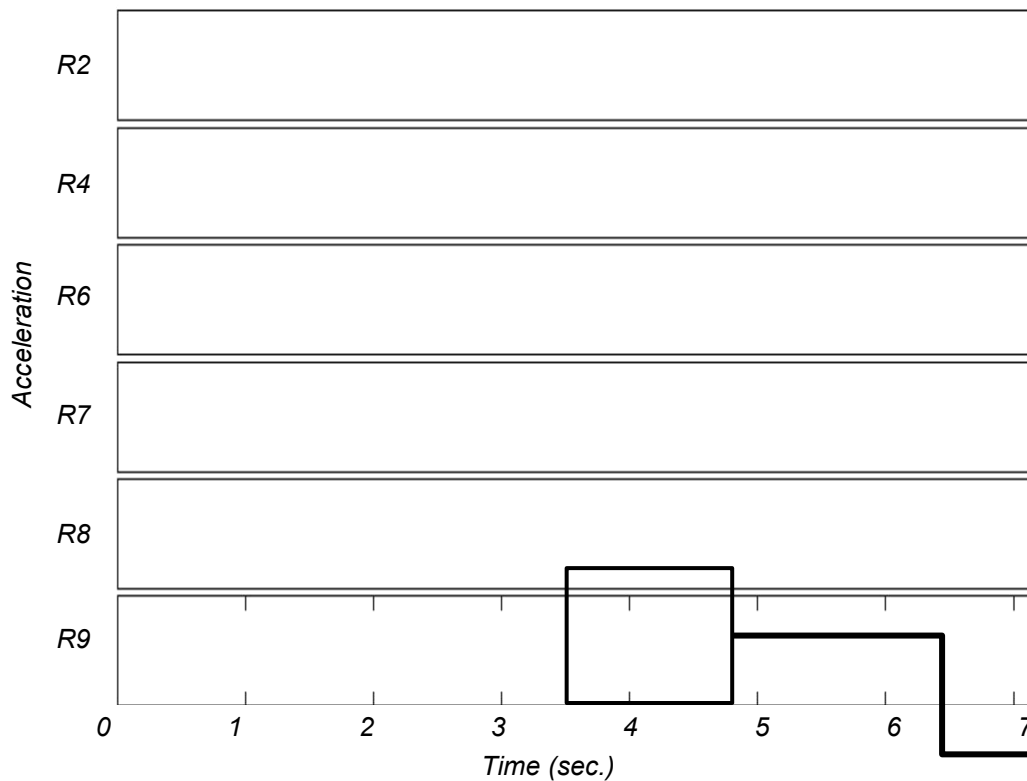
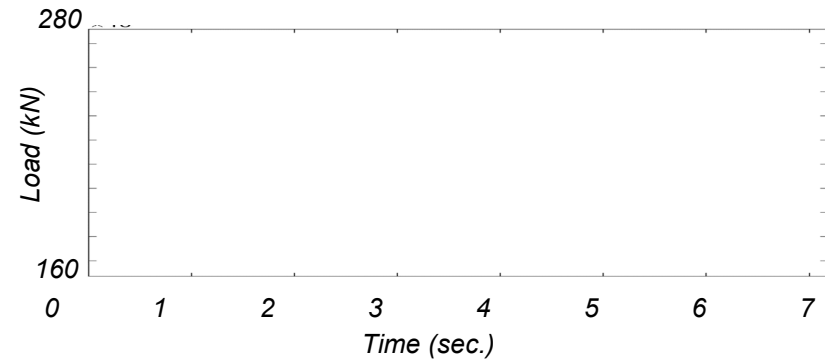
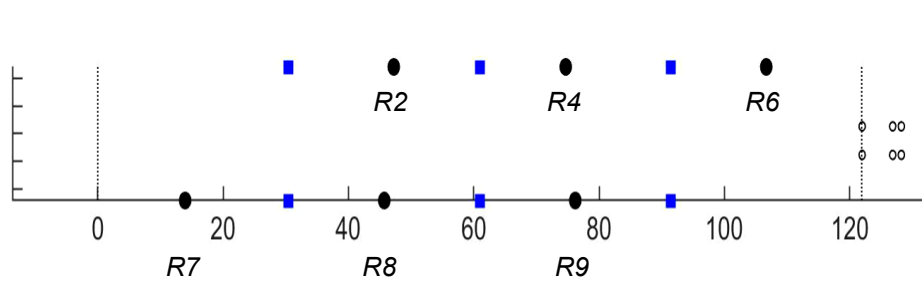
Validation Study – NOBL Testbed



Finite Element Modeling

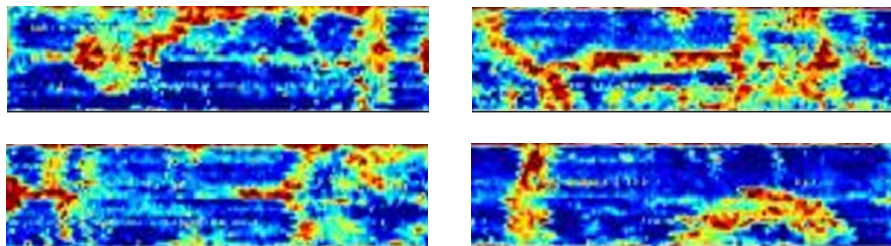
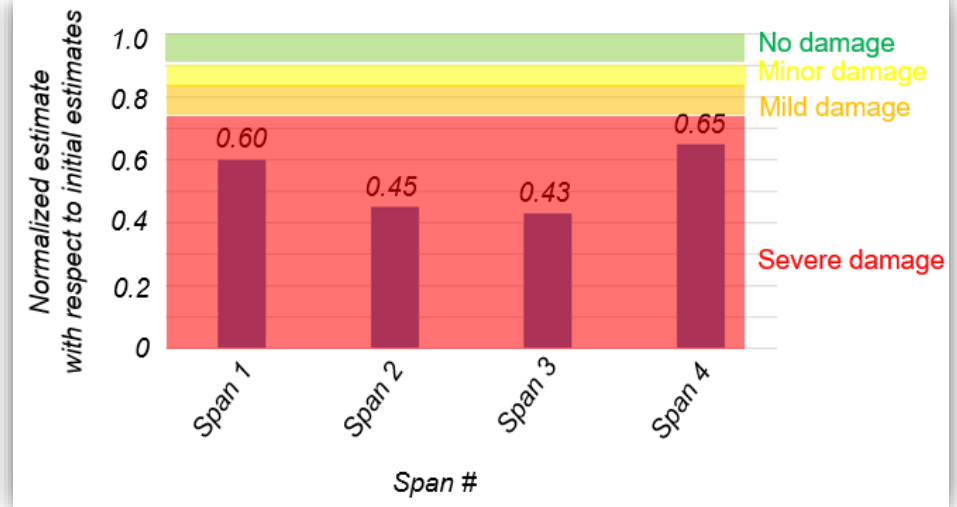
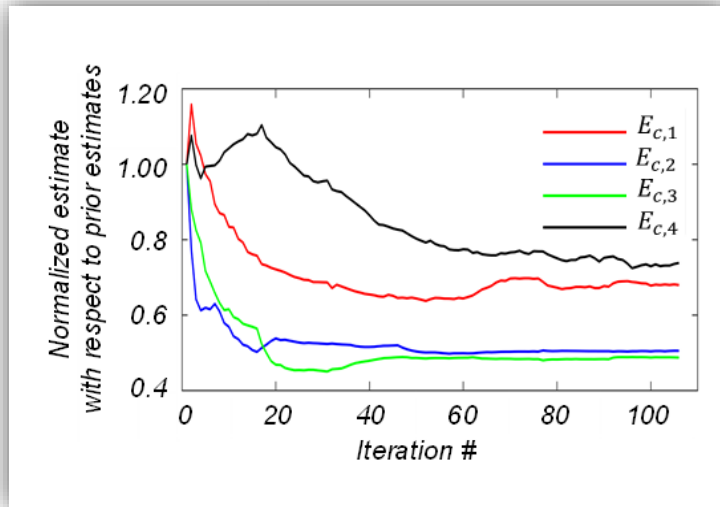
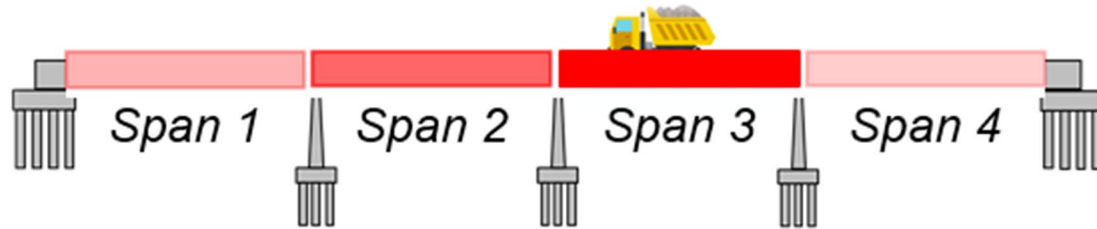


Time-Domain Model Updating

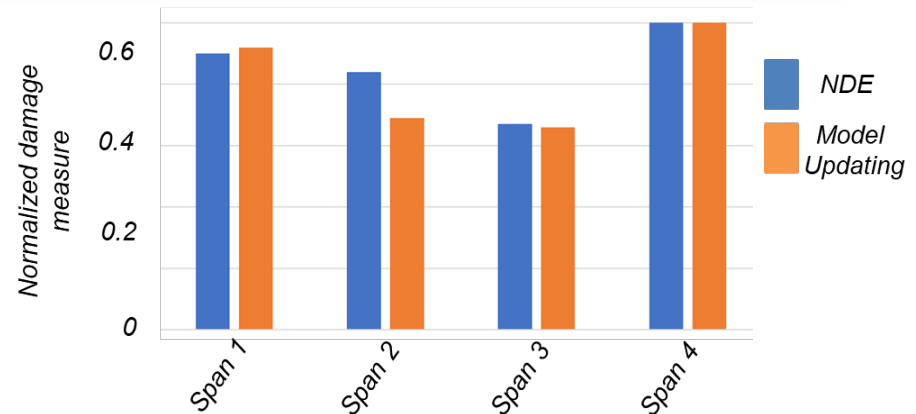


— Measurement — A priori estimates — A posteriori estimates ■ Pier ● Accelerometer

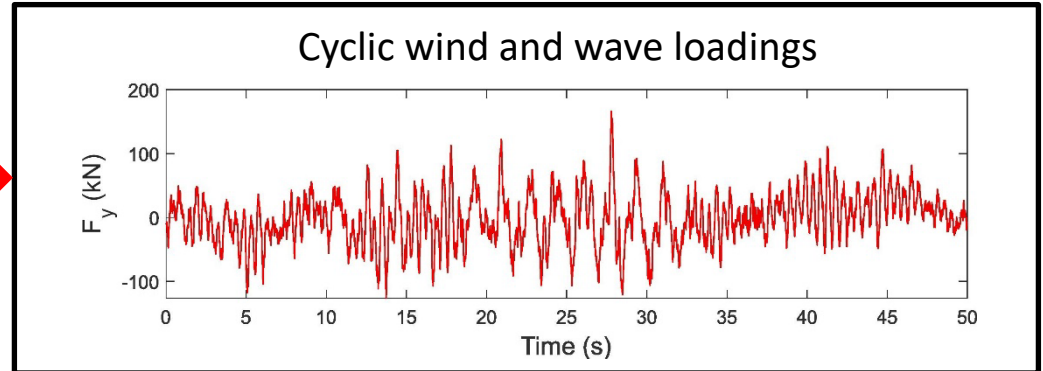
Damage Identification



Less deterioration More deterioration



Operational Monitoring of Offshore Wind Turbines



**Fatigue damage
and failure**



**Expensive operation and maintenance cost
Levelized cost of energy**

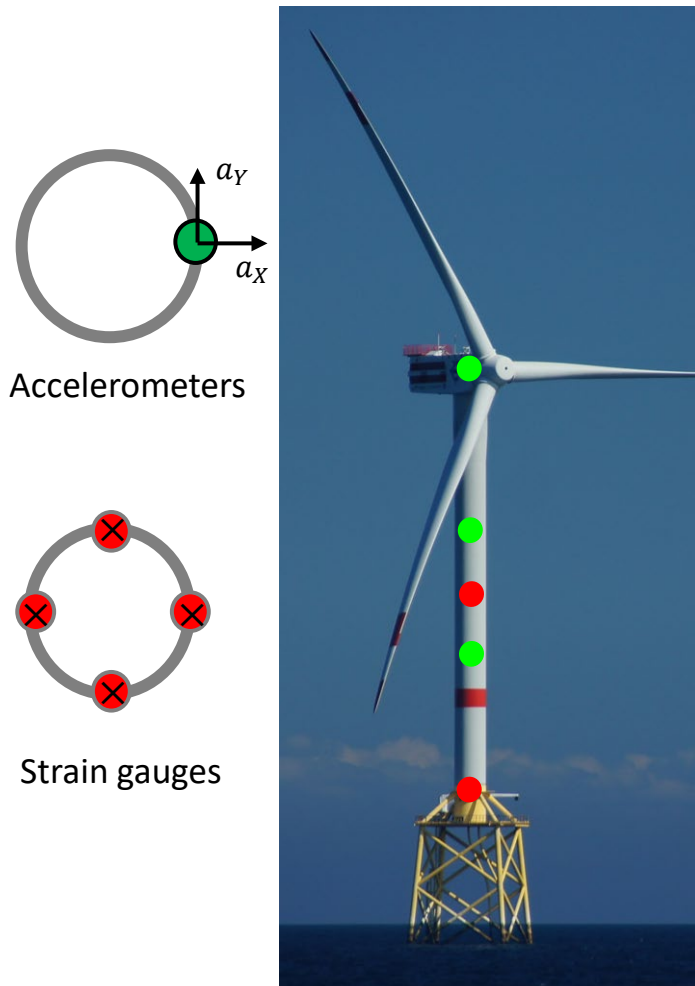


**Condition monitoring for load estimation
through physics-based digital twins**

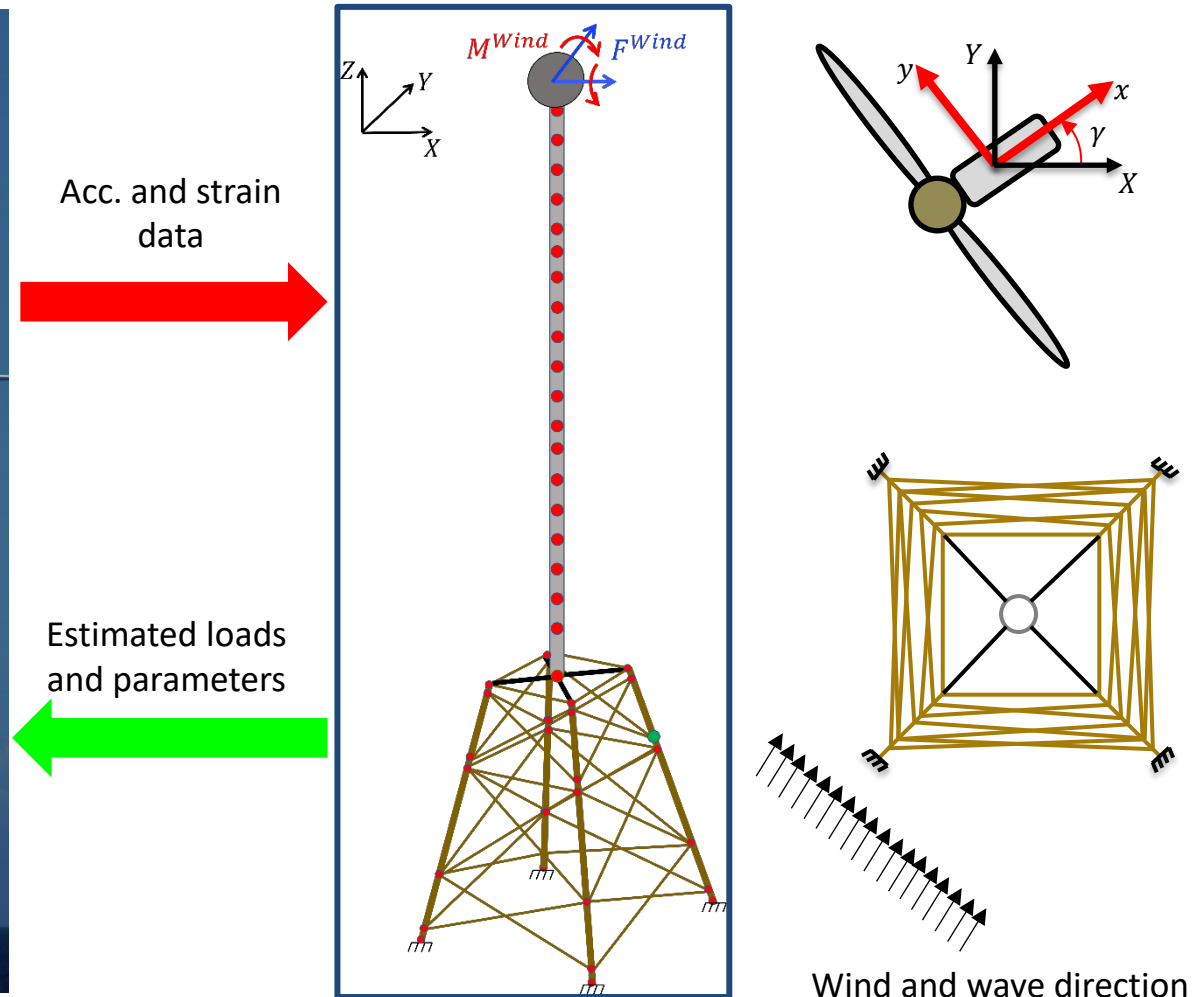


Model Updating Framework

Physical twin



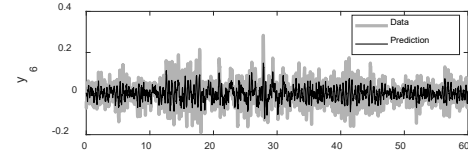
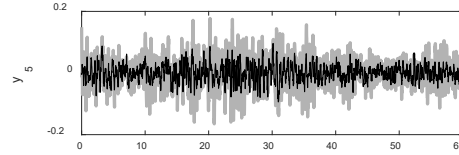
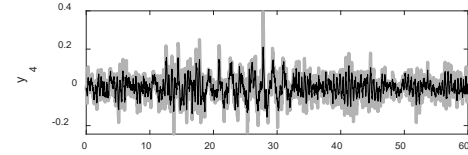
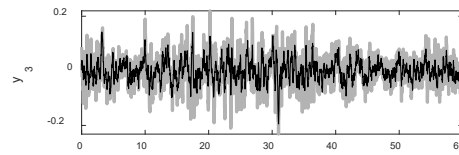
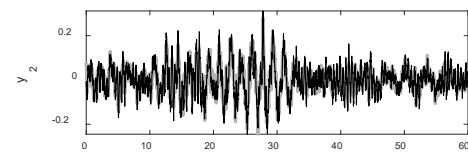
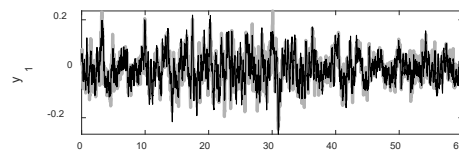
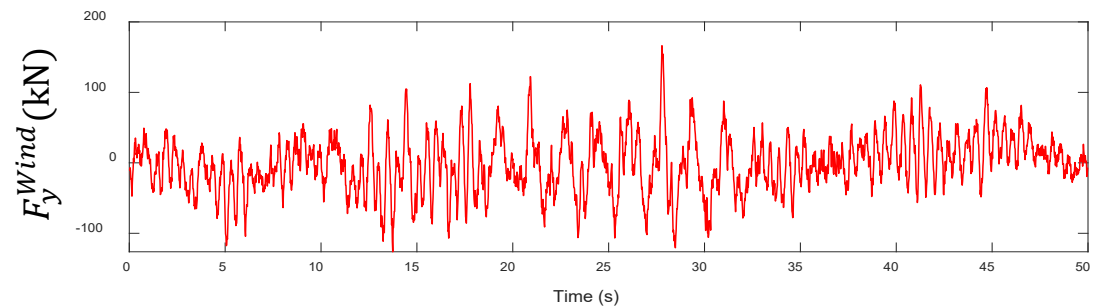
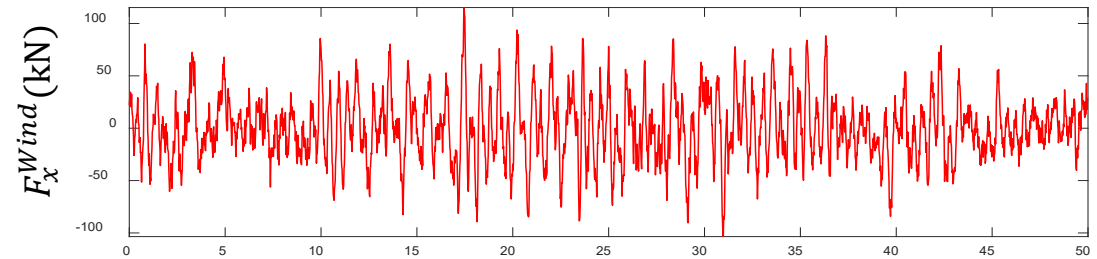
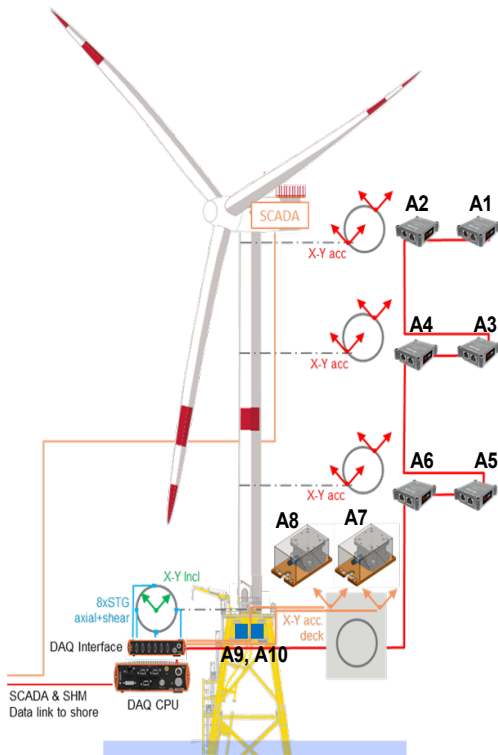
Digital twin



Time-Domain Model Updating



Application to Block Island Wind Turbine Data



M. Song, B. Moaveni, H. Ebrahimi, E. Hines, A. Bajric, "Joint Parameter-input Estimation for Digital Twinning of the Block Island Wind Turbine using Output-only Measurements," *Mechanical Systems and Signal Processing*, 2023.

Limitation: Modeling Error or Model-Form Uncertainty

$$\mathbf{M}(\boldsymbol{\theta})\ddot{\mathbf{u}}_k + \mathbf{C}(\boldsymbol{\theta})\dot{\mathbf{u}}_k + \mathbf{r}_k(\mathbf{u}_{1:k}, \boldsymbol{\theta}) = -\mathbf{M}(\boldsymbol{\theta})\mathbf{L}\ddot{\mathbf{u}}_k^g$$

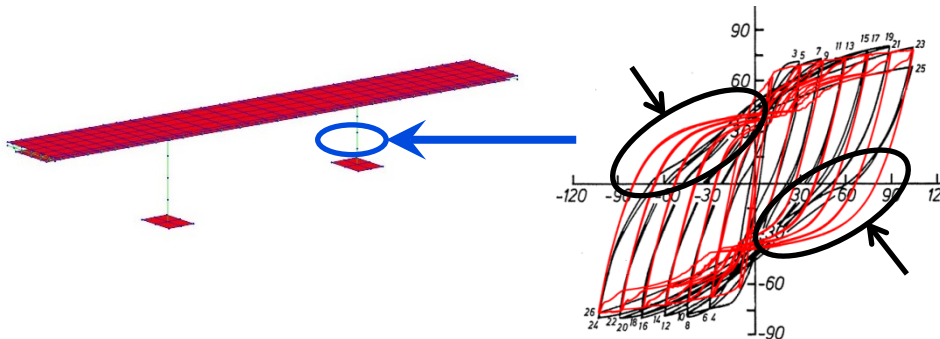
$$(\mathbf{M}(\boldsymbol{\theta}) + \delta\mathbf{M})\ddot{\mathbf{u}}_k + (\mathbf{C}(\boldsymbol{\theta}) + \delta\mathbf{C}_k)\dot{\mathbf{u}}_k + \mathbf{r}_k(\mathbf{u}_{1:k}, \boldsymbol{\theta}) + \delta\mathbf{r}_k = -(\mathbf{M}(\boldsymbol{\theta}) + \delta\mathbf{M})\mathbf{L}\ddot{\mathbf{u}}_k^g + \mathbf{f}_k^{\text{unknown}}$$

$$\mathbf{M}(\boldsymbol{\theta})\ddot{\mathbf{u}}_k + \mathbf{C}(\boldsymbol{\theta})\dot{\mathbf{u}}_k + \mathbf{r}_k(\mathbf{u}_{1:k}, \boldsymbol{\theta}) + \omega_k = -\mathbf{M}(\boldsymbol{\theta})\mathbf{L}\ddot{\mathbf{u}}_k^g$$

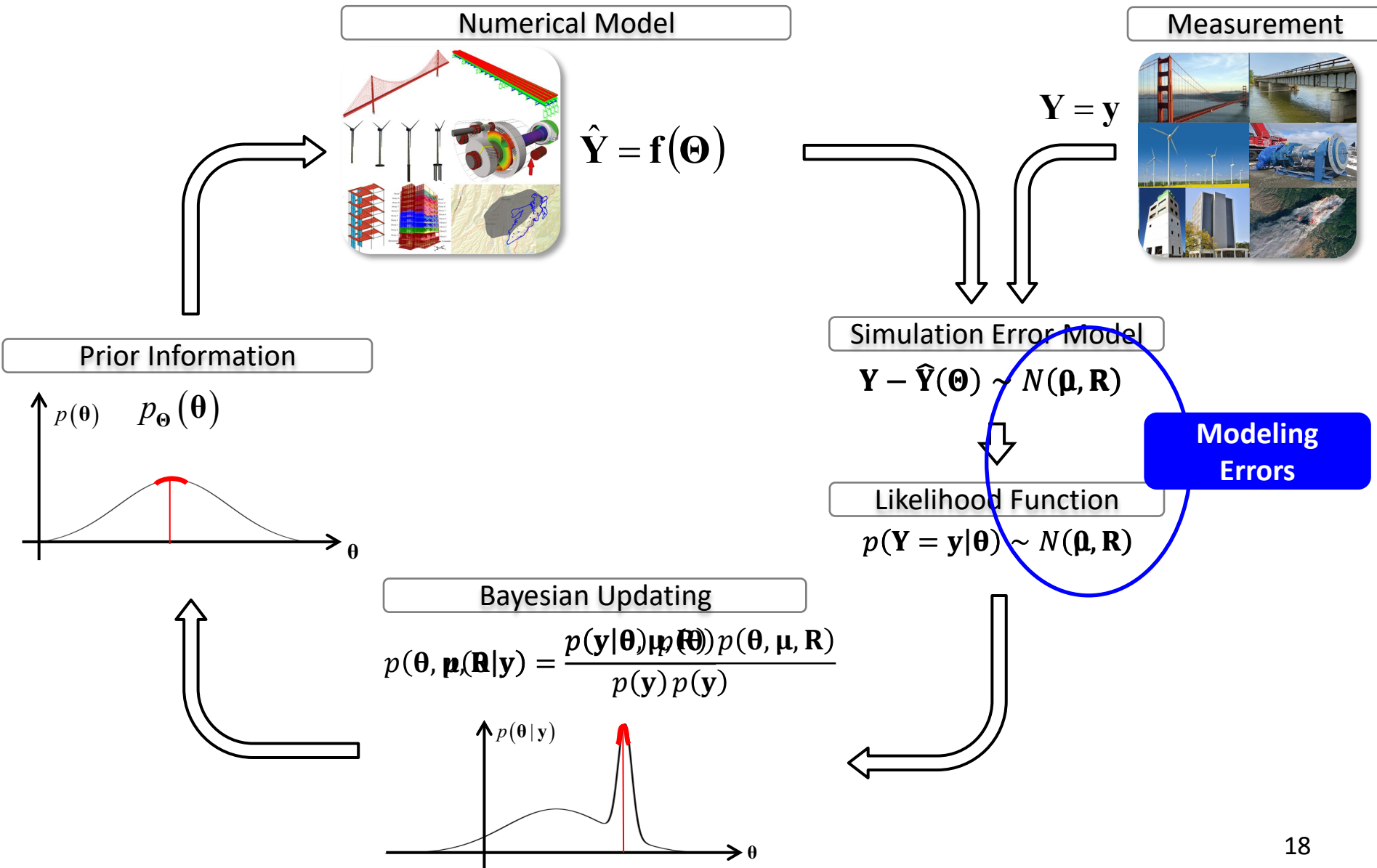
⇒ ω_k : Modeling errors lumped at the structural level

Kennedy and O'Hagan (KOH) method

⇒ Hierarchical approach: Jointly estimate $\boldsymbol{\theta}$ and higher statistics of ω_k @ measurement locations

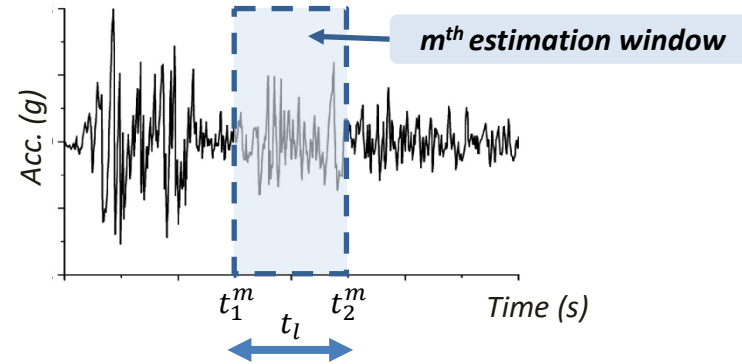


Adaptive Bayesian Inference for Model Updating



Joint Parameter, Input, and Noise Estimation

$$p(\boldsymbol{\Psi}_m, \boldsymbol{\mu}_m, \mathbf{R}_m | \mathbf{y}_{t_1:t_2}) = \frac{p(\mathbf{y}_{t_1:t_2} | \boldsymbol{\Psi}_m, \boldsymbol{\mu}_m, \mathbf{R}_m) p(\boldsymbol{\Psi}_m, \boldsymbol{\mu}_m, \mathbf{R}_m)}{p(\mathbf{y}_{t_1:t_2})}$$



$$\{\hat{\boldsymbol{\Psi}}_m^+, \hat{\boldsymbol{\mu}}_m^+, \hat{\mathbf{R}}_m^+\} = \underset{\boldsymbol{\Psi}_m, \boldsymbol{\mu}_m, \mathbf{R}_m}{\operatorname{argmax}} p(\boldsymbol{\Psi}_m, \boldsymbol{\mu}_m, \mathbf{R}_m | \mathbf{y}_{t_1^m:t_2^m}^m)$$

$$\{\hat{\boldsymbol{\Psi}}_m^+\} = \underset{\boldsymbol{\Psi}_m}{\operatorname{argmax}} p(\boldsymbol{\Psi}_m | \boldsymbol{\mu}_m, \mathbf{R}_m, \mathbf{y}_{t_1^m:t_2^m}^m)$$

$$\begin{aligned} \hat{\boldsymbol{\Psi}}_m^+ &= \hat{\boldsymbol{\Psi}}_m^- + \mathbf{K}_m (\mathbf{y}_{t_1^m:t_2^m}^m - \mathbf{h}_{t_1^m:t_2^m}(\hat{\boldsymbol{\Psi}}_m^-) - \hat{\boldsymbol{\mu}}_{t_1^m:t_2^m}^+) \\ \mathbf{P}_{\boldsymbol{\Psi},m}^+ &= \mathbf{P}_{\boldsymbol{\Psi},m}^- - \mathbf{K}_m \mathbf{P}_{\mathbf{y}\mathbf{y},m} \mathbf{K}_m^T \end{aligned}$$

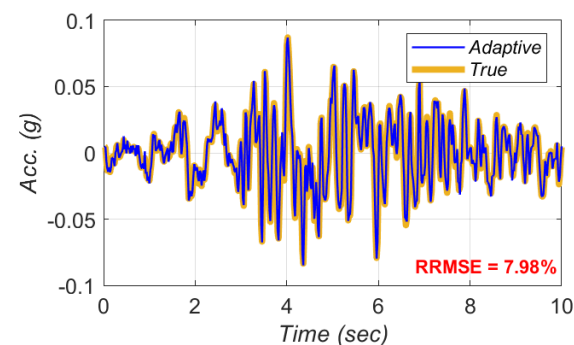
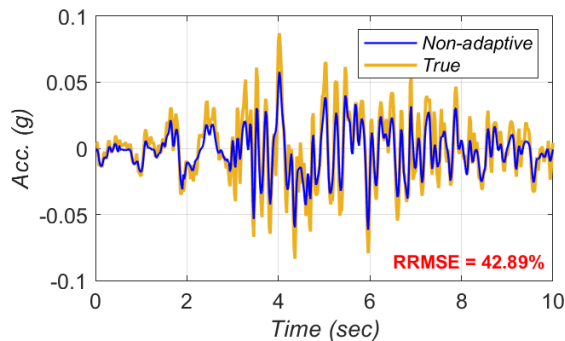
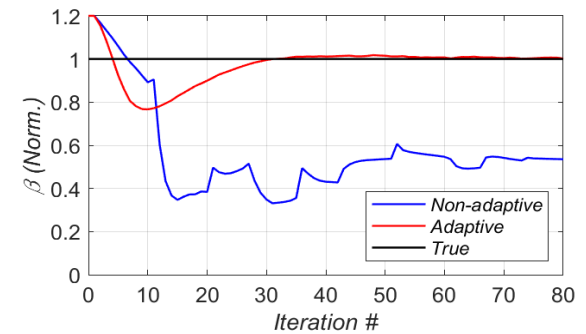
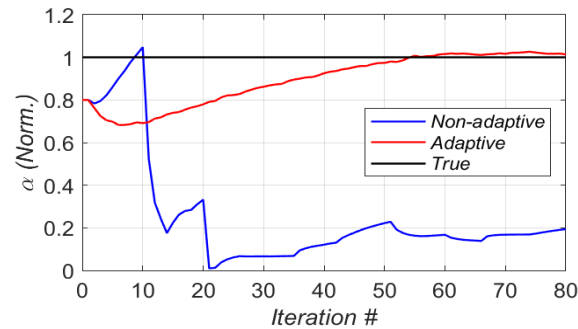
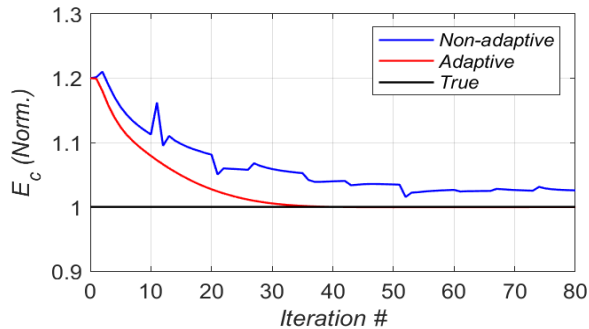
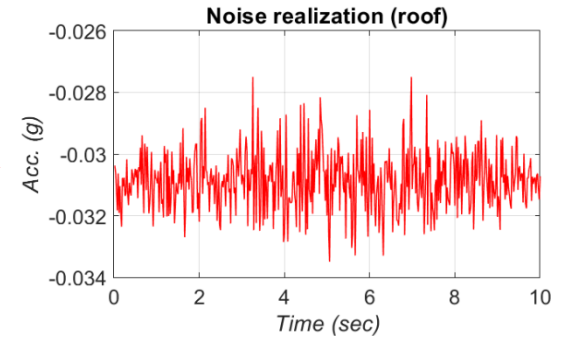
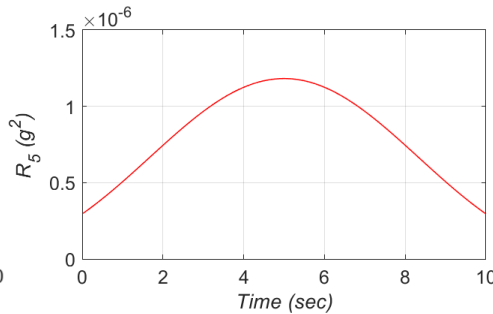
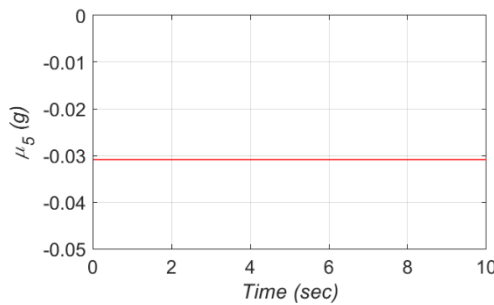
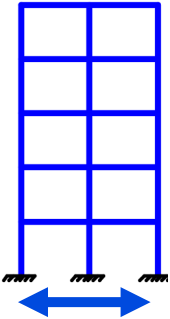
$$\{\hat{\boldsymbol{\mu}}_m^+, \hat{\mathbf{R}}_m^+\} = \underset{\boldsymbol{\mu}_m, \mathbf{R}_m}{\operatorname{argmax}} p(\boldsymbol{\mu}_m, \mathbf{R}_m | \mathbf{y}_{t_1^m:t_2^m}^m)$$

$$\begin{aligned} \hat{\boldsymbol{\mu}}_m^+ &= \frac{\lambda_m^-}{t_l + \lambda_m^-} \hat{\boldsymbol{\mu}}_m^- + \frac{t_l}{t_l + \lambda_m^-} \bar{\boldsymbol{\omega}}_m \\ \hat{\mathbf{R}}_m^+ &= \frac{\mathbf{V}_m^+}{v_m^+ + n_y + 1} \end{aligned}$$

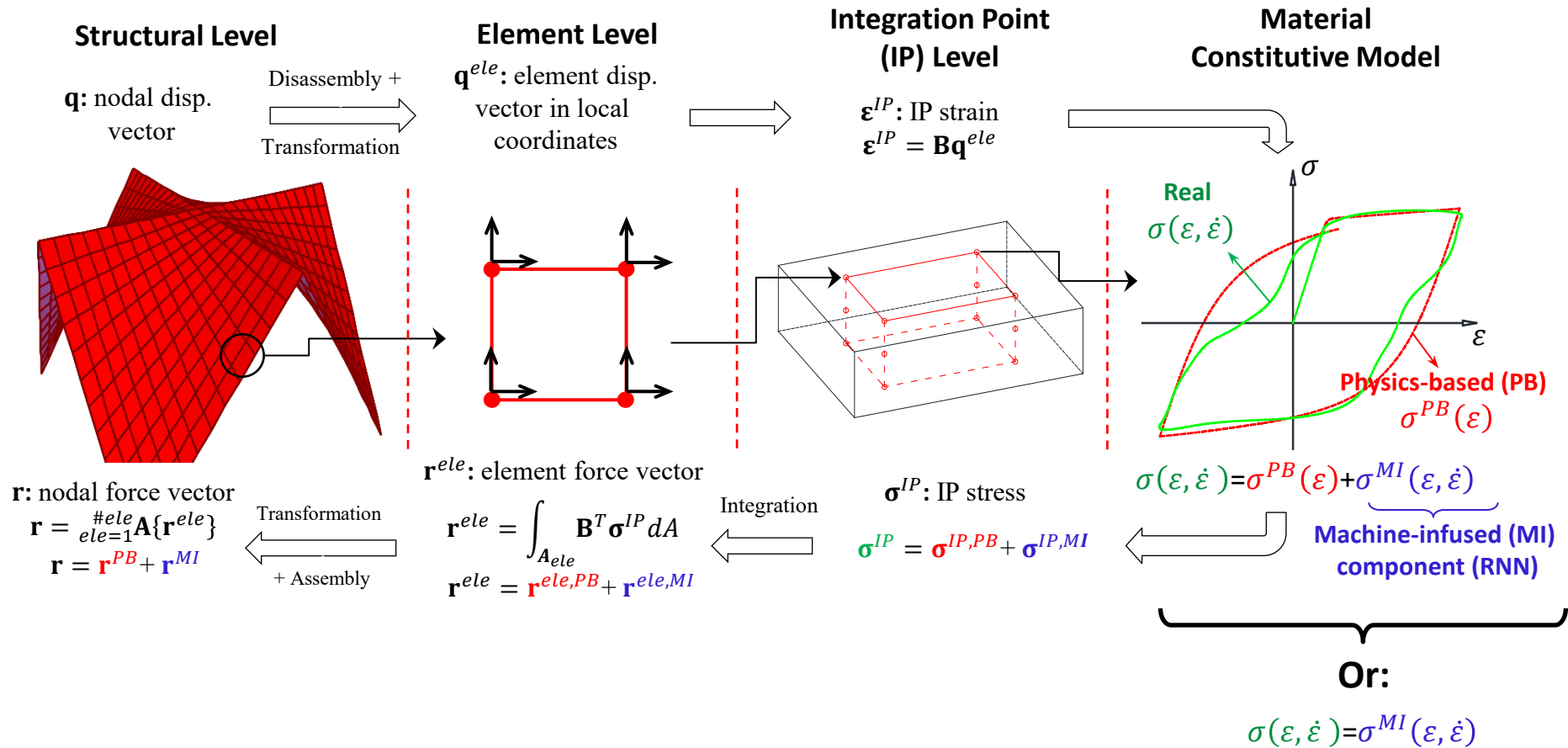
Fixed-Point Iterations

Non-Adaptive vs. Adaptive Estimation

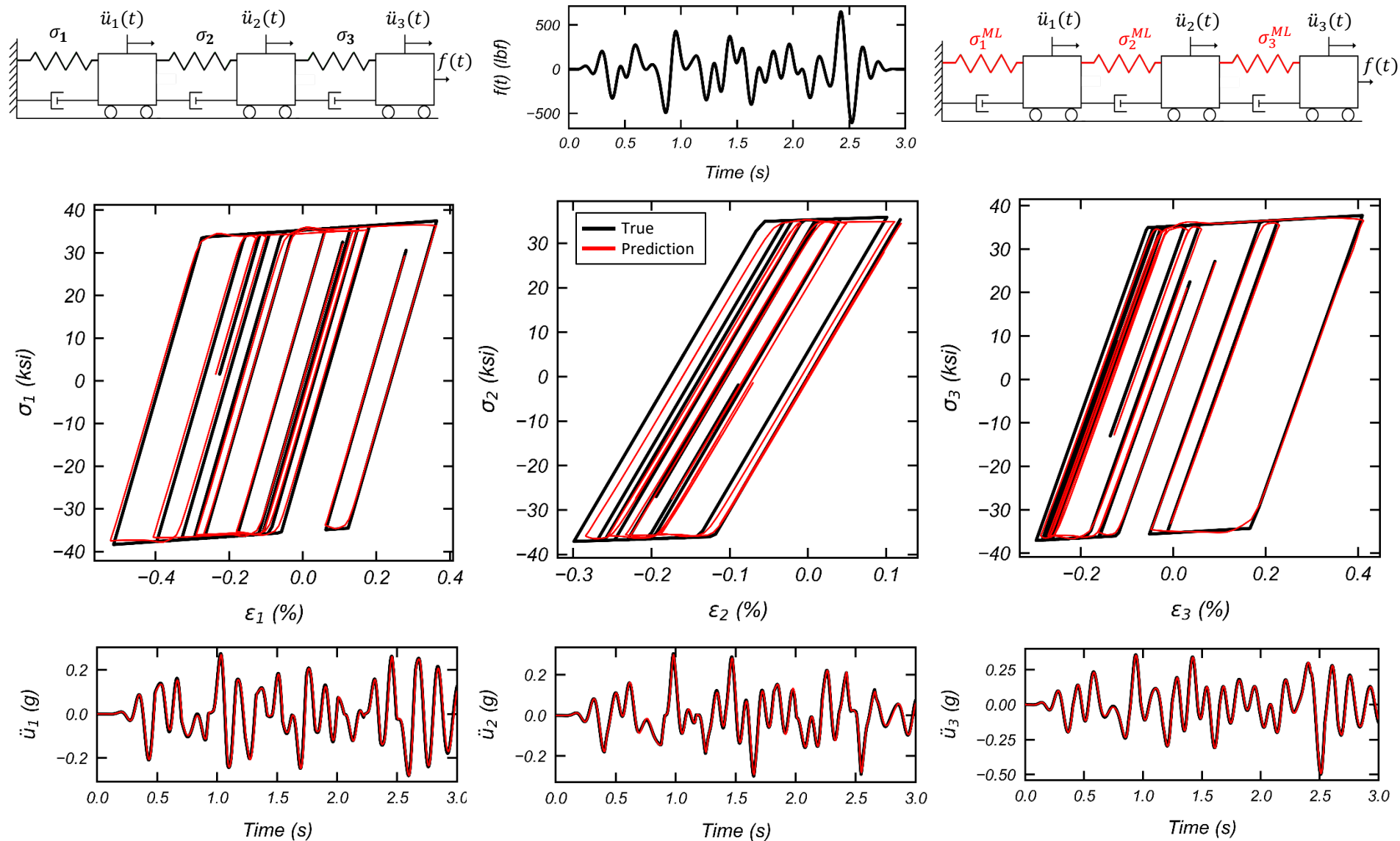
Estimate E_c , α , β & Input Motion with Non-Stationary Non-Zero Mean Noise



Machine-Infused Physics-Based Model Updating



Proof-of-Concept – Learning Material Model





Thank you!

