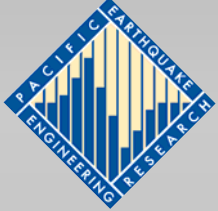


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# Probabilistic Performance-Based Optimum Seismic Design of (Bridge) Structures

PI: Joel P. Conte

*Graduate Student:* Yong Li

Sponsored by the Pacific Earthquake Engineering Research Center

# *Outline*

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## **(I) PEER Performance Based Earthquake Engineering (PBEE) Methodology**

- ❖ Probabilistic **Seismic Hazard** Analysis
- ❖ Probabilistic **Seismic Demand** Hazard Analysis
- ❖ Probabilistic **Seismic Damage** Hazard Analysis
- ❖ Probabilistic **Seismic Loss** Hazard Analysis

## **(II) Parametric PBEE Analysis of SDOF System**

- ❖ **Yield strength  $F_y$**  as varying parameter
- ❖ **Hardening ratio  $b$**  as varying parameter
- ❖ Both **mass  $m$  and initial stiffness  $k_0$**  as varying parameters (with fixed  $k_0/m$ )
- ❖ **Mass  $m$**  as varying parameter
- ❖ **Initial stiffness  $k_0$**  as varying parameter

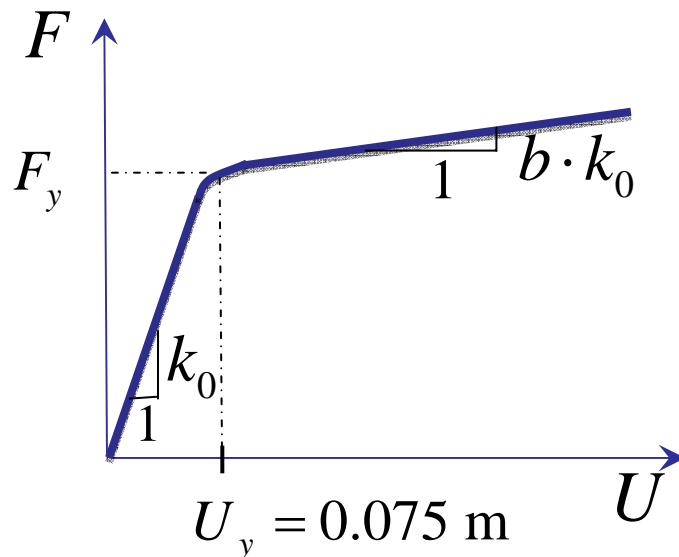
## **(III) Optimization Formulation for Probabilistic Performance Based Optimum Seismic Design**

***(I) PEER PBEE Methodology***  
***(Forward PBEE Analysis)***

# SDOF Bridge Model

- **Single-Degree-of-Freedom Bridge Model:**

- SDOF bridge model with the same initial period as an MDOF model of the Middle Channel Humboldt Bay Bridge previously developed in OpenSees.



**SDOF Model**  
(Menegotto-Pinto)

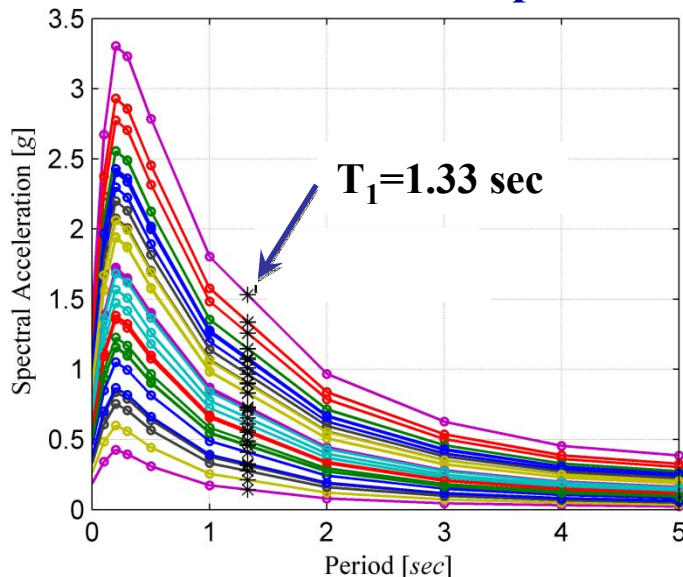


$T_1 = 1.33 \text{ s}$   
 $m = 6,150 \text{ tons}$   
 $k_0 = 137,200 \text{ kN/m}$   
 $F_y = 10,290 \text{ kN}$   
 $b = 0.10$   
 $\xi = 0.02$

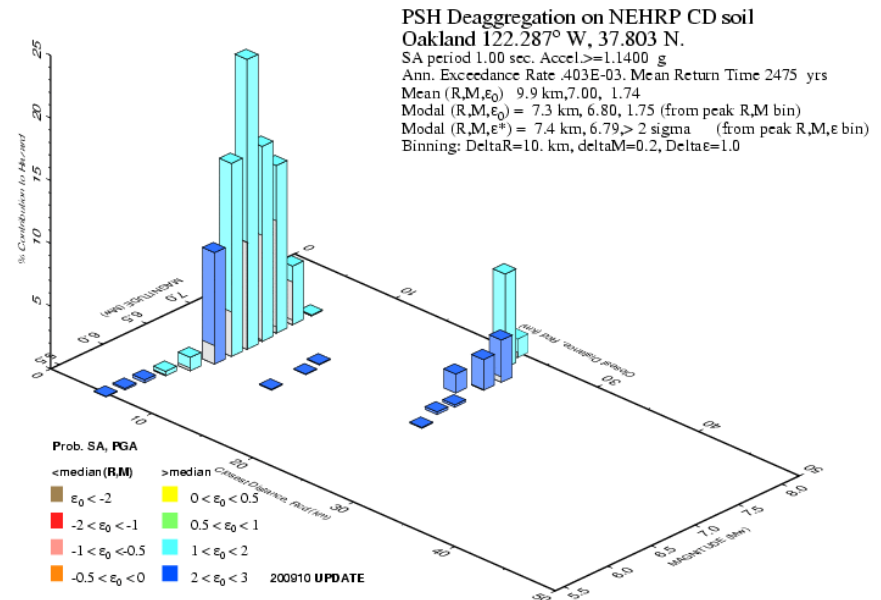
# Site Description and Prob. Seismic Hazard Analysis

- **Site Location:** Oakland (37.803N, 122.287W)
- **Site Condition:**  $V_{s30} = 360\text{m/s}$  (Class C-D)
  - **Uniform Hazard Spectra** are obtained for **30 hazard levels** from the **USGS 2008 Interactive Deaggregation** software/website (beta version)
- **Uniform Hazard Spectra and M-R Deaggregation:**

**Uniform Hazard Spectra**



**Deaggregation ( $T_1 = 1 \text{ sec}$ , 2% in 50 years)**

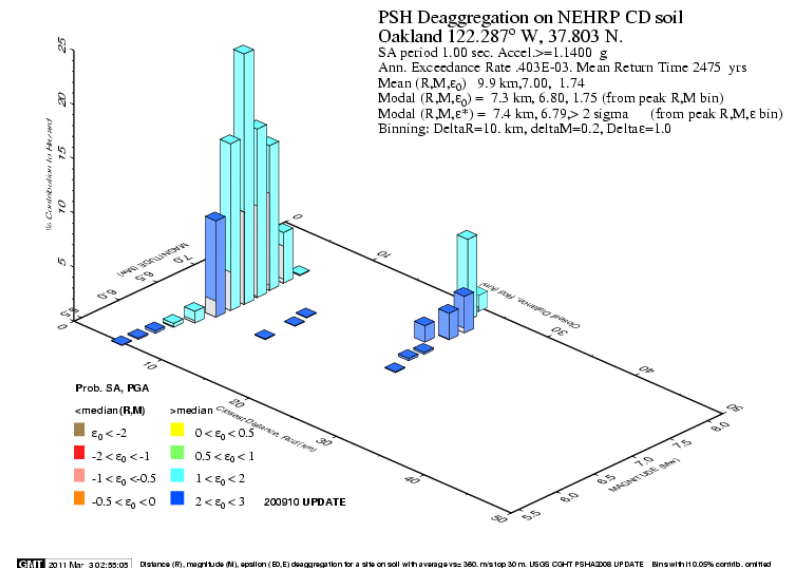


# Earthquake Record Selection

## • Earthquake Record Selection:

➤ **Records were selected from NGA database** based on geological and seismological conditions (e.g., fault mechanism), M-R deaggregation of probabilistic seismic hazard results, and local site condition (e.g.,  $V_{s30}$ )

- NGA Excel Flatfile (3551 records)
- Mechanism: Strike-slip (➔1004 records)
- Magnitudes: 5.9 to 7.3 (➔652 records)
- Distance: 0 – 40 km (➔193 records)
- $V_{s30}$ : C-D range (➔146 records)



➤ **146 horizontal ground motion components were selected from NGA database**

# Probabilistic Seismic Hazard Analysis

- Seismic Hazard Curve (for single/scalar Intensity Measure  $IM$ ):

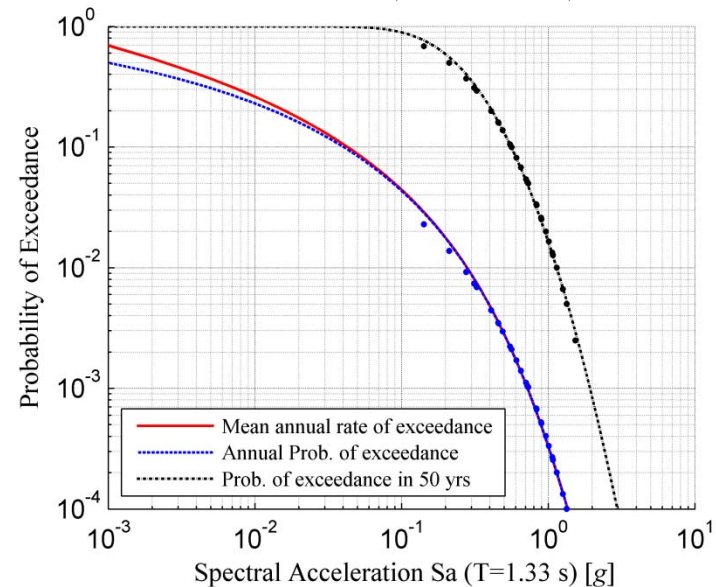
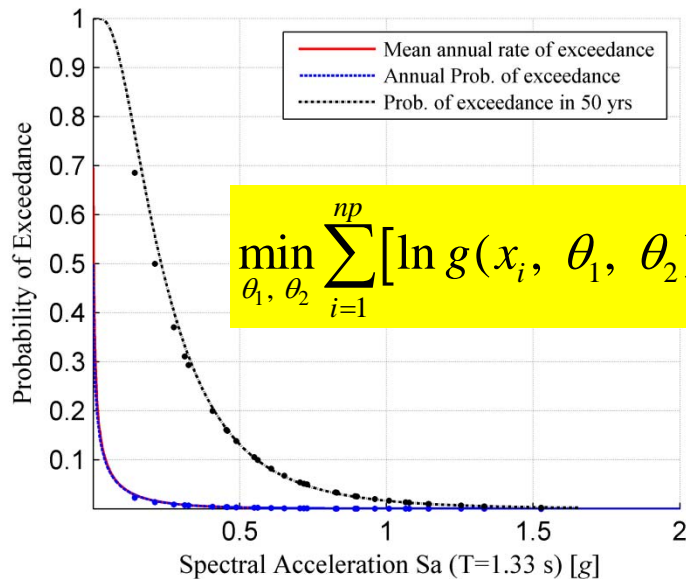
$$v_{IM}(im) = \sum_{i=1}^{N_{flt}} v_i \cdot \int_{R_i} \int_{M_i} P[IM > im | m, r] \cdot f_{M_i}(m) \cdot f_{R_i}(r) \cdot dm \cdot dr$$

$$\square \rightarrow S_a(T_1, \xi=5\%)$$

- Least Square Fitting of 30 Data Points ( $IM_i, MARE_i$ ) from USGS:

➤ Data Points:  $(x_i, y_i) = (IM_i, PE50_i)$   $i = 1, 2, \dots, 30$

➤ Fitting Function (lognormal CCDF):  $g(x, \theta_1, \theta_2) = 1 - \Phi\left(\frac{\ln x - \theta_1}{\theta_2}\right)$



# Probabilistic Seismic Demand Hazard Analysis

- **Demand Hazard Curve:**

$$v_{EDP}(edp) = \int_{IM} P[EDP > edp | IM] dv_{IM}(im)$$

Complementary CDF of EDP given IM

Seismic hazard curve

- **Probabilistic Seismic Demand Analysis conditional on IM:**

➤ “**Cloud Method**” based on following assumptions:

1. Linear regression after **ln** transformation of predictor and response variables:

$$\hat{\mu}_{EDP|\ln IM} = E[\ln EDP | IM = im] = \hat{a} + \hat{b} \ln IM$$

2. Variance of EDP independent of IM:

$$\hat{S}^2 = \frac{\sum_{i=1}^n \left( \ln edp_i - (\hat{a} + \hat{b} \ln im_i) \right)^2}{n - 2}$$

3. Probability distribution of EDP|IM assumed to be **Lognormal**:

$$EDP | IM \sim LN(\lambda, \zeta) \quad \text{where } \lambda = \hat{\mu}_{\ln EDP|IM} = \hat{a} + \hat{b} \ln IM \text{ and}$$

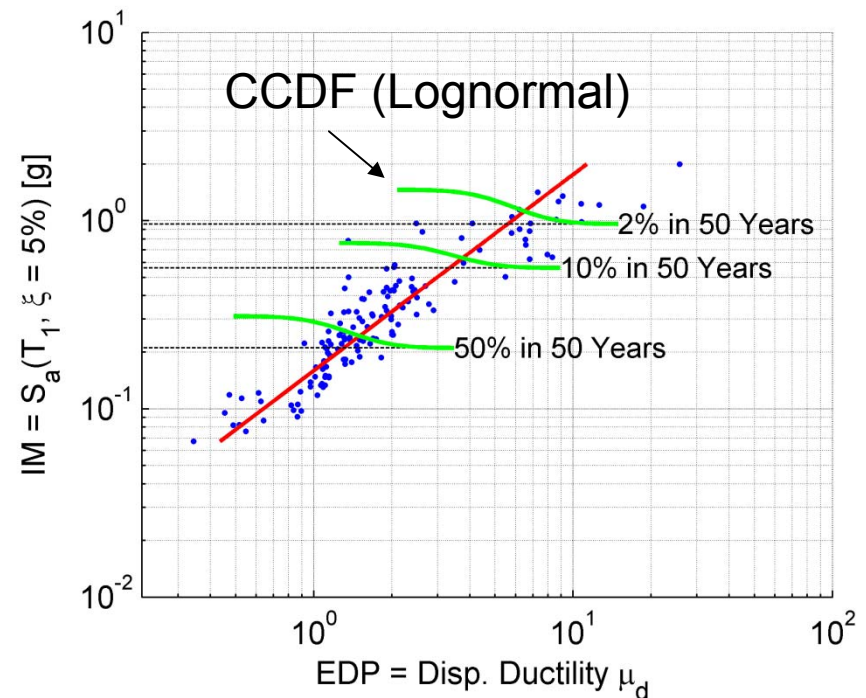
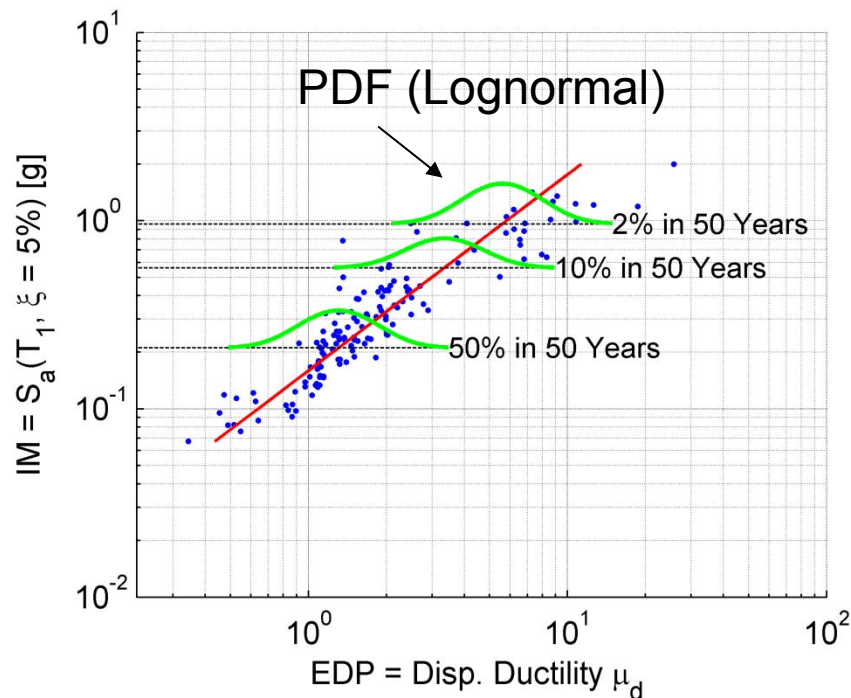
$$\zeta = \hat{\sigma}_{\ln EDP|IM} = \sqrt{\hat{S}^2}$$



# Probabilistic Seismic Demand Hazard Analysis

## • EDP: Displacement Ductility

- Regression analysis results and PDF/CCDF for EDP|IM shown at three hazard levels commonly used in Earthquake Engineering:



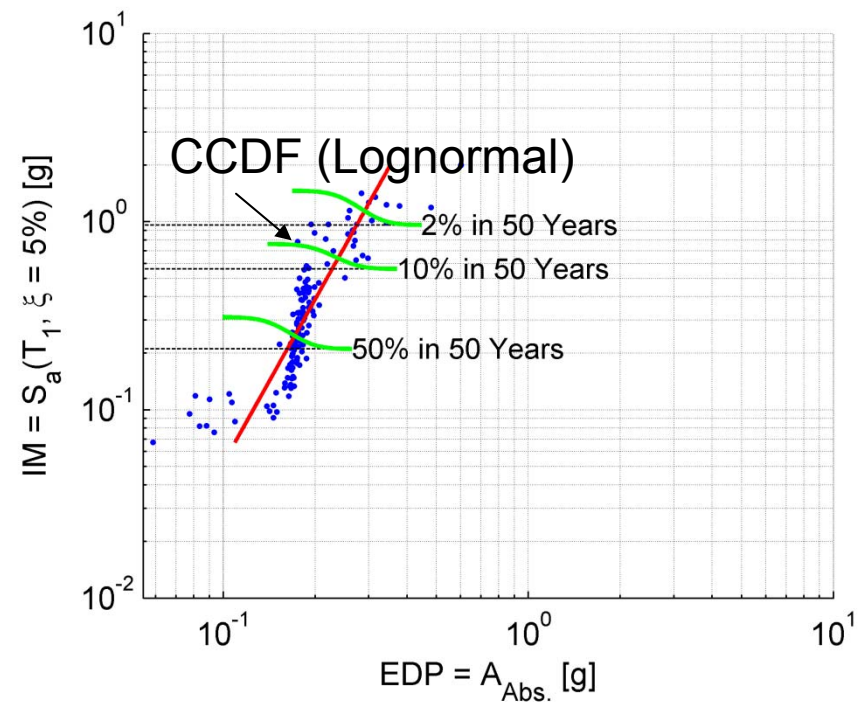
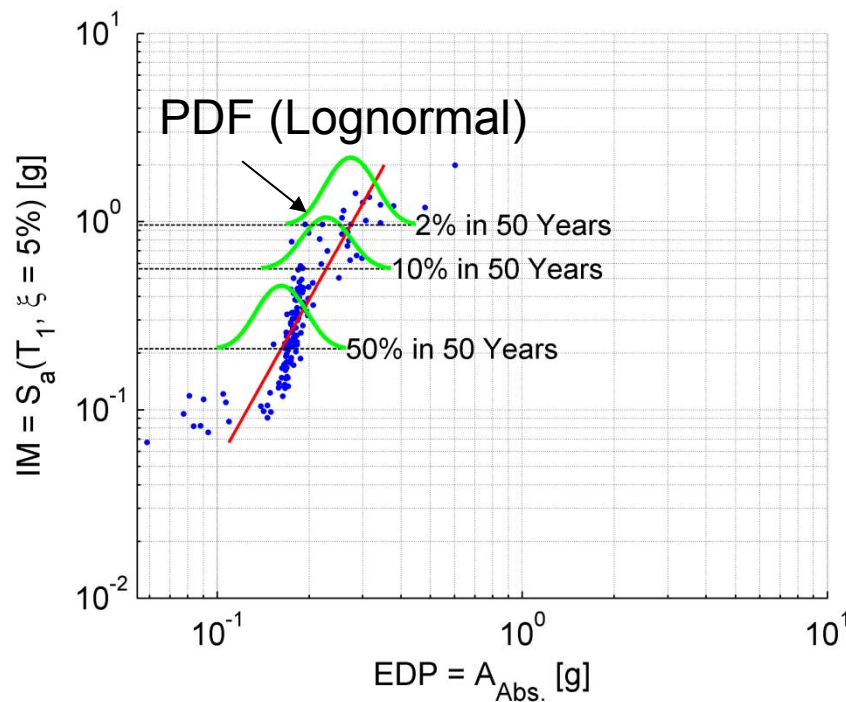
Where the displacement ductility is defined as:

$$\mu_d = \text{Max}_{0 < t < t_d} \left( \frac{u(t)}{U_y} \right)$$

# Probabilistic Seismic Demand Hazard Analysis

## • EDP: Peak Absolute Acceleration

- Regression analysis results and PDF/CCDF for EDP|IM shown at three hazard levels commonly used in Earthquake Engineering:



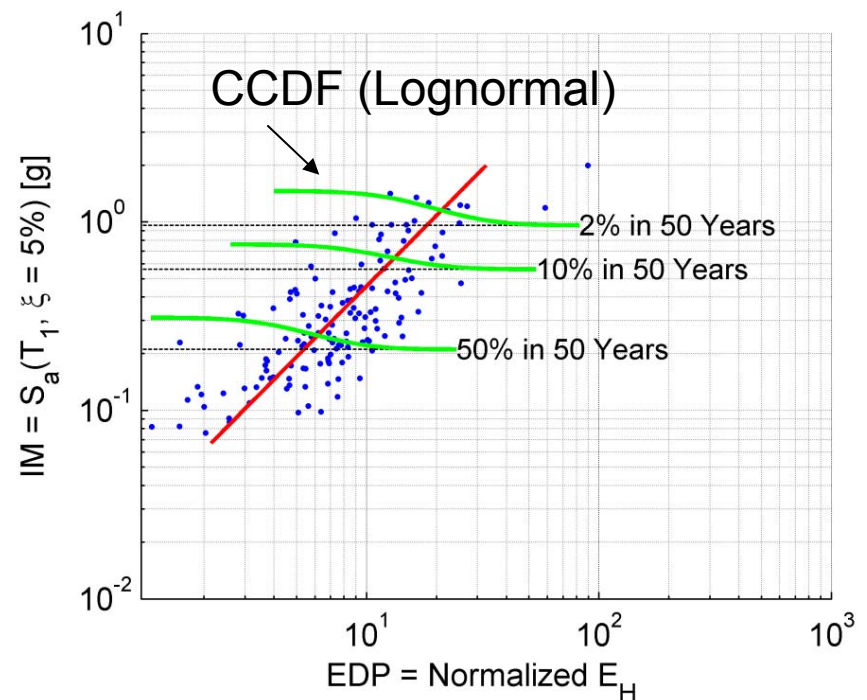
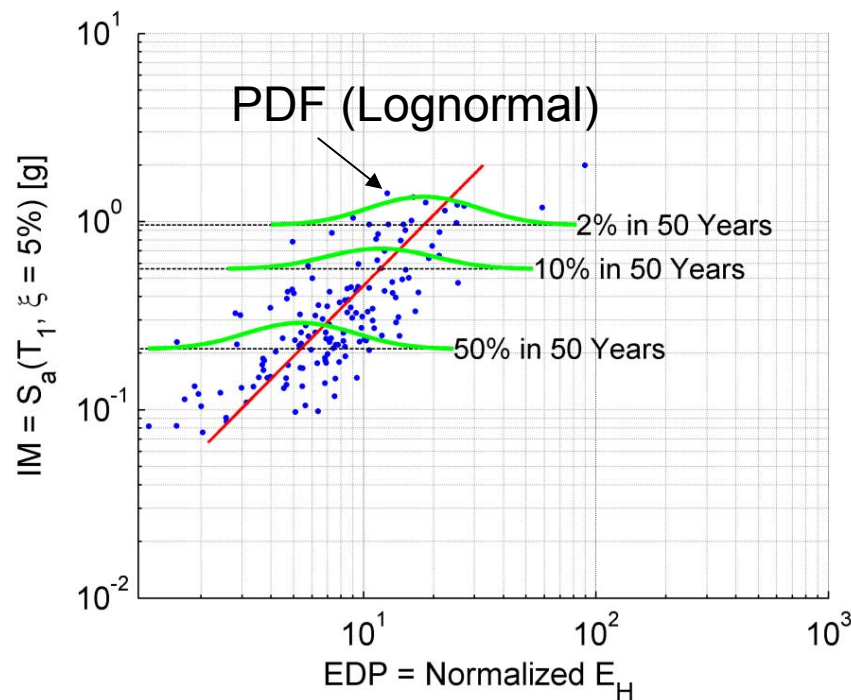
Where the peak absolute acceleration is defined as:

$$A_{Abs.} = \text{Max}_{0 < t < t_d} \left( \frac{\ddot{u}(t)}{g} \right)$$

# Probabilistic Seismic Demand Hazard Analysis

- **EDP: Normalized Hysteretic Energy Dissipated**

- Regression analysis results and PDF/CCDF for EDP|IM shown at three hazard levels commonly used in Earthquake Engineering:



Where the Normalized hysteretic energy is defined as:

$$E_H = \frac{\int_0^{t_d} R(t) du(t) - E_E}{F_y U_y}$$

# Probabilistic Seismic Demand Hazard Analysis

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- Demand Hazard Curve:

$$v_{EDP}(edp) = \int_{IM} P[EDP > edp | IM = im] |dv_{IM}(im)|$$

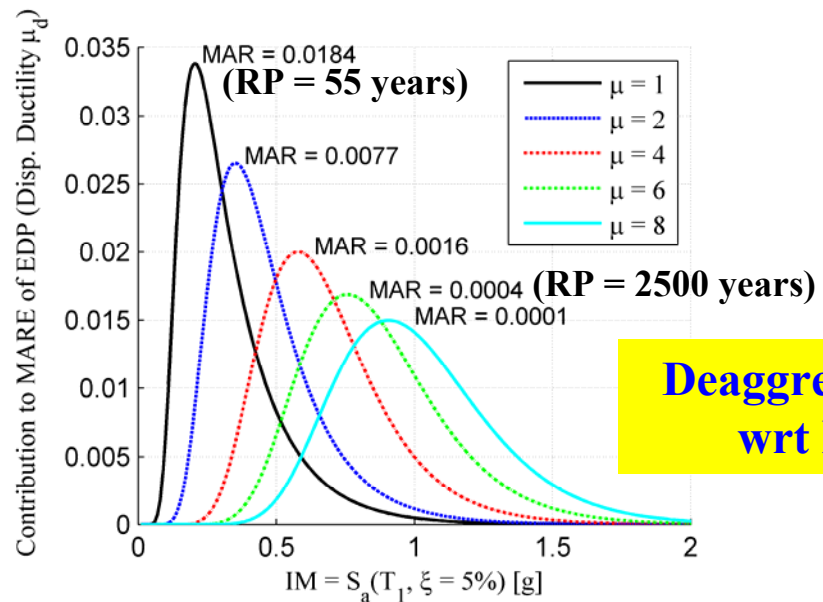
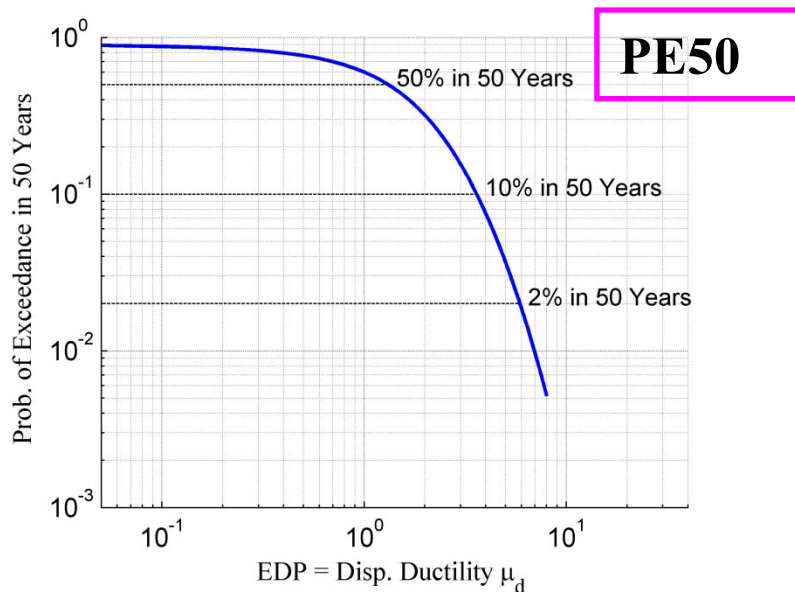
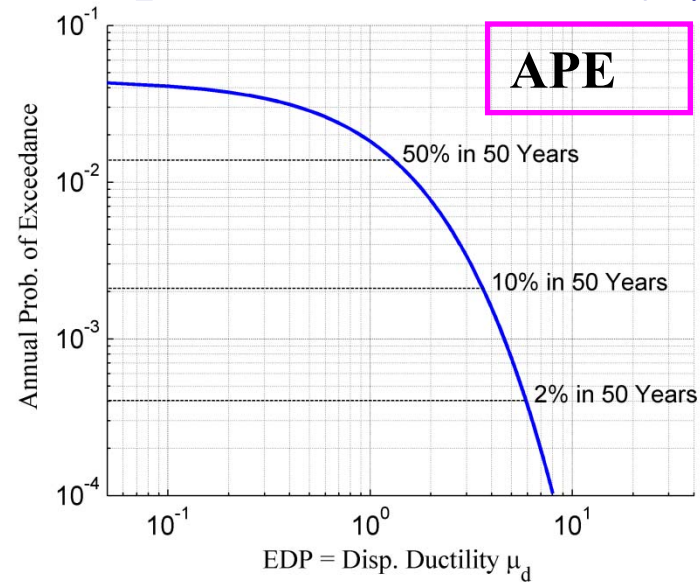
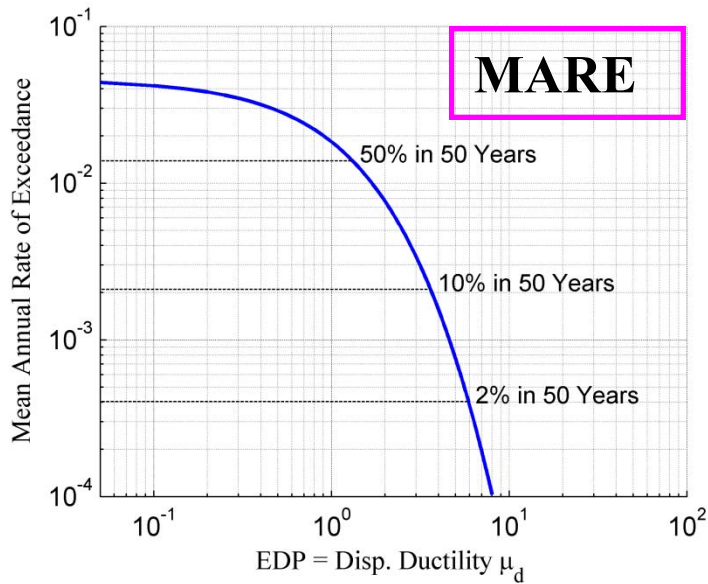
- Deaggregation of  $v_{EDP}(edp)$  with respect to IM:

$$\begin{aligned} v_{EDP}(edp) &= \int_{IM} P[EDP > edp | IM] \left| \frac{dv_{IM}(im)}{dim} \right| dim \\ &= \sum_i P[EDP > edp | IM = im_i] \left| \frac{dv_{IM}(im_i)}{\Delta(im)_i} \right| \cdot \Delta(im)_i \end{aligned}$$

Contribution of bin  
 $IM = im_i$  to  $v_{EDP}(edp)$

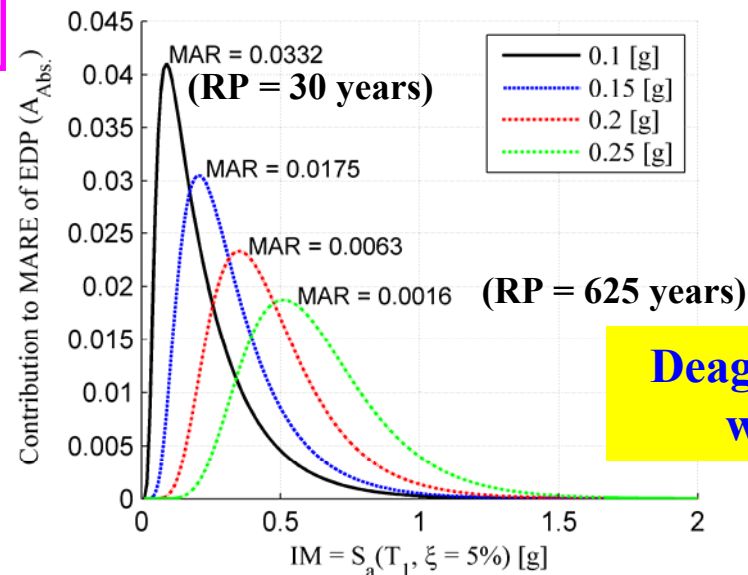
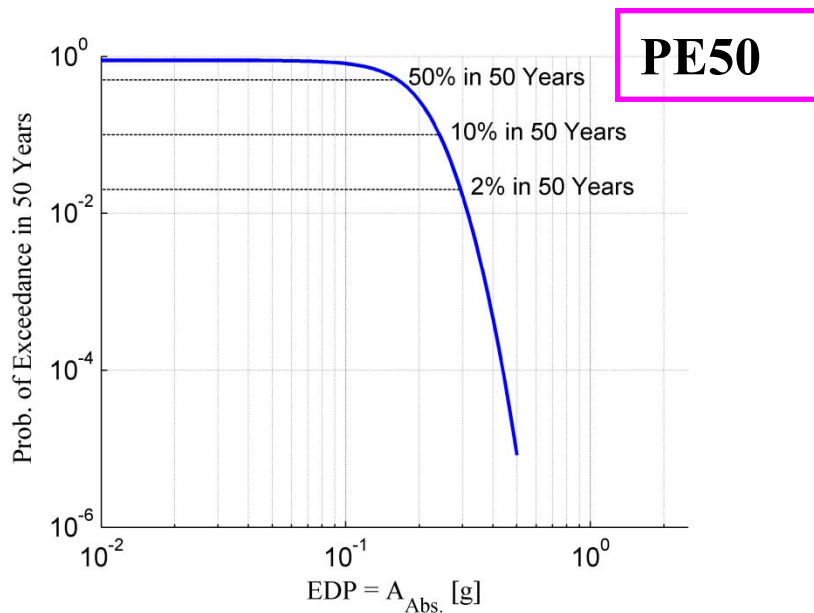
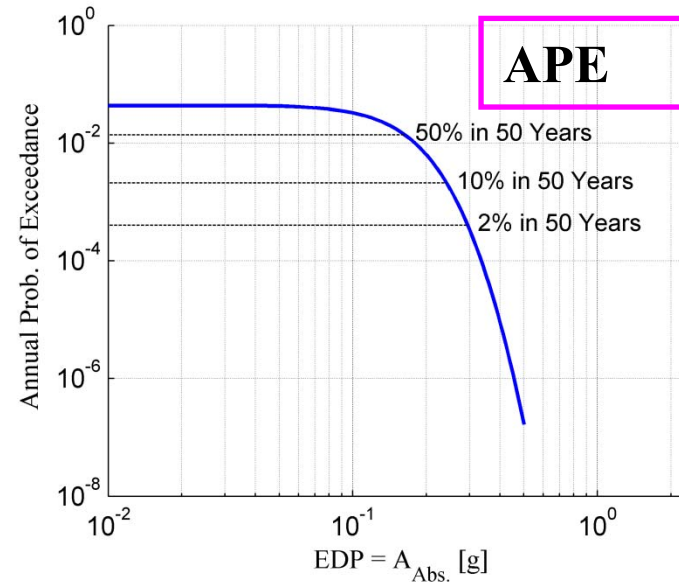
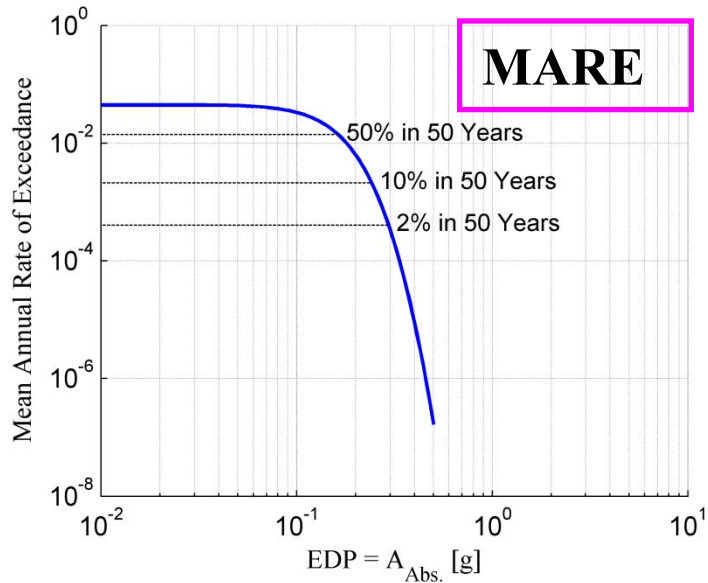
# Probabilistic Seismic Demand Hazard Analysis

- Demand Hazard Curve for **EDP = Displacement Ductility  $\mu_d$** :



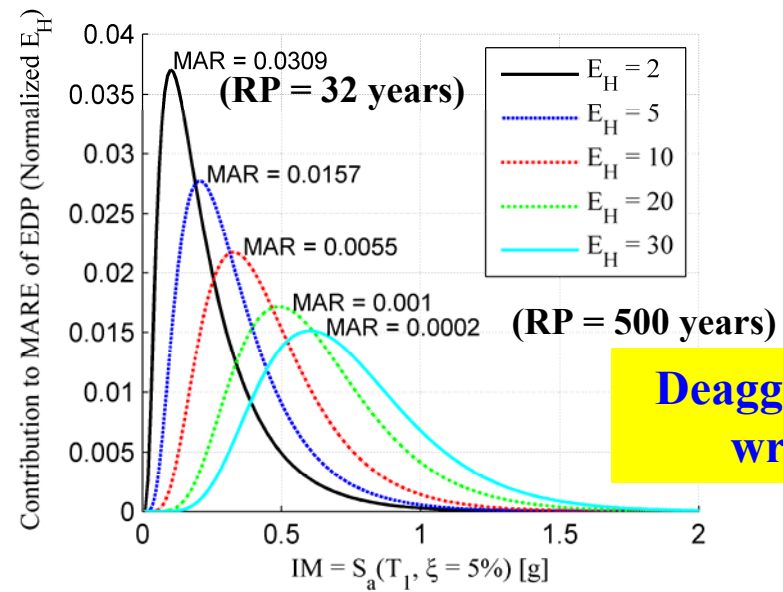
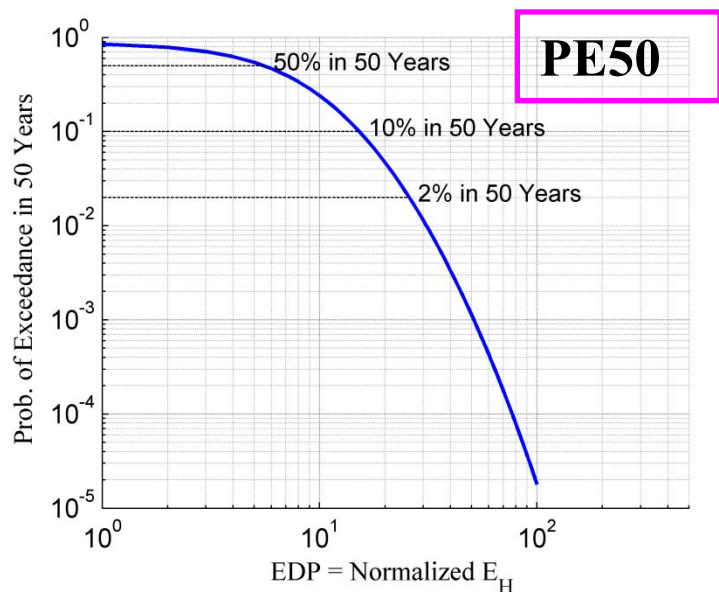
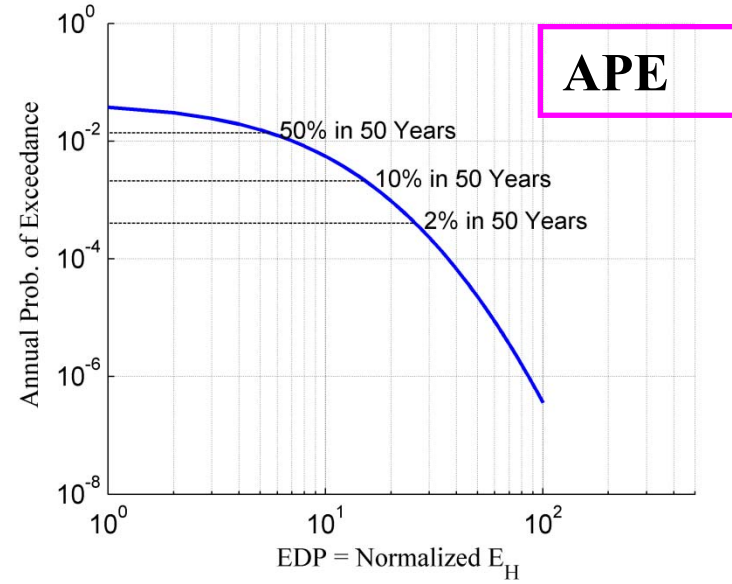
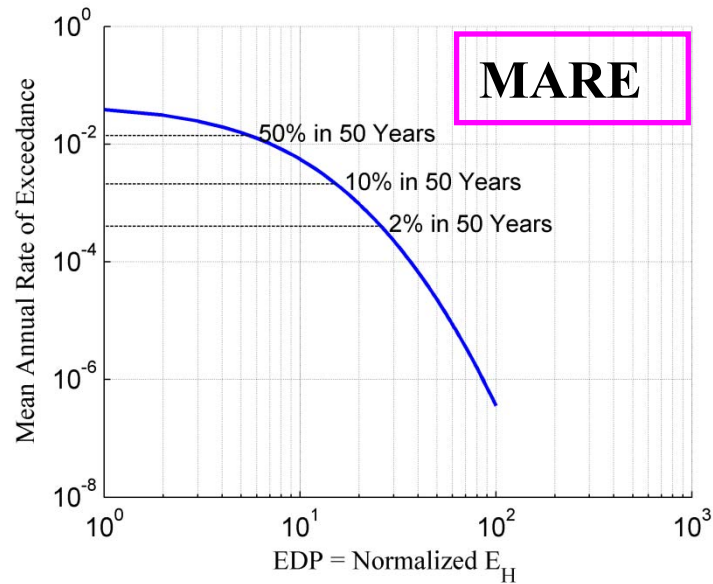
# Probabilistic Seismic Demand Hazard Analysis

- Demand Hazard Curve for **EDP = Peak Absolute Acceleration**:



# Probabilistic Seismic Demand Hazard Analysis

- Demand Hazard Curve for EDP = Normalized  $E_H$  Dissipated:



# Probabilistic Capacity (Fragility) Analysis

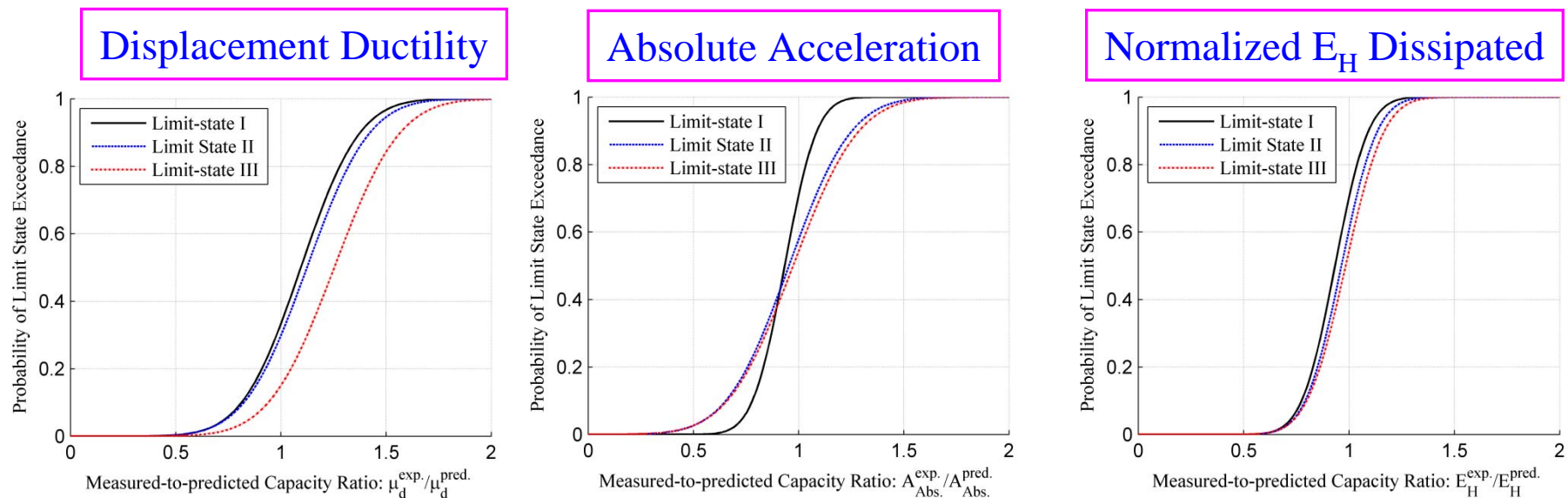
## • Fragility Curves (postulated & parameterized):

- Defined as the probability of the structure/component exceeding  $k^{\text{th}}$  limit-state given the demand.

$$P[DM > ls_k | EDP = edp]$$

↖ Damage Measure  
↘ k-th limit state

- Developed based on analytical and/or empirical capacity models and experimental data.





# Probabilistic Capacity (Fragility) Analysis

- Fragility Curve Parameters:

Associated EDP	Limit-states	Predicted Capacity	Measured-to-predicted capacity ratio (Normal distributed)	
			Mean	c.o.v
Displacement Ductility	I	$\mu = 2$	1.095	0.201
	II	$\mu = 6$	1.124	0.208
	III	$\mu = 8$	1.254	0.200
Peak Absolute Acceleration [g]	I	$A_{Abs.} = 0.10$	0.934	0.128
	II	$A_{Abs.} = 0.20$	0.952	0.246
	III	$A_{Abs.} = 0.25$	0.973	0.265
Normalized Hysteretic Energy Dissipated	I	$E_H = 5$	0.934	0.133
	II	$E_H = 20$	0.965	0.140
	III	$E_H = 30$	0.983	0.146

# Probabilistic Damage Hazard Analysis

- **Damage Hazard (MAR of limit-state exceedance):**

$$V_{DS_k} = \int_{EDP} P[DM > ls_k | EDP = edp] |dv_{EDP}(edp)|$$

Fragility Analysis

- **Deaggregation of Damage Hazard:**

➤ with respect to EDP:

Contribution of bin  
EDP =  $edp_i$  to  $V_{DS_k}$

$$V_{DS_k} = \sum_i P[DM > ls_k | EDP = edp_i] \frac{|dv_{EDP}(edp_i)|}{\Delta(edp)_i} \Delta(edp)_i$$

➤ with respect to IM:

$$V_{DS_k} = \sum_i P[DM > ls_k | EDP] \int_{IM} dP[EDP > edp | IM] \frac{|dv_{IM}(im_i)|}{\Delta(im)_i} \Delta(im)_i$$

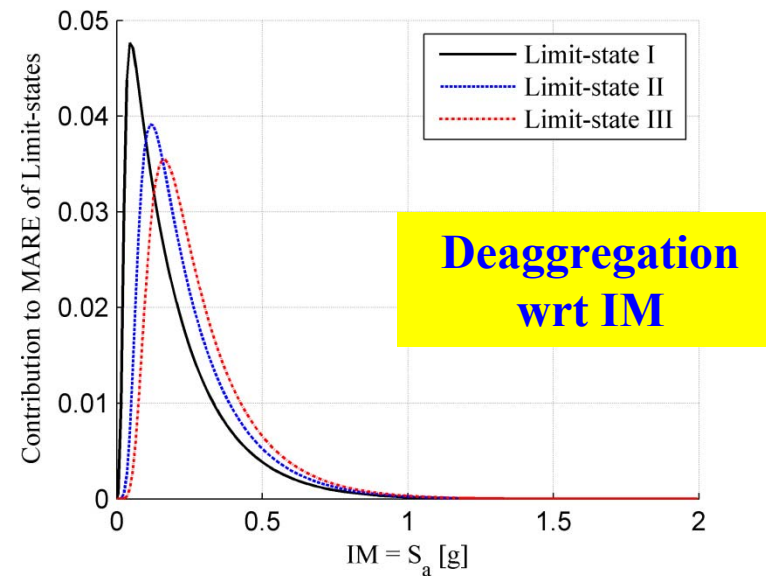
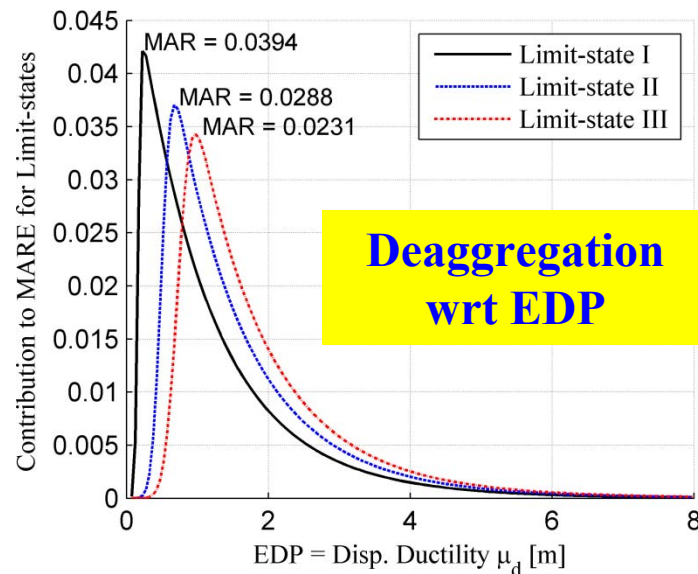
Contribution of bin  
IM =  $im_i$  to  $V_{DS_k}$

# Probabilistic Damage Hazard Analysis

- Associated EDP = Displacement Ductility:

Associated EDP	Limit States	MARE	RP	PE50
Displacement Ductility	I ( $\mu_d = 2$ )	0.0394	26 Years	86%
	II ( $\mu_d = 6$ )	0.0288	35 Years	76%
	III ( $\mu_d = 8$ )	0.0231	44 Years	68%

- Deaggregation Results:

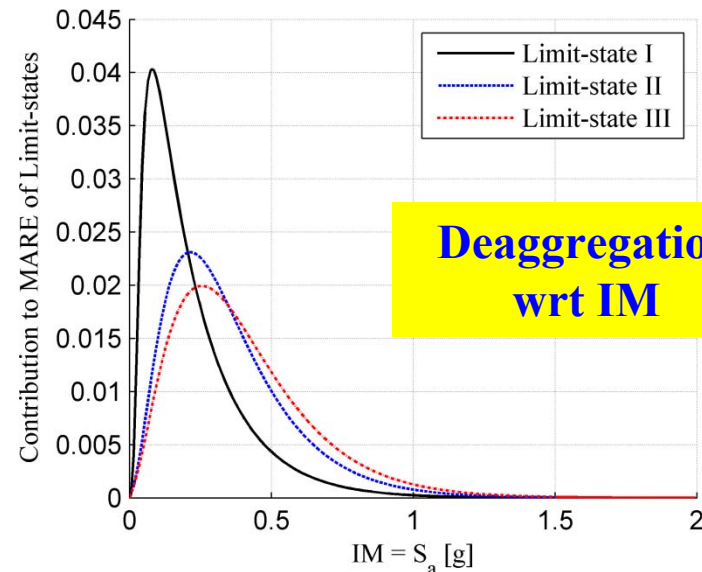
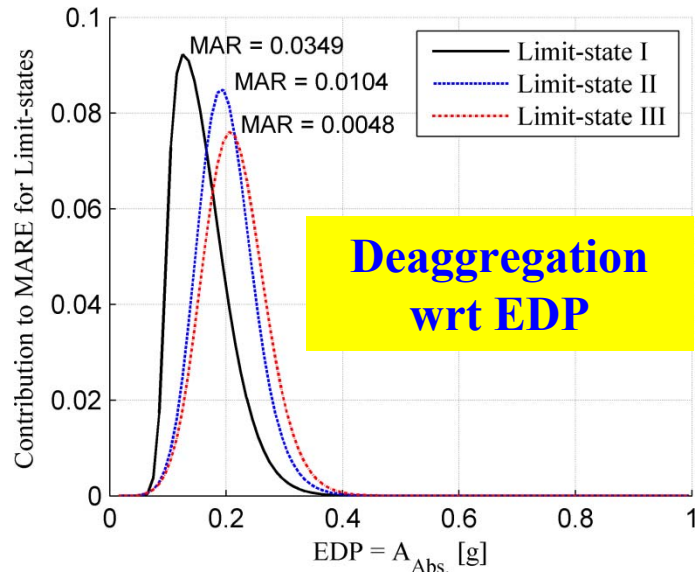


# Probabilistic Damage Hazard Analysis

- Associated EDP = Peak Absolute Acceleration:

Associated EDP	Limit States	MARE	RP	PE50
Absolute Acceleration	I ( $A_{Abs} = 0.1g$ )	0.0349	29 Years	82%
	II ( $A_{Abs} = 0.2g$ )	0.0104	96 Years	41%
	III ( $A_{Abs} = 0.25g$ )	0.0048	208 Years	21%

- Deaggregation Results:

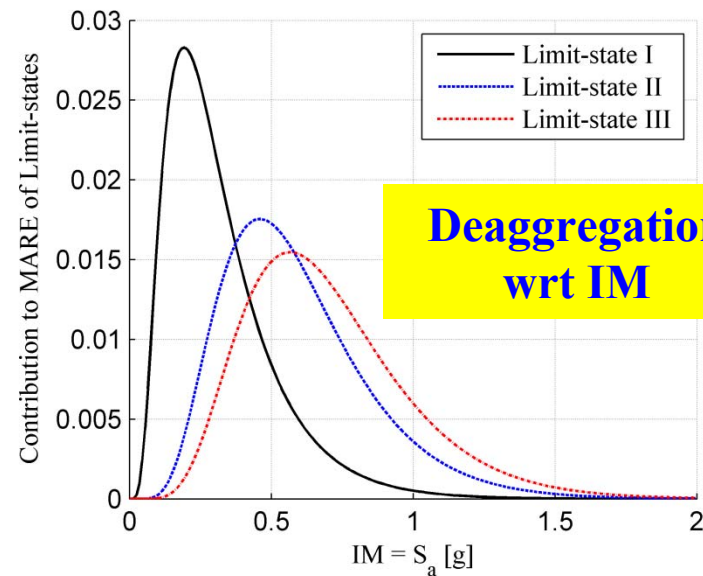
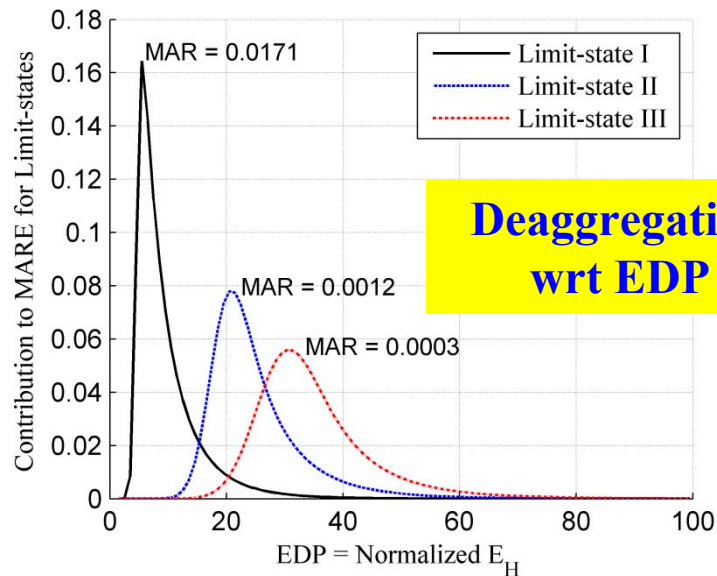


# Probabilistic Damage Hazard Analysis

- Associated EDP = Normalized Hysteretic Energy Dissipated:

Associated EDP	Limit States	MARE	RP	PE50
Normalized Hysteretic Energy Dissipated	I ( $E_H = 5$ )	0.0171	58 Years	57%
	II ( $E_H = 20$ )	0.0012	833 Years	6%
	III ( $E_H = 30$ )	0.0003	3330 Years	1.5%

- Deaggregation Results:



# Probabilistic Loss Hazard Analysis

- **Loss Hazard Curve:**

- Component-wise Loss Hazard Curve:

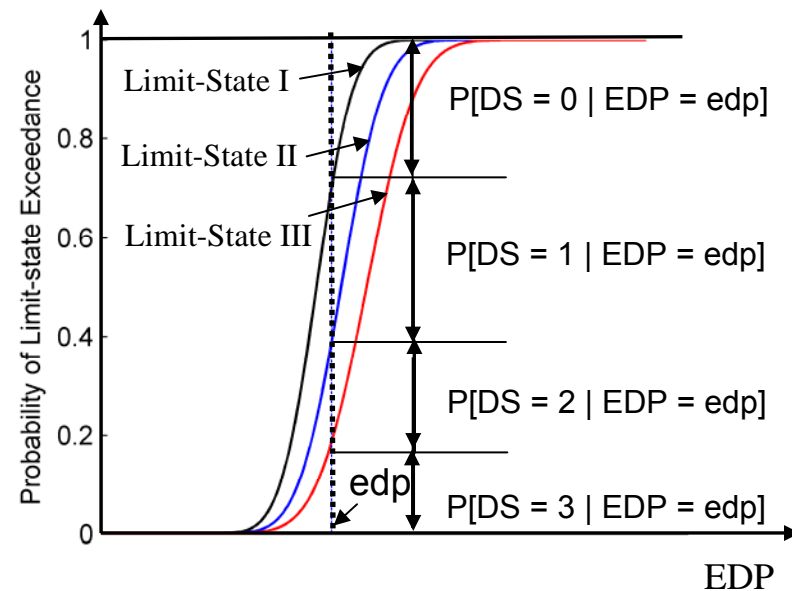
$$v_{L_j}(l) = \int_{DM} P[L_j > l | DM] |dv_{DM}| = \sum_{k=1}^{nls_j} P[L_j > l | DS = k] |v_{DS_k} - v_{DS_{k+1}}|$$

$nls_j$  = number of limit states for component  $j$

- Only discrete damage states are used in practice.

$v_{DS_k}$  = MAR of exceeding the  $k$ -th limit-state and

$v_{DS_{nls+1}} = 0$



# Probabilistic Loss Hazard Analysis

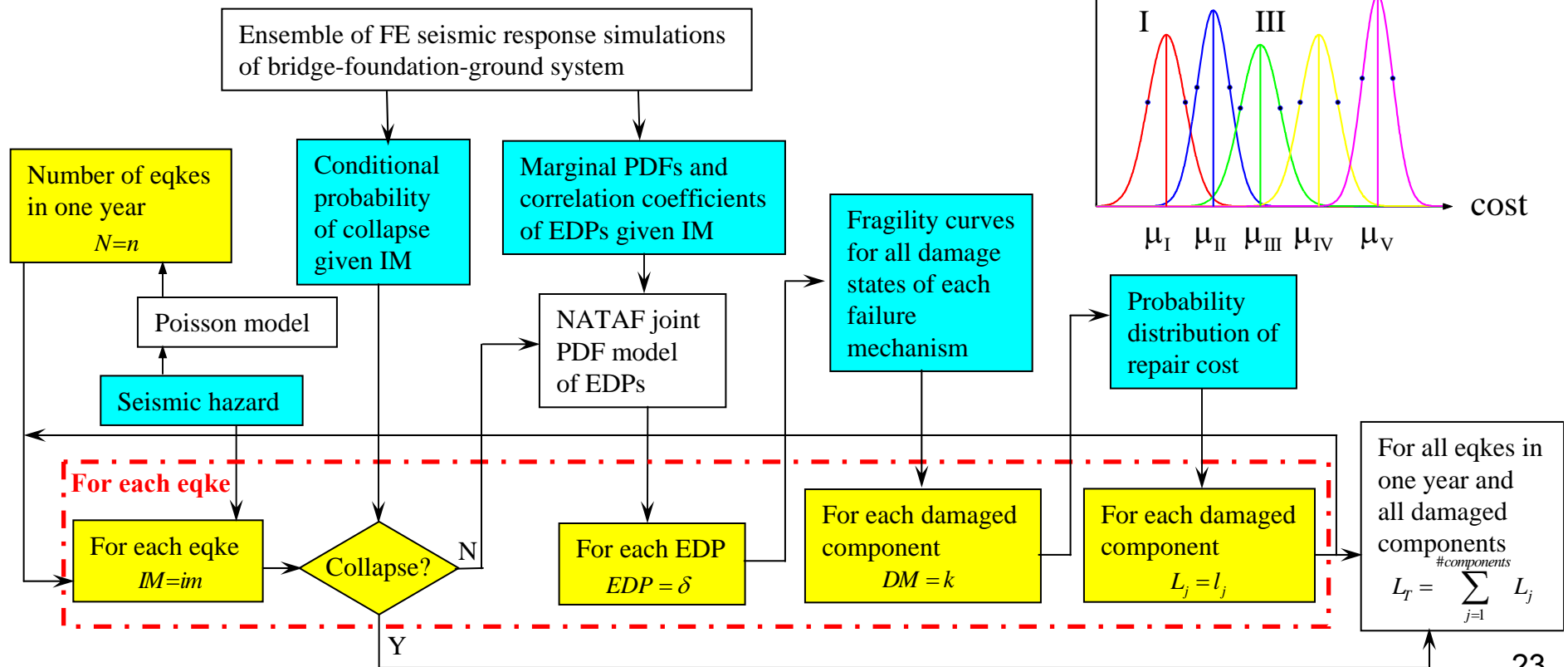
- **Multilayer Monte Carlo Simulation:**

- **Total Loss (Repair/Replacement Cost):**

$$L_T = \sum_{j=1}^{\#components} L_j$$

- Computation of  $\nu_{L_T}(l)$  requires n-fold integration of the joint PDF of the component losses, which is very difficult to obtain.

- ➔  $\nu_{L_T}(l)$  is estimated using multilayer MCS



# *Probabilistic Loss Hazard Analysis*

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- Repair/Replacement Cost Parameters:**

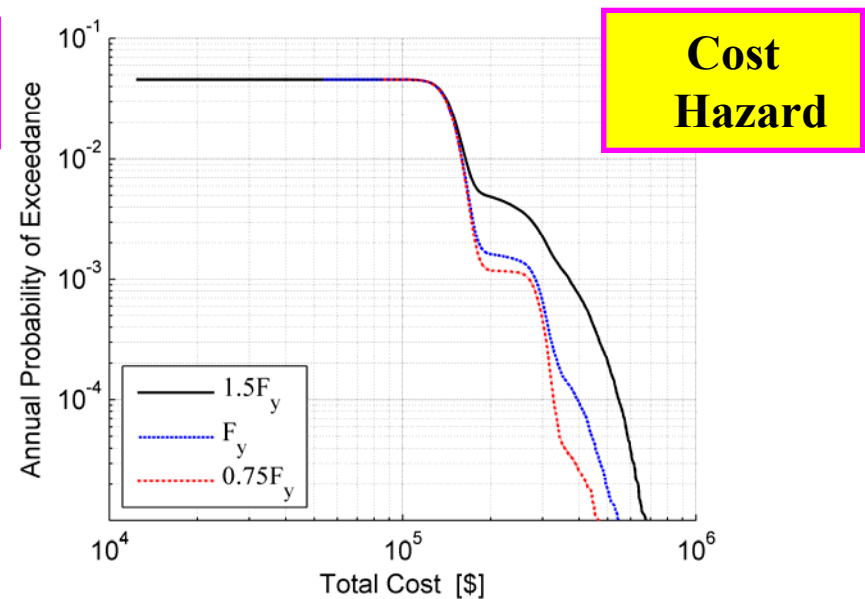
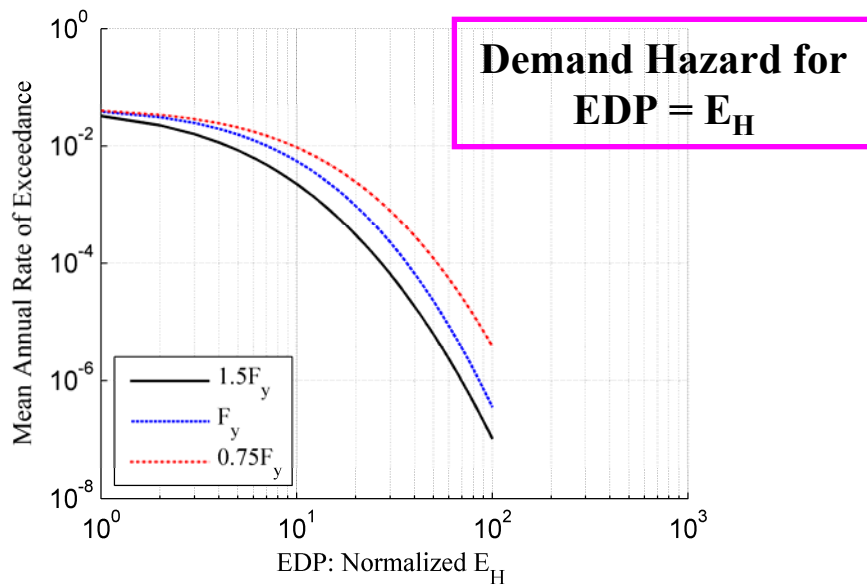
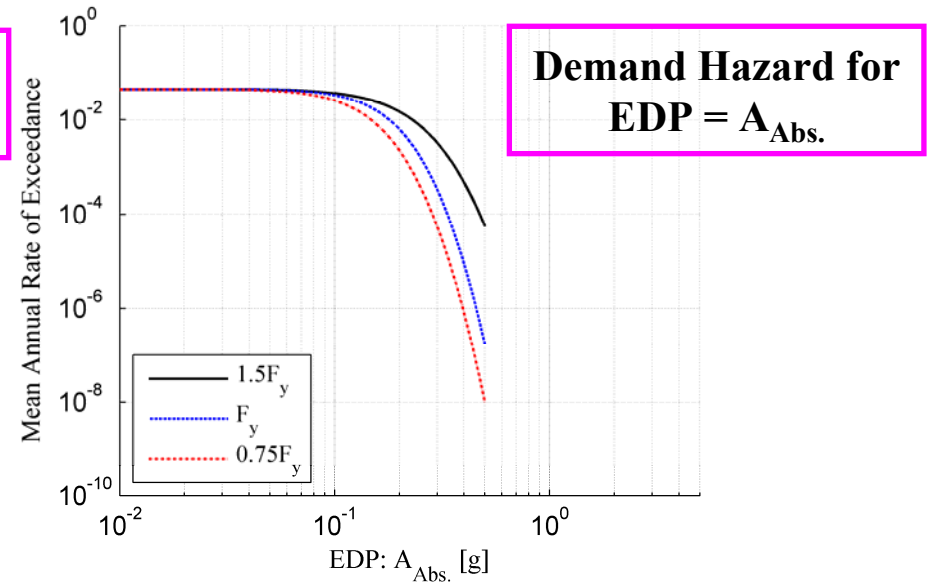
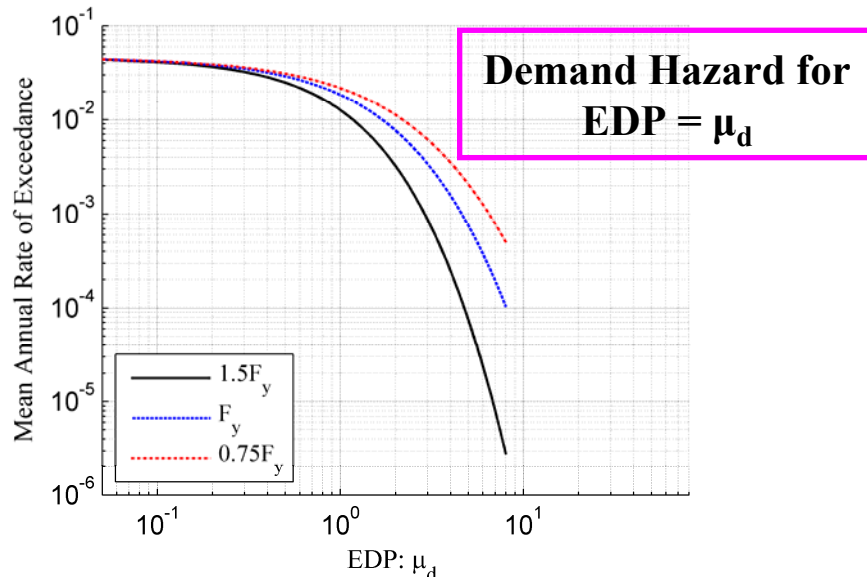
<b>Failure Associated EDP</b>	<b>Limit-states</b>	<b>Repair/Replacement Cost (Normal distributed)</b>	
		<b>Mean (\$)</b>	<b>c.o.v.</b>
Displacement Ductility	I	146,500	0.12
	II	246,400	0.25
	III	350,000	0.32
Peak Absolute Acceleration [g]	I	55,000	0.11
	II	100,000	0.20
	III	500,000	0.28
Normalized Hysteretic Energy Dissipated	I	55,650	0.13
	II	110,000	0.22
	III	520,000	0.28



*(II) Parametric PBEE Analysis of  
SDOF System*

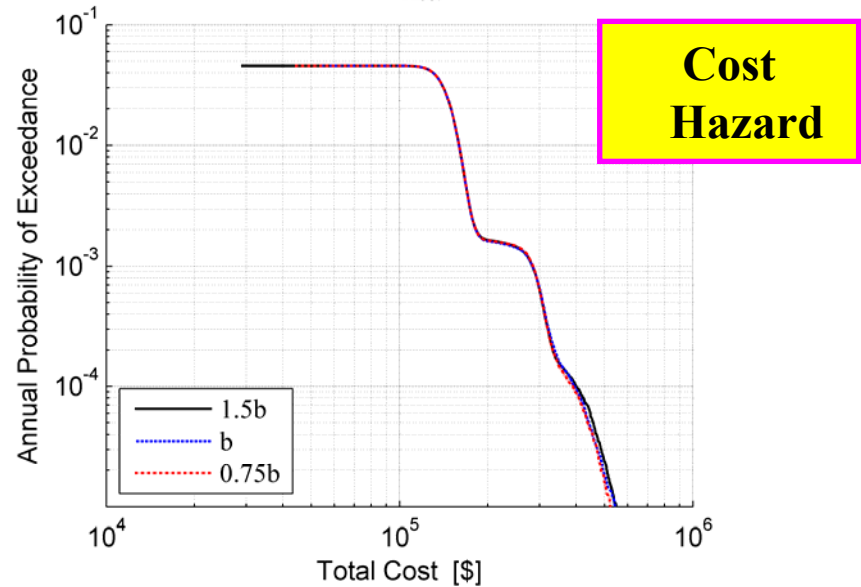
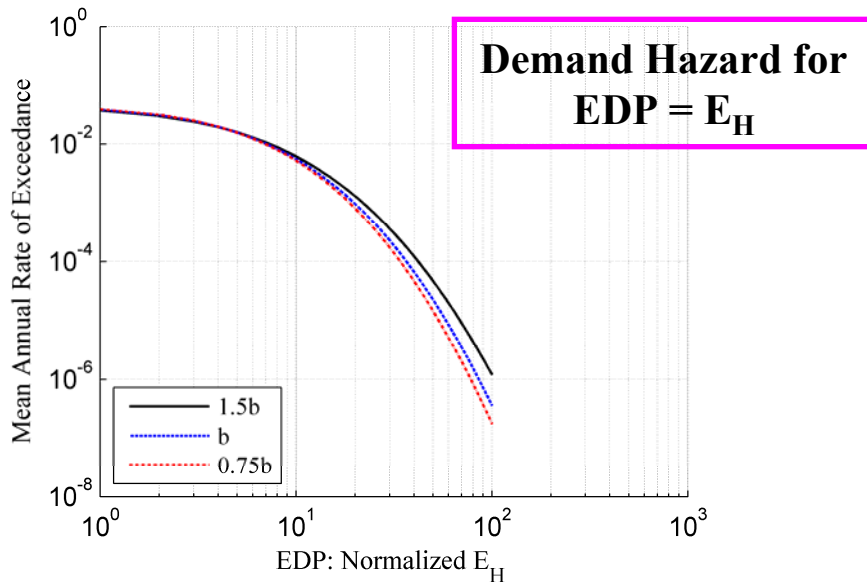
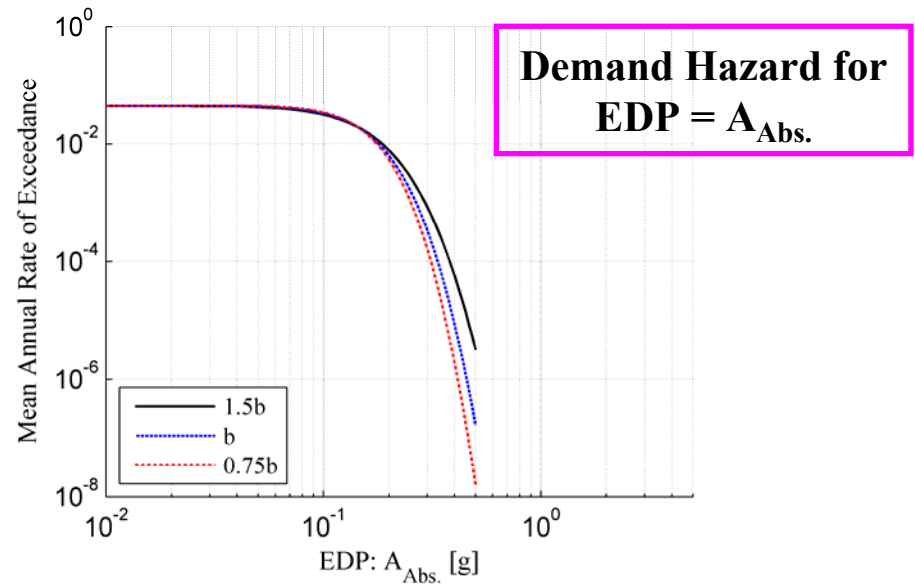
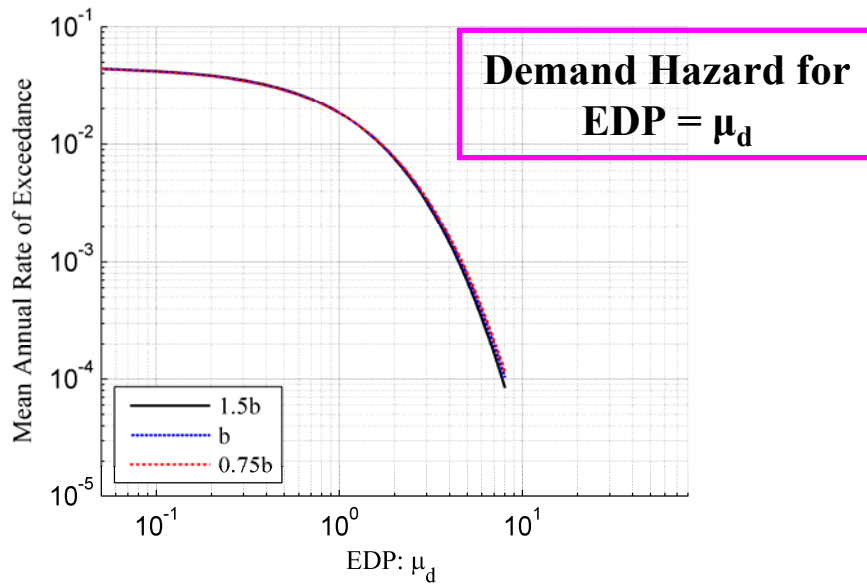
# Parametric PBEE Analysis

- Varying Parameter: Yield Strength  $F_y$



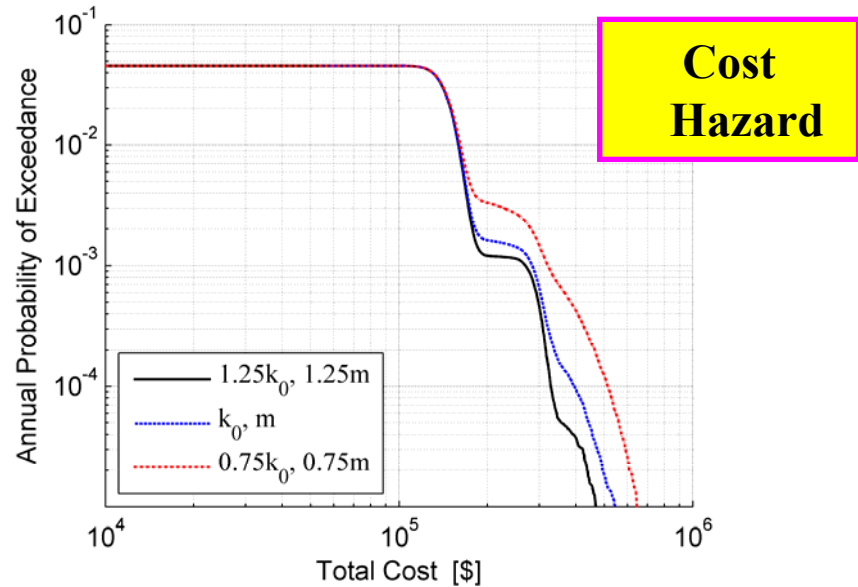
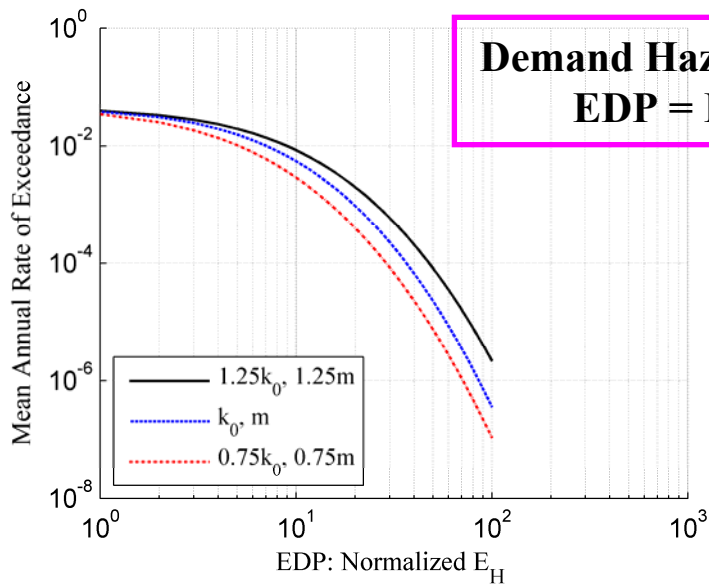
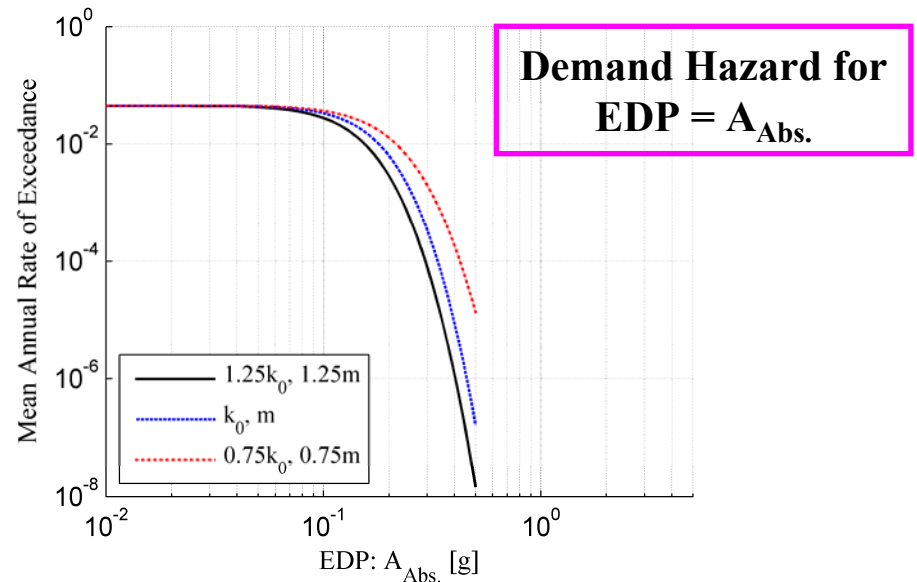
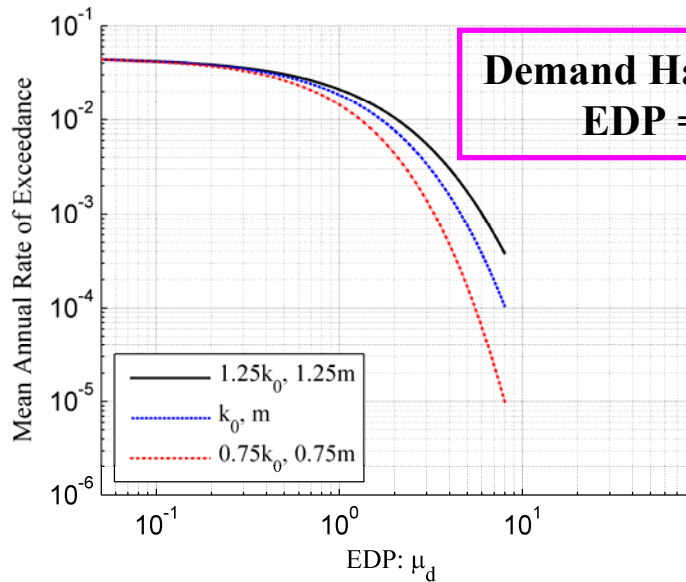
# Parametric PBEE Analysis

## • Varying Parameter: Hardening Ratio $b$



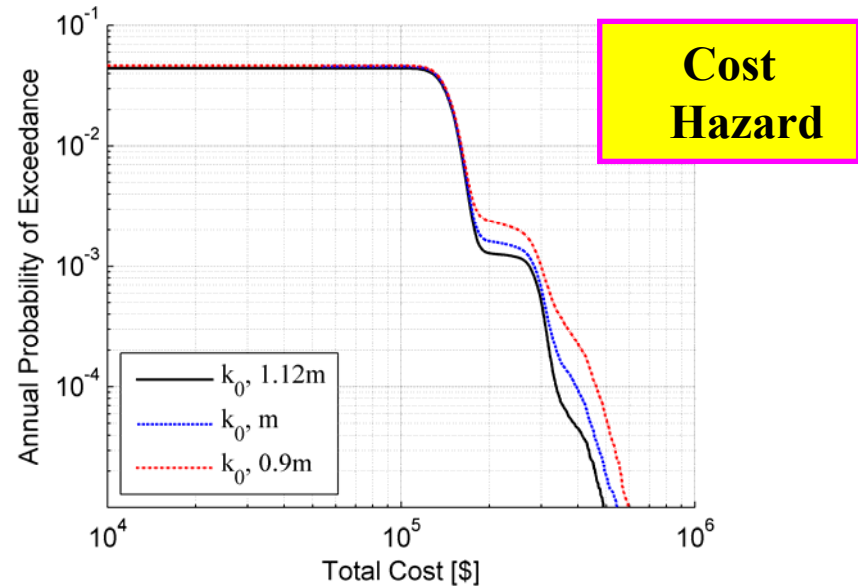
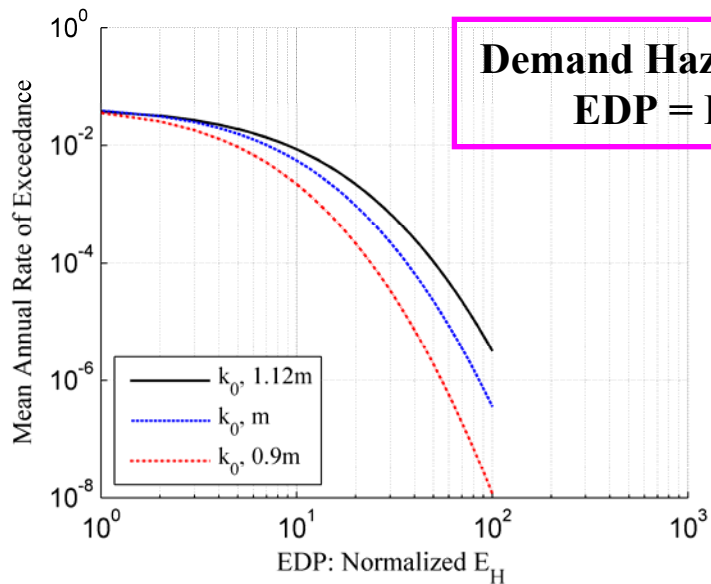
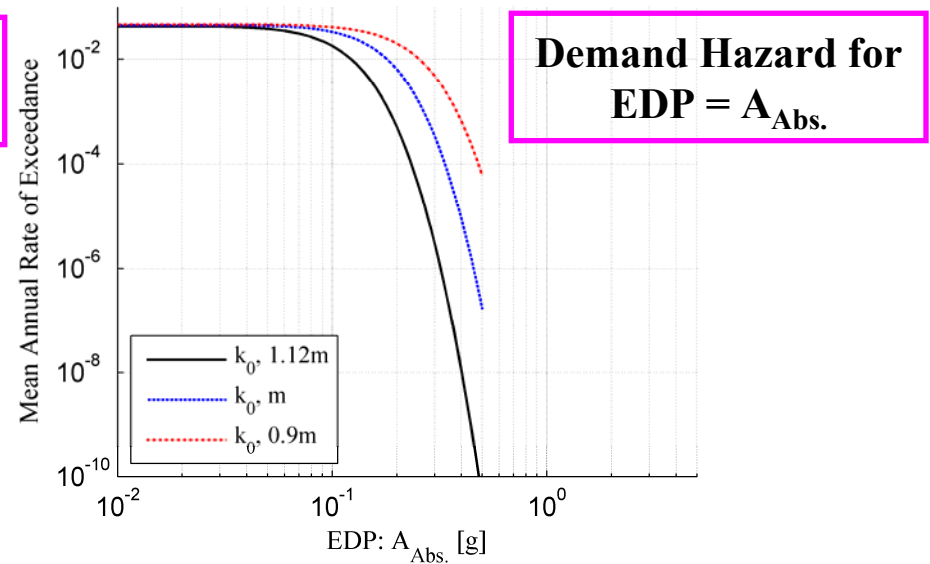
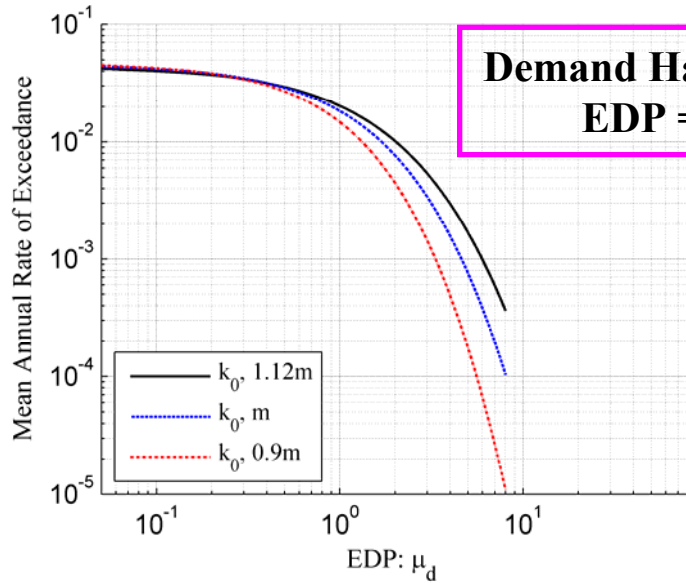
# Parametric PBEE Analysis

- Varying Parameters: Both Mass  $m$  and Initial Stiffness  $k_0$  (period fixed):



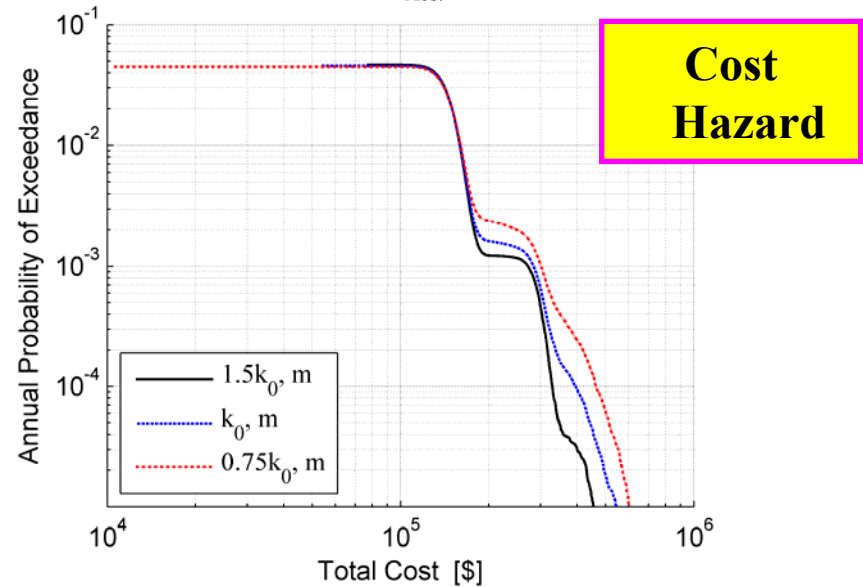
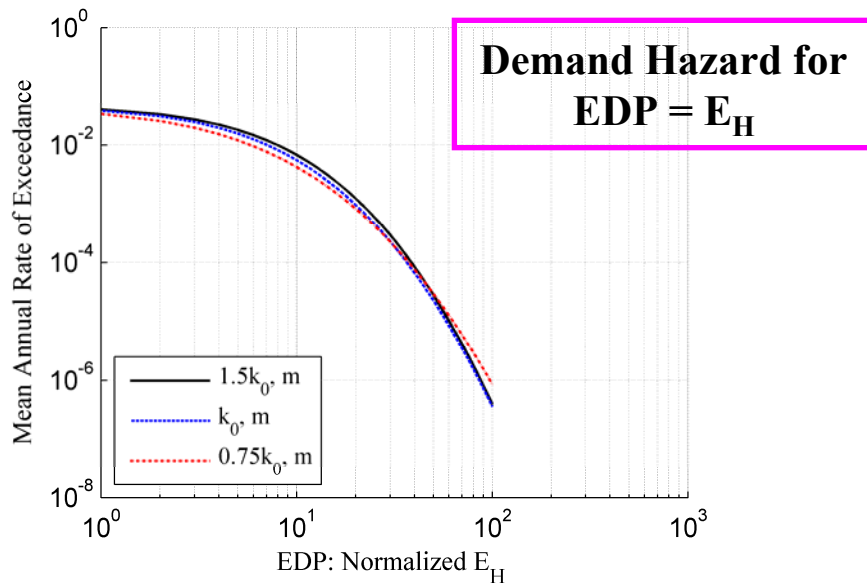
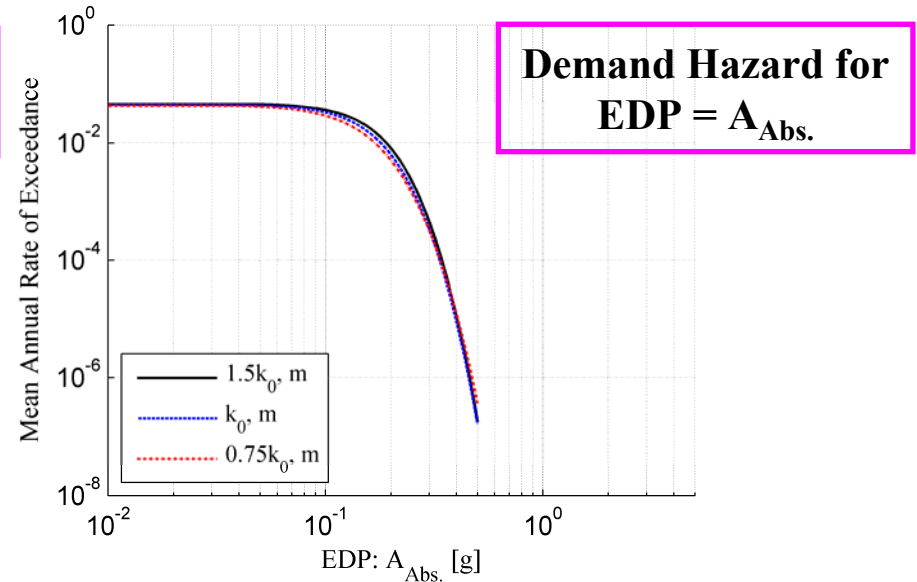
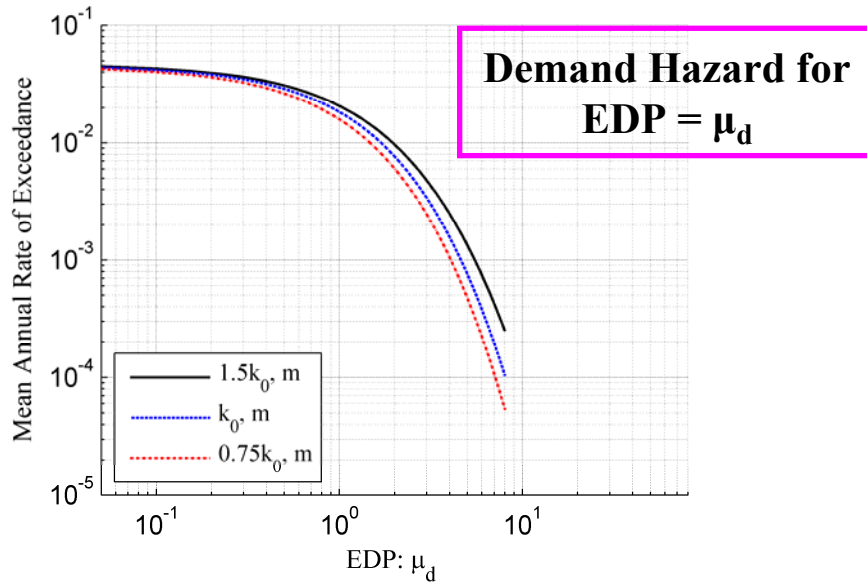
# Parametric PBEE Analysis

- Varying Parameter: Mass  $m$



# Parametric PBEE Analysis

- Varying Parameter: Initial Stiffness  $k_0$



***(III) Optimization Formulation for  
Probabilistic Performance Based  
Optimum Seismic Design  
(Inverse PBEE Analysis)***

# Optimization Formulation of PBEE

- Optimization Problem Formulation:

- **Objective (Target/Desired)** Loss Hazard Curve:  $v_{L_T}^{Obj}(l)$

- Objective function:  $f(k_0, F_y, b, \dots) = \sum_i |v_{L_T}(l_i, k_0, F_y, b, \dots) - v_{L_T}^{Obj}(l_i)|^2$

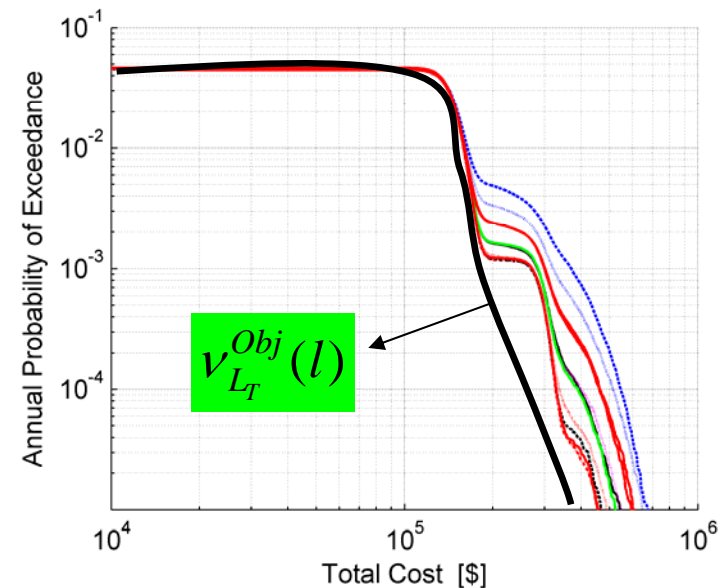
- Optimization Problem:

$$\text{Minimize}_{\{k_0, F_y, b, \dots\}} f(k_0, F_y, b, \dots)$$

subject to:

$$h(k_0, F_y, b, \dots) = 0$$

$$g(k_0, F_y, b, \dots) \leq 0$$



- Optimization Performed using extended **OpenSees-SNOPT** Framework (ongoing work).

Gu, Q., Barbato, M., Conte, J. P., Gill, P. E., and McKenna, F., "OpenSees-SNOPT Framework for Finite Element-Based Optimization of Structural and Geotechnical Systems," Journal of Structural Engineering, ASCE, under review, 2011.



*Thank you !*

*Any Questions?*