Real-time dynamic hybrid testing for soil-structure interaction analysis

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2009-10-15
CONTENTS

1. Introduction of soil-structure interaction
2. RTDHT system in Tsinghua University
3. Soil-structure interaction model
4. SSI-RTDHT test setup
5. SSI-RTDHT test results
6. CONCLUSION
1 Introduction of soil-structure interaction

Soil and structure should be considered as a whole system in dynamic analysis.

<table>
<thead>
<tr>
<th>Direct method</th>
<th>Substructure method</th>
</tr>
</thead>
<tbody>
<tr>
<td>• the structure and soil are treated as a whole system.</td>
<td>• the structure and the soil are treated as two different substructures.</td>
</tr>
<tr>
<td>• The region of the soil adjacent to the structure-soil interface is also explicitly modeled.</td>
<td>• Each substructure can be analyzed using a best-suited computational technique.</td>
</tr>
<tr>
<td>• Artificial boundary must be introduced so as to cover the unbounded soil domain.</td>
<td>• Combining the force-displacement relationship of the soil with the discretized motion equation of the structure, results in the final system of equation of the total dynamic system.</td>
</tr>
</tbody>
</table>
Considering SSI in shaking-table tests is still a problematic option.

1 Introduction of soil-structure interaction

Model box with finite region of soil

(Chen, et al. 2005)

Model boundary with damping material

(Li, et al. 2003)
1 Introduction of soil-structure interaction

Combining the numerical calculations of semi-infinite soil together with superstructure large scale model testing.
2 RTDHT system in Tsinghua University

Distributed real-time calculation system

- Host PC – Ordinary computer with Windows operating system
- Target PC – Ordinary computer without Windows operating system (but xPC kernel)
- Software – Matlab, Simulink, Real-time workshop, xPC target

Shared common RAM network

- SCRAMNet card SC150
- Optical cable
- high-speed: 16.7 Mb/s
- ultra-low-latency: 250 ns

Shaking table loading system

- Area: 1.5m × 1.5 m
- Bearing capacity: 2 t
- Max acceleration: 3.6 g (bare table), 1.2 g (full loaded)
- Frequency: 0~50 Hz
- MTS 469D controller: Mathworks’ Simulink is integrated into the controller platform
3 Soil-structure interaction model

Lumped parameter model used in this paper

Luan Maotian & Lin Gao 1996

\[ M_f \ddot{U}_f + C_f \dot{U}_f + K_f U_f = P_f \]
3 Soil-structure interaction model

Lumped parameter model used in this paper

\[ M_f \ddot{U}_f + C_f \dot{U}_f + K_f U_f = P_f \]

\[
M_f = M_{fe} \begin{bmatrix} m_{f1}^* & m_{f2}^* \end{bmatrix} \quad \Rightarrow \quad M_{fe} = K_s \left( \frac{r_0}{c_s} \right)^2
\]

\[
C_f = C_{fe} \begin{bmatrix} c_{f1}^* + c_{f2}^* & -c_{f2}^* \\ -c_{f2}^* & c_{f2}^* + c_{f3}^* \end{bmatrix} \quad \Rightarrow \quad C_{fe} = K_s \cdot \frac{r_0}{c_s}
\]

\[
K_f = K_{fe} \begin{bmatrix} k_{f1}^* + k_{f2}^* & -k_{f2}^* \\ -k_{f2}^* & k_{f2}^* + k_{f3}^* \end{bmatrix} \quad \Rightarrow \quad K_{fe} = K_s = \pi \rho c_s^2
\]

\[
U_f = \begin{bmatrix} U_{f1} \\ U_{f2} \end{bmatrix}^T \quad \quad \quad P_f = \begin{bmatrix} P_{f1} \\ P_{f2} \end{bmatrix}^T
\]
3 Soil-structure interaction model

**Dynamic stiffness coefficient**

If the system is excited by harmonic force, the external force and displacement vector can be rewritten as follows:

\[
P_f = \begin{bmatrix} P_{f0} \\ 0 \end{bmatrix}^T e^{i\omega t}
\]

\[
U_f = \begin{bmatrix} U_{f1,0} \\ U_{f2,0} \end{bmatrix}^T e^{i\omega t}
\]

\[
K(\omega) = P_{f0}/U_{f1,0}
\]

\[
K(a_0)/K_s = k(a_0) + i\alpha_0 c(a_0)
\]

\[
= k_{f1}^* + k_{f2}^* - a_0^2 m_{f1}^* - \frac{(k_{f2}^* - a_0^2 c_{f2}^*) (k_{f2}^* + k_{f3}^* - a_0^2 m_{f2}^*) + 2a_0^2 k_{f2}^* c_{f2}^* (c_{f2}^* + c_{f3}^*)}{(k_{f2}^* + k_{f3}^* - a_0^2 m_{f2}^*)^2 + a_0^2 (c_{f2}^* + c_{f3}^*)^2}
\]

\[
+ i\alpha_0 \left[ c_{f1}^* + c_{f2}^* - \frac{2k_{f2}^* c_{f2}^* (k_{f2}^* + k_{f3}^* - a_0^2 m_{f2}^*) - (c_{f2}^* + c_{f3}^*) (k_{f2}^* - a_0^2 c_{f2}^*)^2}{(k_{f2}^* + k_{f3}^* - a_0^2 m_{f2}^*)^2 + a_0^2 (c_{f2}^* + c_{f3}^*)^2} \right]
\]
3 Soil-structure interaction model

Dynamic stiffness coefficient

\[
K(a_0)/K_s = k(a_0) + i a_0 c(a_0)
\]

\[
= \left[ k_{f1} + k_{f2} - a_0^2 m_{f1} - \frac{(k_{f2}^2 - a_0^2 c_{f2}^2)(k_{f3}^2 + a_0^2 m_{f3}^2) + 2 a_0^3 k_{f2} c_{f2} (c_{f2}^* + c_{f3}^*)}{(k_{f2}^2 + k_{f3}^2 - a_0^2 m_{f2}^2)} + a_0^2 (c_{f2}^* + c_{f3}^*)^2 \right] + i a_0 \left[ c_{f1}^* + c_{f2}^* - \frac{2 k_{f2} c_{f2} (k_{f2}^* + k_{f3}^* - a_0^2 m_{f2}^*) - (c_{f2}^* + c_{f3}^*) (k_{f2}^* - a_0^2 c_{f2}^*)}{(k_{f2}^* + k_{f3}^* - a_0^2 m_{f2}^*)^2 + a_0^2 (c_{f2}^* + c_{f3}^*)^2} \right]
\]

Table 1 Parameters of the lumped parameter model

<table>
<thead>
<tr>
<th>(m_{f1}^*)</th>
<th>(m_{f2}^*)</th>
<th>(k_{f1}^*)</th>
<th>(k_{f2}^*)</th>
<th>(k_{f3}^*)</th>
<th>(c_{f1}^*)</th>
<th>(c_{f2}^*)</th>
<th>(c_{f3}^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.035</td>
<td>0.150</td>
<td>1.423</td>
<td>0.716</td>
<td>-0.202</td>
<td>1.724</td>
<td>0.126</td>
<td>0.815</td>
</tr>
</tbody>
</table>
4 SSI-RTDHT test setup

1 Motion equation

\[
\begin{bmatrix}
  m_1 & \cdots & \cdots & \cdots & m_n \\
  \vdots & \ddots & \ddots & \ddots & \vdots \\
  \vdots & \ddots & m_{f1} & \cdots & m_{f2} \\
  \vdots & \ddots & \ddots & \ddots & \vdots \\
  m_1 & \cdots & \cdots & \cdots & m_n \\
\end{bmatrix}
\begin{bmatrix}
  \ddot{u}_1 \\
  \vdots \\
  \ddot{u}_{f1} \\
  \ddot{u}_{f2} \\
  \dddot{u}_1 \\
\end{bmatrix}
+ \begin{bmatrix}
  c_1 & \cdots & \cdots & \cdots & -c_1 \\
  \vdots & \ddots & \ddots & \ddots & \vdots \\
  \vdots & \ddots & -c_n & \cdots & \cdots \\
  \vdots & \ddots & \ddots & \ddots & \vdots \\
  c_1 & \cdots & \cdots & \cdots & -c_1 \\
\end{bmatrix}
\begin{bmatrix}
  u_1 \\
  \vdots \\
  u_{f1} \\
  u_{f2} \\
  u_1 \\
\end{bmatrix}
= \begin{bmatrix}
  0 \\
  \vdots \\
  0 \\
  0 \\
  0 \\
\end{bmatrix}
\]
4 SSI-RTDHT test setup

2 Concretize

- Soil properties:
- Mass density $\rho = 2000 \text{ kg/m}^3$

- Soft Soil:
  - $c_s = 200 \text{ m/s}$
  - $K_{fe} = 2.513 \times 10^8 \text{ N/m}$
  - $C_{fe} = 3.016 \times 10^7 \text{ Ns/m}$
  - $M_{fe} = 3.619 \times 10^6 \text{ kg}$

- Hard Soil:
  - $c_s = 800 \text{ m/s}$
  - $K_{fe} = 4.021 \times 10^9 \text{ N/m}$
  - $C_{fe} = 1.2064 \times 10^8 \text{ Ns/m}$
  - $M_{fe} = 3.619 \times 10^6 \text{ kg}$
4 SSI-RTDHT test setup

3 Substructuring

1 Physical substructure

\[
\begin{bmatrix}
    m_1 & \{\ddot{u}_1\} \\
    m_2 & \{\ddot{u}_2\}
\end{bmatrix}
+ \begin{bmatrix}
    c_1 & -c_1 \\
    -c_1 & c_1
\end{bmatrix}
\begin{bmatrix}
    \{\dot{u}_1\} \\
    \{\dot{u}_2\}
\end{bmatrix}
+ \begin{bmatrix}
    k_1 & -k_1 \\
    -k_1 & k_1
\end{bmatrix}
\begin{bmatrix}
    \{u_1\} \\
    \{u_2\}
\end{bmatrix}
= \begin{bmatrix}
    0 \\
    -T
\end{bmatrix}
\]

2 Numerical substructure

\[
\begin{bmatrix}
    m_{f1} & \{\ddot{u}_{f1}\} \\
    m_{f2} & \{\ddot{u}_{f2}\}
\end{bmatrix}
+ \begin{bmatrix}
    c_{f1} + c_{f2} & -c_{f2} \\
    -c_{f2} & c_{f2} + c_{f3}
\end{bmatrix}
\begin{bmatrix}
    \{\dot{u}_{f1}\} \\
    \{\dot{u}_{f2}\}
\end{bmatrix}
+ \begin{bmatrix}
    k_{f1} + k_{f2} & -k_{f2} \\
    -k_{f2} & k_{f2} + k_{f3}
\end{bmatrix}
\begin{bmatrix}
    \{u_{f1}\} \\
    \{u_{f2}\}
\end{bmatrix}
= \begin{bmatrix}
    p_0^g + T \\
    0
\end{bmatrix}
\]

\[T = c_2 (\ddot{u}_2 - \ddot{u}_{f1}) + k_2 (u_2 - u_{f1})\]
4 SSI-RTDHT test setup

4 Model and measurements

NI DAQ
For data acquisition

Strain gauges bridge
For shear force measurement

LVDT
For displacement measurement

Accelerometer
For acceleration measurement
4 SSI-RTDHT test setup

5 Procedure of SSI-RTDHT

Free field $u_f(t)$

Calculate effective driving force $p_0^E(t)$

Calculate displacement at soil structure interface $u(t + \Delta t)$ (Numerical substructure)

Measure shear force $T(t)$

Delay compensation $u'(t + \Delta t)$

Drive the shaking table (Physical substructure)

$t = t + \Delta t$

$t \geq N\Delta t$

End
5 SSI-RTDHT test results

1. Far boundary FEM

The mesh boundary is far enough so that the effect of the wave reflection and scattering from boundary on the structure-soil interface response can be avoided during the calculation duration.

\[
D \geq \frac{cT}{2} \quad L \geq 2D \quad a \leq \frac{c}{10f}
\]

---

### Table 3 Finite element analysis parameters

<table>
<thead>
<tr>
<th>Wave velocity $c$/m·s⁻¹</th>
<th>Calculation duration $T$/s</th>
<th>Highest frequency component / Hz</th>
<th>Depth $D$/m</th>
<th>Element size $a$/m</th>
<th>Node number</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>5</td>
<td>10</td>
<td>500</td>
<td>2</td>
<td>125751</td>
</tr>
</tbody>
</table>
5 SSI-RTDHT test results

2. Artificial wave

![Graph showing artificial wave and response spectrum]
3. Accuracy verification of lumped parameter model

Peak comparison
LPM: 1.226 g
FEM: 1.147 g
Error: 6.9%

Peak comparison
LPM: 0.793 g
FEM: 0.773 g
Error: 2.6%
5 SSI-RTDHT test results

3. Accuracy verification of lumped parameter model

![Graph showing displacement vs time with peak comparison]

- LPM: 9.751 mm
- FEM: 9.594 mm
- Error: 1.6%
3. Accuracy verification of lumped parameter model

- **Top storey shear force**:
  - LPM: 635.1 kN
  - FEM: 597.3 kN
  - Error: 6.33%

- **Bottom storey shear force**:
  - LPM: 688.7 kN
  - FEM: 689.6 kN
  - Error: 0.13%
5 SSI-RTDHT test results

4. Accuracy verification of SSI-RTDHT

Peak comparison
SSI-RTDHT: 1.317 g
FEM: 1.147 g
Error: 14.8 %

Peak comparison
SSI-RTDHT: 1.179 g
FEM: 0.773 g
Error: 52.5 %
5 SSI-RTDHT test results

4. Accuracy verification of SSI-RTDHT

- Peak comparison
  - SSI-RTDHT: 10.39 mm
  - FEM: 9.594 mm
  - Error: 8.3%
5 SSI-RTDHT test results

4. Accuracy verification of SSI-RTDHT

Peak comparison
SSI-RTDHT: 659.7 kN
FEM: 597.3 kN
Error: 10.4 %

Peak comparison
SSI-RTDHT: 728.2 kN
FEM: 689.6 kN
Error: 5.6 %
5 SSI-RTDHT test results

5. SSI effect of different types of soil

- **Peak comparison**
  - SOFT 0.40 g
  - RIGID 0.55 g
  - REDUCT: 27.3 %

- **Peak comparison**
  - SOFT 0.35 g
  - RIGID 0.27 g
  - REDUCT: 22.9 %
5. SSI-RTDHT test results

5. SSI effect of different types of soil

- Rigid foundation
- \( C_s = 200 \text{ m/s} \)
- \( C_s = 800 \text{ m/s} \)

**Peak comparison**

<table>
<thead>
<tr>
<th>Type</th>
<th>Force</th>
<th>Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft</td>
<td>196 kN</td>
<td>24.6%</td>
</tr>
<tr>
<td>Rigid</td>
<td>260 kN</td>
<td></td>
</tr>
<tr>
<td>Reduct</td>
<td>260 kN</td>
<td>38.6%</td>
</tr>
</tbody>
</table>

Graphs showing top and bottom storey shear force over time.
6 CONCLUSION

1. The lumped parameter model used in this paper can give a high accuracy approximation of the infinite half space elastic foundation.
2. Comparing the SSI-RTDHT test results with the FEM results shows that SSI-RTDHT can produce satisfying results in SSI analysis.
3. Comparing the tests results under rigid foundation, hard soil foundation, and soft soil foundation show that soil-structure interaction can affect the response of the structure: softer the soil is, more obvious the SSI effect becomes.

Using the idea of RTDHT, the infinite soil foundation model calculation and superstructure testing are combined together. Radiation damping of the infinite foundation can be included in shaking-table tests.
THE END