Stability for Bridge Cable and Cable-Deck Interaction

Dr. Maria Rosaria Marsico   D.J. Wagg   S.A. Neild
Introduction

The interaction between cables and structure can lead to complex vibration response.

Cables are highly non-linear due to the interaction of geometric non-linear effects and extremely lightly damping.
Experimental part

One possible solution is

RTDS: Real-time dynamic substructuring

The nonlinear components of the structure are physically tested.

The dynamic response of the remaining part of the structure is computed numerically.

The interface between physical and numerical components is provided by a set of transfer systems, typically actuators.
Modelling: the cable-deck interaction

- $u$, axial displacements
- $v$, out-of-plane displacements
- $w$, in-plane displacements

The cable is rigidly supported at the upper end and an oscillator is included at the lower end to simulate the deck mode.
Equation to capture the modal behaviour

- The sag is small compared to the length of the cable
- The dynamics along the cable (u) are insignificant
- The amplitude of vibration is small compared with the sag such that the compatibility equation can be linearized

The mode shapes for the linearized system in both the in-plane and out-of-plane directions were calculated.
Deck

The deck is modelled as a single degree-of-freedom system, so the resulting equation of motion is

\[ M\ddot{\delta} + C\dot{\delta} + K\delta + T\sin(\theta) = F_e \]
**Equation for 3 modes and deck mode**

Other modes are assumed to be insignificant. The oscillator is excited vertically by an external force of amplitude $F$ and frequency $\Omega$.

<table>
<thead>
<tr>
<th>Equation Description</th>
<th>Mode Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ddot{y}_1 + 2\xi_1 \omega_1 \dot{y}<em>1 + \omega_1^2 y_1 + W</em>{11} \dot{y}<em>1^2 + W</em>{12} y_1 (\dot{y}_2^2 + \dot{z}_2^2) + N_1 \delta y_1 = 0$</td>
<td>the first out-of-plane mode</td>
</tr>
<tr>
<td>$\ddot{y}_2 + 2\xi_2 \omega_2 \dot{y}<em>2 + \omega_2^2 y_2 + W</em>{21} y_2 \dot{y}<em>1^2 + W</em>{22} y_2 (\dot{y}_2^2 + \dot{z}_2^2) + N_2 \delta y_2 = 0$</td>
<td>the second out-of-plane mode</td>
</tr>
<tr>
<td>$\ddot{z}_2 + 2\xi_2 \omega_2 \dot{z}<em>2 + \omega_2^2 z_2 + W</em>{21} z_2 \dot{y}<em>1^2 + W</em>{22} z_2 (\dot{y}_2^2 + \dot{z}_2^2) + N_2 \delta z_2 = B \delta$</td>
<td>the first in-plane mode</td>
</tr>
<tr>
<td>$\ddot{\delta} + 2\xi_\delta \omega_\delta \dot{\delta} + \omega_\delta^2 \delta + W_\delta (\dot{y}_1^2 + 4\dot{y}_2^2 + 4\dot{z}_2^2) = F \cos(\Omega t)$</td>
<td>the equation of motion for the deck</td>
</tr>
</tbody>
</table>

The forcing frequency $\Omega$ and the oscillator (i.e. global mode) frequency $\omega_\delta$ are close to twice the first out-of-plane natural frequency.
Analysis

- 1.98 m long, steel cable inclined 20°
- 0.8 mm diameter
- mass of 0.67 kg/m (attaching lead weights at 60 mm intervals)
- T static 205 N

<table>
<thead>
<tr>
<th>Configuration</th>
<th>M [Kg]</th>
<th>C [Kg/s]</th>
<th>K [KN/m]</th>
<th>ωg [rad/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>50</td>
<td>147</td>
<td>8.78·2π</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>50</td>
<td>132</td>
<td>8.35·2π</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>50</td>
<td>162</td>
<td>9.19·2π</td>
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</tbody>
</table>

CABLE NATURAL FREQUENCIES

<table>
<thead>
<tr>
<th></th>
<th>ωy2 [rad/s]</th>
<th>ωz1 [rad/s]</th>
<th>ωz2 [rad/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exper.</td>
<td>4.40·2π</td>
<td>4.71·2π</td>
<td>8.76·2π</td>
</tr>
<tr>
<td>Theor.</td>
<td>4.40·2π</td>
<td>4.71·2π</td>
<td>8.81·2π</td>
</tr>
</tbody>
</table>
Substructuring Experiments

INSTRUMENTATION
a. load cell
b. LVDT displacement transducer
c. high speed vision system

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Localized stability

In order to calculate the first out-of-plane stability boundary, we have to solve simultaneously both the equations for the second in-plane mode and for the first out-of-plane mode.
Stability Boundaries for three Different Global Modes

Each line represents the stability boundary of the semi-trivial solution, which physically corresponds to the point at which the cable starts to have out-of-plane response when both the input and previous response were in-plane.
Experimental analysis

EXPERIMENTAL AND THEORETICAL STABILITY BOUNDARIES. FIRST OUT-OF-PLANE MODE. (a) $q=0.96$; (b) $q=1$; (c) $q=1.04$

Analytical model

Unstable experimental points *
Stable experimental points ○
Input and Output Systems
Analytical model for a new cable

$2^{nd}$ in plane  \( \mu = 0.03 \)

\( l = 5.40 \) m
\( \alpha = 22.6^0 \)
\( \phi = 0.8 \) mm
\( T \text{ static} = 286 \) N
Theoretical stability boundaries

FORCING AMPLITUDE vs FREQUENCY PLANE

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Test #10: $\Delta = 2.16\text{m}, f = 6.735\text{rad/sec}, \text{delay 17.1}$
Experimental stability boundaries
Conclusions

- The in-plane and out-of-plane modes for a cable are considered to know its stability boundaries.
- A series of real-time dynamic substructuring experiments were performed in order to experimentally validate this observation.
- Analytical and experimental results show good agreement.
THANK YOU FOR YOUR KIND ATTENTION

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