be certain.

MTS

Overview of Real-time Hybrid Simulation Techniques

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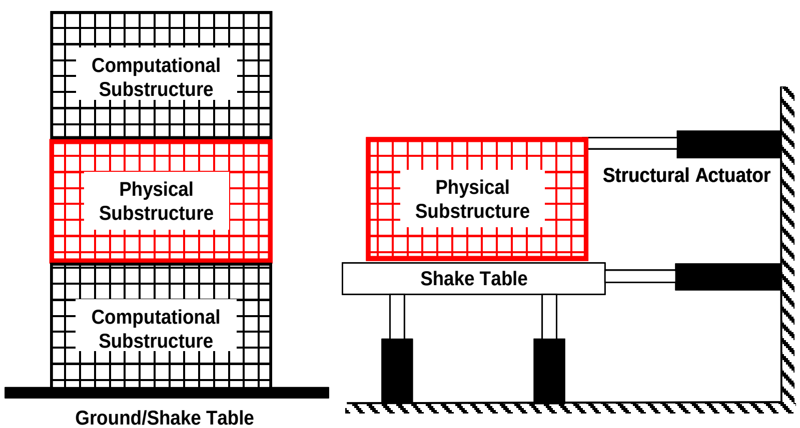
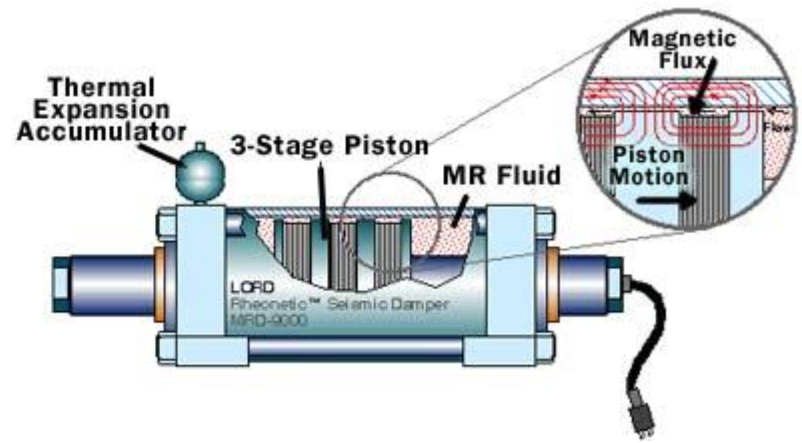
Outline

- MTS Real-time Hybrid Simulation Solution
- Real-time Hybrid System Dynamical Stability Analysis
  - Single DOF Hybrid System Analysis
  - Multiple DOF Hybrid System Analysis
  - Demo Examples
- Real-time Hybrid Simulation with Reduced Order Model (ROM)
- Summary

Motivation for Real-time Hybrid Simulation

- Physical substructures also contribute damping and inertia effects to the overall structure
- Requires high-force, dynamic actuators and large hydraulic pumping systems
- Control methods and system modeling tools are current topic of advanced research





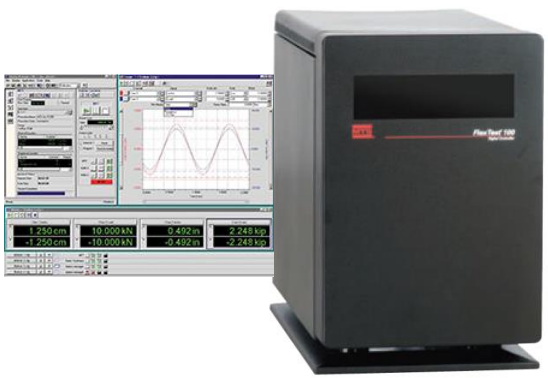
$\underbrace{\text{M,} \quad \text{C \&} \quad \text{K}}$

Test Specimen Rate Dependent

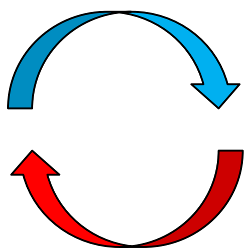
MTS Real-Time Hybrid Simulation Flow Chart

Powerful Controller & Software

- Ultra-low latency flow of data via SCRAMNet
- Command and feedback exchange linked directly to real-time test control processor
- Tuning and control techniques to ensure accurate force and motion
- Safety limits provided for the hybrid simulation

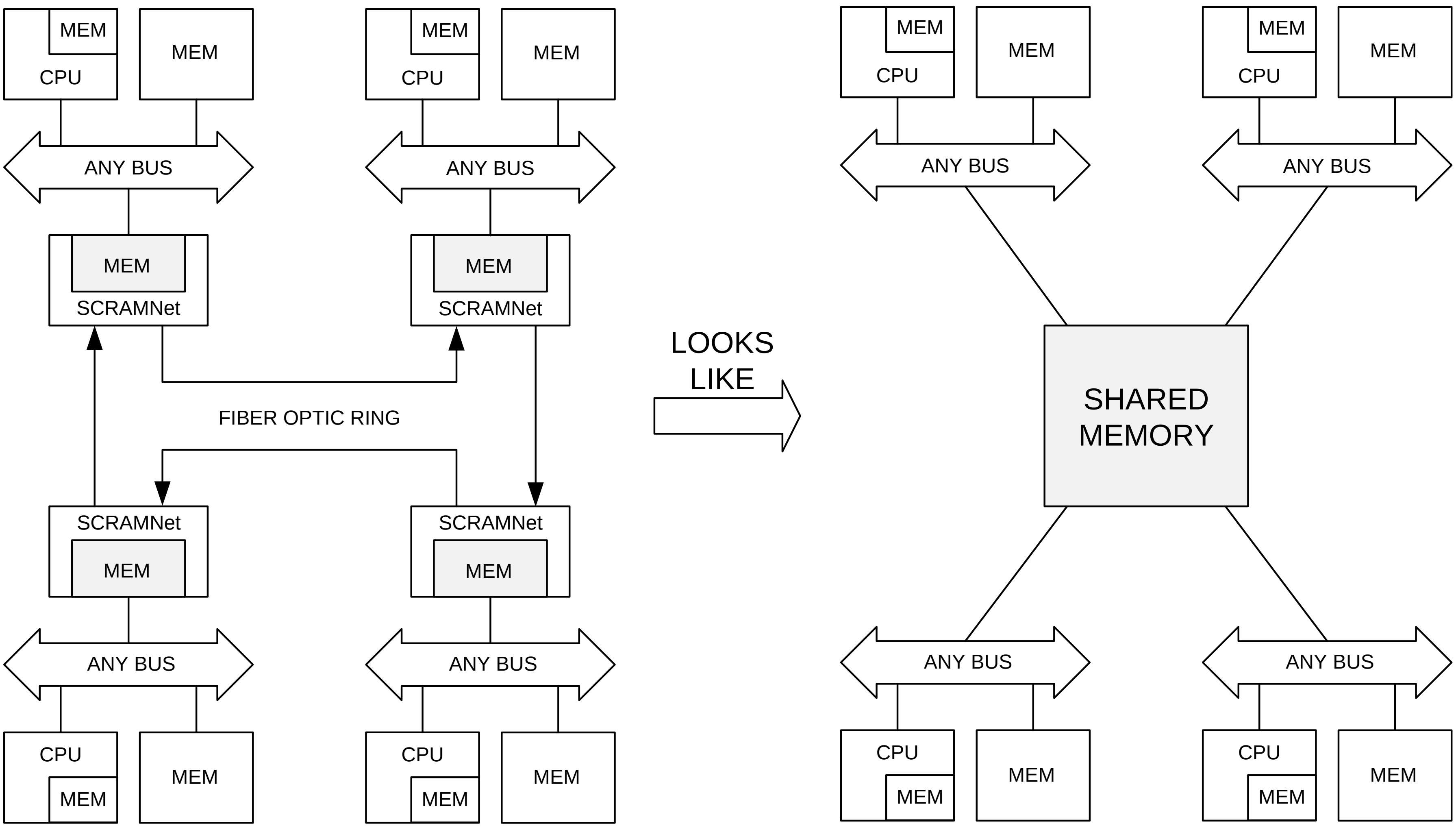


MTS FlexTest Controller
External Mode

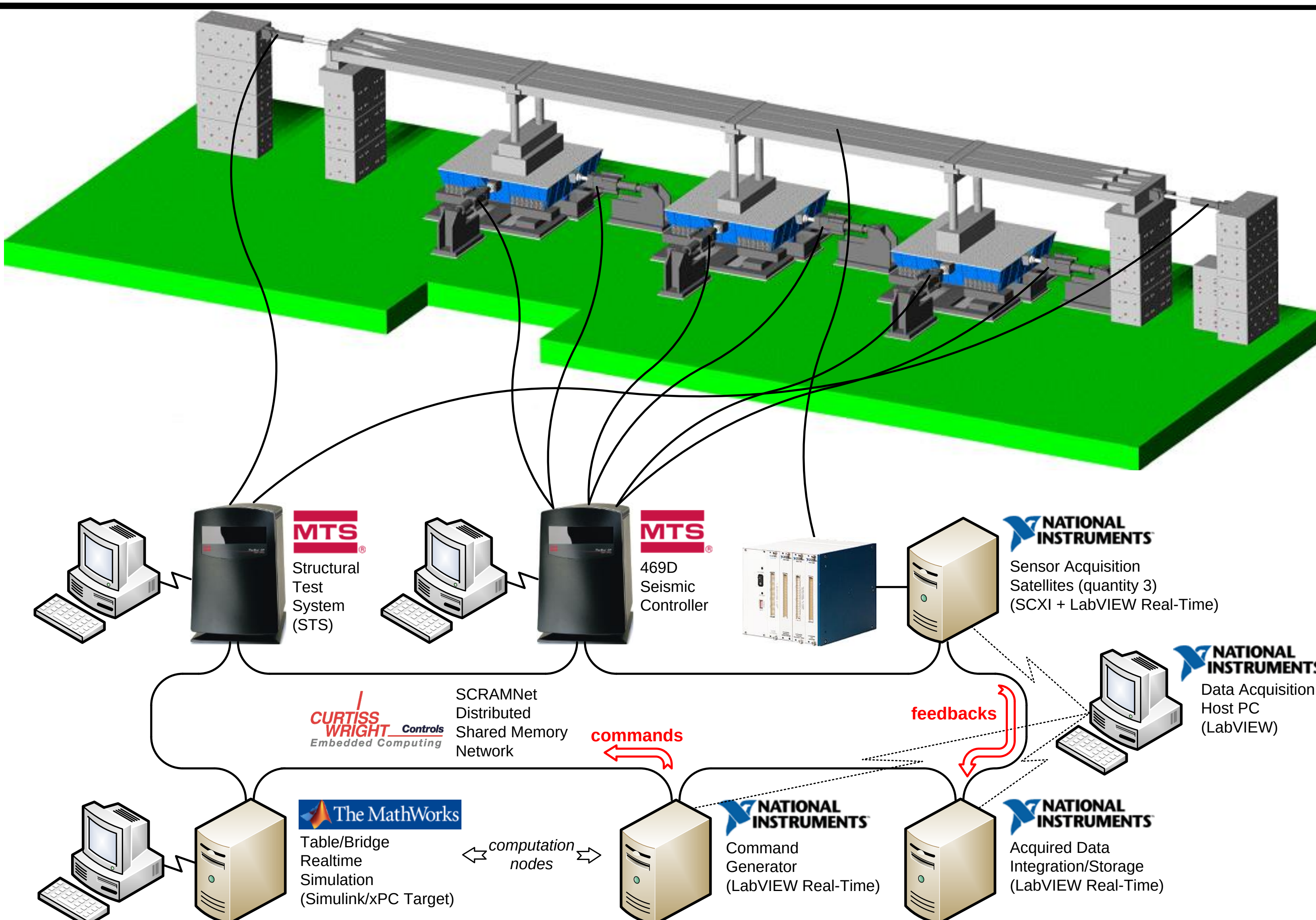


Programmable Realtime
Target PC

SCRAMNet Reflective Memory



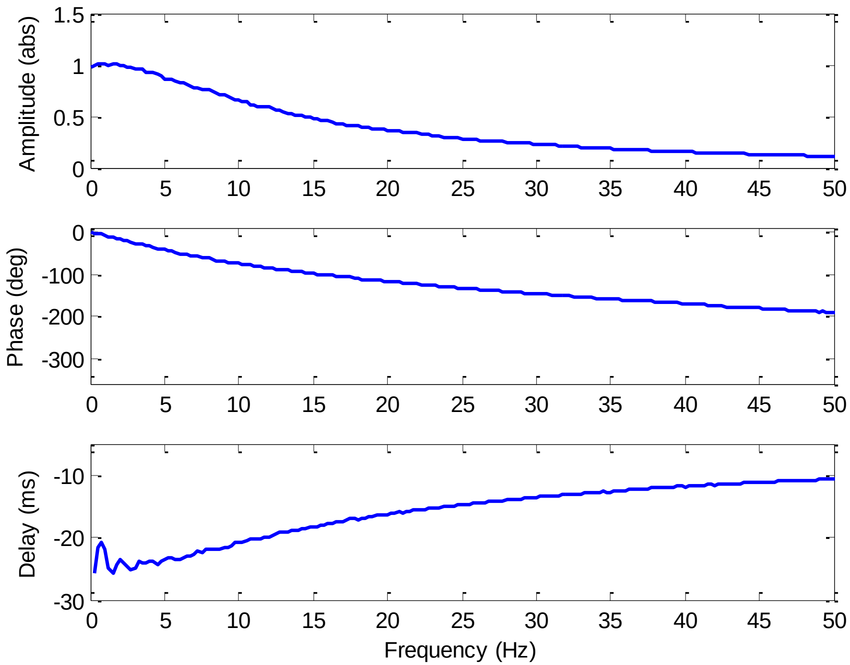
Shared Memory Network

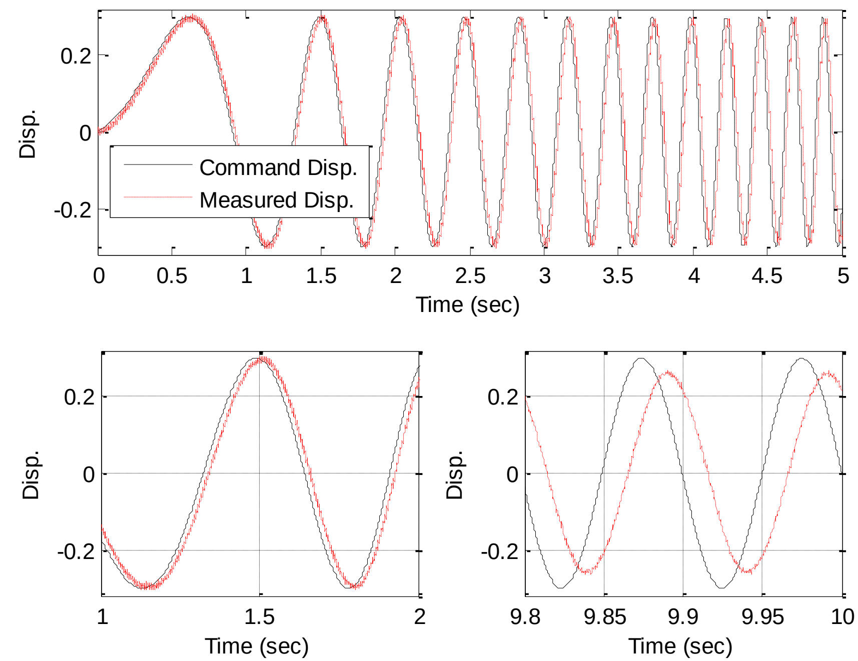


Real-time Hybrid System Dynamical Stability Analysis

Realistic Control System Behavior

- Typical control system response
  - Amplitude Roll-off
  - Phase lag/delay





Phase lag/Delay is a major concern in real-time hybrid simulation as it introduces negative damping!

Single DOF Hybrid System Analysis

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Reference system equation of motion (EOM)

$$M\ddot{x} + C\dot{x} + Kx = -M\ddot{x}_g$$

$$\boxed{(M_n + M_e)\ddot{x} + (C_n + C_e)\dot{x} + (K_n + K_e)x = -(M_n + M_e)\ddot{x}_g}$$

Hybrid system equation of motion (EOM)

$$M_n\ddot{x} + C_n\dot{x} + K_n x + M_e\ddot{x}_e + C_e\dot{x}_e + K_e x_e = -(M_n + M_e)\ddot{x}_g$$

Assume *Δ* and *δt* represent the amplitude and phase errors: $x = A\sin\omega t, x_e = \Delta A\sin\omega(t - \delta t)$

Linearized approximation using: $\sin(\omega\delta t) \approx \omega\delta t, \cos(\omega\delta t) \approx 1$

$$x_e = \Delta A\sin\omega(t - \delta t) \approx \Delta A(\sin\omega t - \omega\delta t\cos\omega t) = \Delta(x - \delta t\dot{x})$$

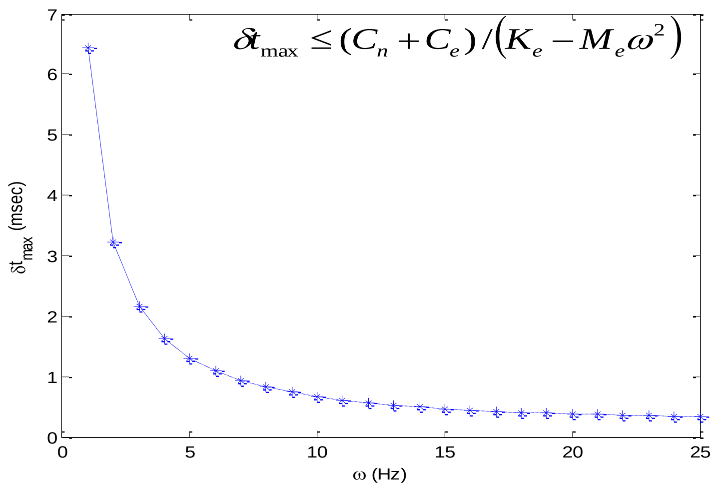
$$\dot{x}_e \approx \Delta(\dot{x} - \delta t\ddot{x})$$

$$\ddot{x}_e \approx \Delta(\ddot{x} + \omega^2\delta t\dot{x})$$

$$\boxed{(M_n + \Delta M_e - \Delta\delta t C_e)\ddot{x} + [C_n + \Delta C_e + \boxed{\Delta\delta t}\boxed{M_e\omega^2 - K_e}]\dot{x} + (K_n + \Delta K_e)x = -(M_n + M_e)\ddot{x}_g}$$

Negative damping depends on the actuator control error, and the substructure partition.

$$\left(M_n + \Delta M_e - \Delta\delta t C_e\right)\ddot{x} + \left[C_n + \Delta C_e + \Delta\delta t\left(M_e\omega^2 - K_e\right)\right]\dot{x} + \left(K_n + \Delta K_e\right)x = -\left(M_n + M_e\right)\ddot{x}_g$$



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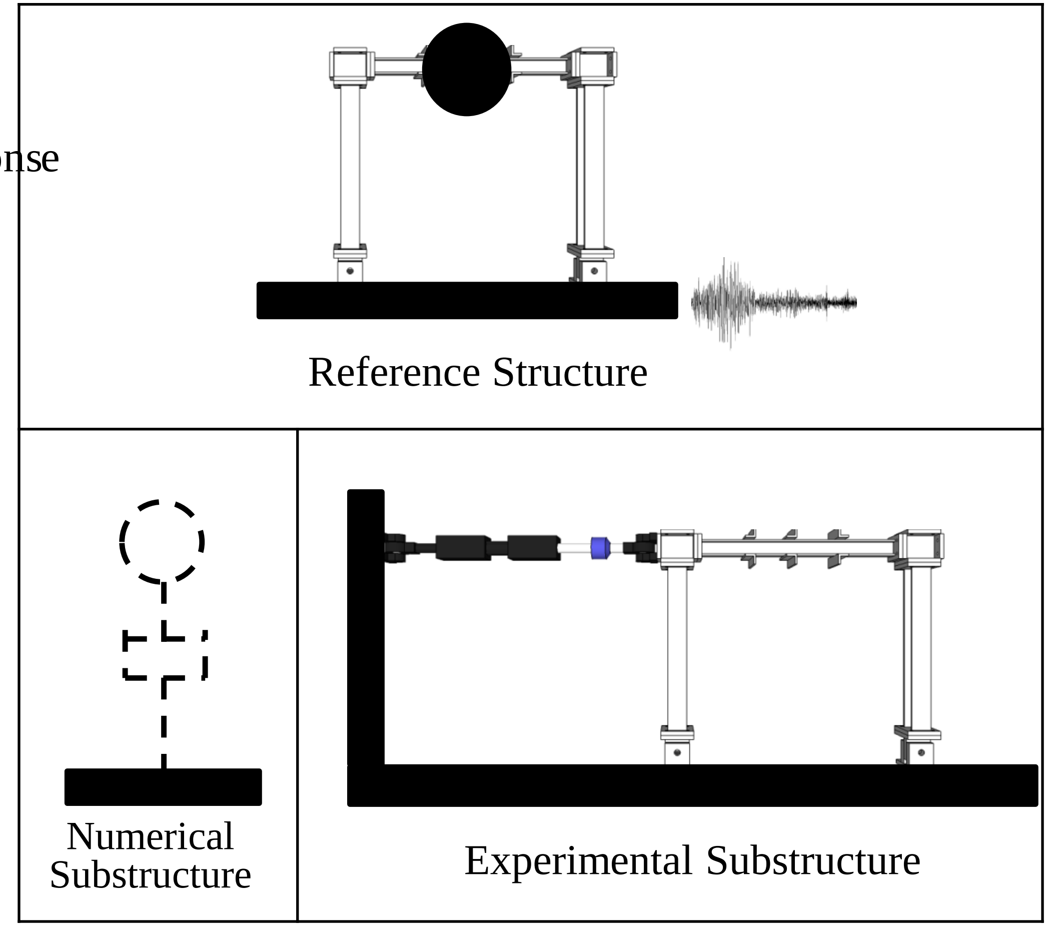

- worst case partition

$K_e = K_{total}$, $M_e = 0$

$K_n = 0$, $M_n = M_{total}$

$M_e\omega^2 - K_e$ causes max negative damping

Reference system equation of motion (EOM)

$$\underline{\underline{M}} \cdot \ddot{\underline{x}}(t) + \underline{\underline{C}} \cdot \dot{\underline{x}}(t) + \underline{\underline{K}} \cdot \underline{x}(t) = \underline{F}(t)$$

Assume classical damping, the EOM can be solved in the modal coordinates (q) for the uncoupled modal equations:

$$\underline{\underline{M_q}} \cdot \ddot{\underline{q}}(t) + \underline{\underline{C_q}} \cdot \dot{\underline{q}}(t) + \underline{\underline{K_q}} \cdot \underline{q}(t) = \underline{F_q}(t)$$

$$\underline{q}(t) = \begin{bmatrix} q_1(t) \\ q_2(t) \\ \vdots \\ q_N(t) \end{bmatrix} = \begin{bmatrix} A_1 \sin(\omega_1 t + \theta_1) \\ A_2 \sin(\omega_2 t + \theta_2) \\ \vdots \\ A_N \sin(\omega_N t + \theta_N) \end{bmatrix}$$

The solution in the natural coordinate (x) is:

$$\underline{x}(t) = \underline{\underline{\Phi}} \cdot \underline{q}(t)$$

the matrix $\underline{\underline{\Phi}}$ is the modal matrix and each column is the eigenvector of the corresponding mode shape.

Hybrid system equation of motion (EOM)

$$\underline{\underline{M}}_n \cdot \ddot{\underline{x}}(t) + \underline{\underline{C}}_n \cdot \dot{\underline{x}}(t) + \underline{\underline{K}}_n \cdot \underline{x}(t) + \underline{\underline{M}}_e \cdot \ddot{\underline{x}}_e(t) + \underline{\underline{C}}_e \cdot \dot{\underline{x}}_e(t) + \underline{\underline{K}}_e \cdot \underline{x}_e(t) = \underline{\underline{F}}(t)$$

Assume in the modal coordinates the amplitude and phase error Δ_i and δt_i for the i^{th} mode:

$$\underline{x}_e(t) = \begin{bmatrix} x_{e,1}(t) \\ x_{e,2}(t) \\ \vdots \\ x_{e,N}(t) \end{bmatrix} = \underline{\underline{\Phi}} \begin{bmatrix} q_{e,1}(t) \\ q_{e,2}(t) \\ \vdots \\ q_{e,N}(t) \end{bmatrix} = \underline{\underline{\Phi}} \begin{bmatrix} \Delta_1 A_1 \sin[\omega_1(t-\delta_1)+\theta_1] \\ \Delta_2 A_2 \sin[\omega_2(t-\delta_2)+\theta_2] \\ \vdots \\ \Delta_N A_N \sin[\omega_N(t-\delta_N)+\theta_N] \end{bmatrix}$$

Obtain the linearized approximation using: $\cos(\omega_i\delta_i) \approx 1, \sin(\omega_i\delta_i) \approx \omega_i\delta_i$

$$\underline{x}_e(t) = \underline{\underline{\Phi}}\,\underline{\underline{\Delta}}[\underline{q}(t) - \underline{\underline{\delta}}\dot{\underline{q}}(t)]$$

$$\dot{\underline{x}}_e(t) = \underline{\underline{\Phi}}\,\underline{\underline{\Delta}}[\dot{\underline{q}}(t) - \underline{\underline{\delta}}\ddot{\underline{q}}(t)]$$

$$\ddot{\underline{x}}_e(t) = \underline{\underline{\Phi}}\,\underline{\underline{\Delta}}[\ddot{\underline{q}}(t) + \underline{\underline{\omega}}^2\,\underline{\underline{\delta}}\dot{q}(t)]$$

Hybrid system equation of motion (EOM)

EOM in the modal coordinates:

$$[\underline{\underline{M_n}} \cdot \underline{\underline{\Phi}} + \underline{\underline{M_e}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Delta}} - \underline{\underline{C_e}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Delta}} \cdot \underline{\underline{\delta}}]\ddot{\underline{q}}(t) + [\underline{\underline{C_n}} \cdot \underline{\underline{\Phi}} + \underline{\underline{M_e}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Delta}} \cdot \underline{\underline{\omega^2}} \cdot \underline{\underline{\delta}} + \underline{\underline{C_e}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Delta}} - \underline{\underline{K_e}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Delta}} \cdot \underline{\underline{\delta}}]\dot{\underline{q}}(t)$$

$$+[\underline{\underline{K_n}} \cdot \underline{\underline{\Phi}} + \underline{\underline{K_e}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Delta}}]\underline{q}(t) = \underline{F}(t)$$

Convert EOM back to the natural coordinates using $\underline{q}(t) = \underline{\underline{\Phi^{-1}}} \cdot \underline{x}(t)$

$$[\underline{\underline{M_n}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Phi^{-1}}} + \underline{\underline{M_e}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Delta}} \cdot \underline{\underline{\Phi^{-1}}} - \underline{\underline{C_e}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Delta}} \cdot \underline{\underline{\delta}} \cdot \underline{\underline{\Phi^{-1}}}]\ddot{\underline{x}}(t)$$

$$+[\underline{\underline{C_n}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Phi^{-1}}} + \underline{\underline{M_e}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Delta}} \cdot \underline{\underline{\omega^2}} \cdot \underline{\underline{\delta}} \cdot \underline{\underline{\Phi^{-1}}} + \underline{\underline{C_e}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Delta}} \cdot \underline{\underline{\Phi^{-1}}} \boxed{- \underline{\underline{K_e}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Delta}} \cdot \underline{\underline{\delta}} \cdot \underline{\underline{\Phi^{-1}}}}]\dot{\underline{x}}(t)$$

$$+[\underline{\underline{K_n}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Phi^{-1}}} + \underline{\underline{K_e}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Delta}} \cdot \underline{\underline{\Phi^{-1}}}]\underline{x}(t) = \underline{F}(t)$$

Hybrid System Stability Demo Example - 1

3-Story Prototype Structure

Story Height 1.8m

Bay width 2.8m

Table 1: Cross-seccional dimension of the Structural Specimen

Structural Members	Dimension (mm x mm x mm x mm)
Column	H 145 x 145 x 8 x 10
Beam	H 140 x 100 x 8 x 10
Braces	H 100 x 100 x 6 x 10

$$K_{ref} = \begin{bmatrix} 184.2 & -115.2 & 14.3 \\ -115.2 & 201.3 & -94.9 \\ 14.3 & -94.9 & 81.7 \end{bmatrix} \frac{kN}{mm}$$

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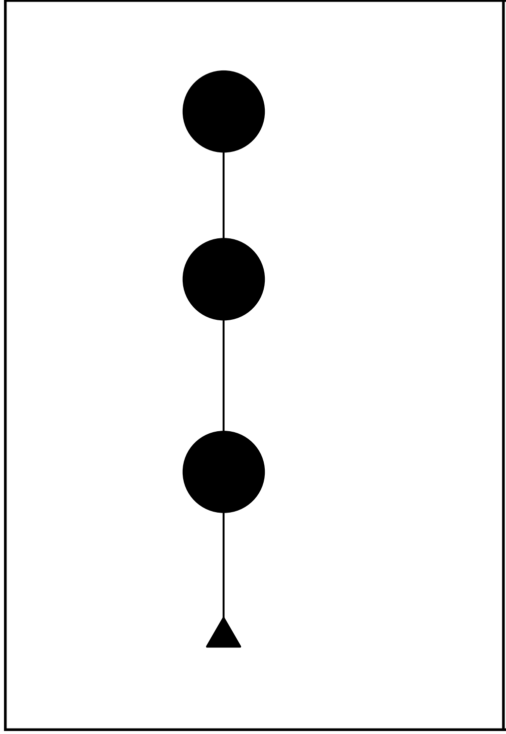
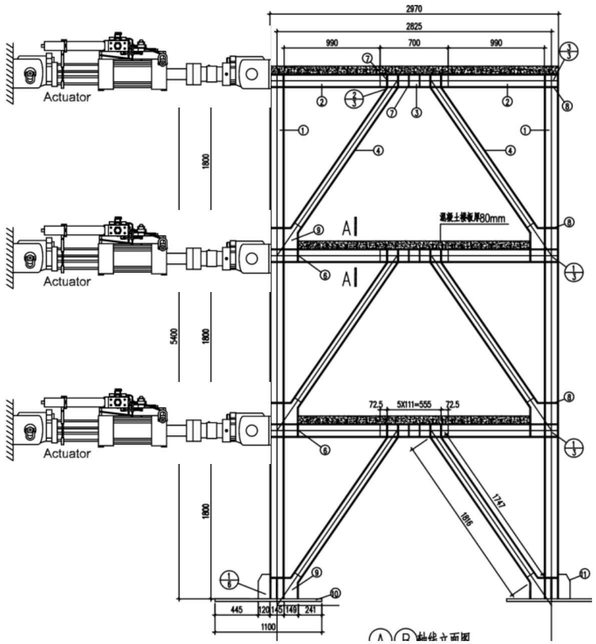
Hybrid Configuration (Worst Case Substructure Partition)





Numerical Substructure

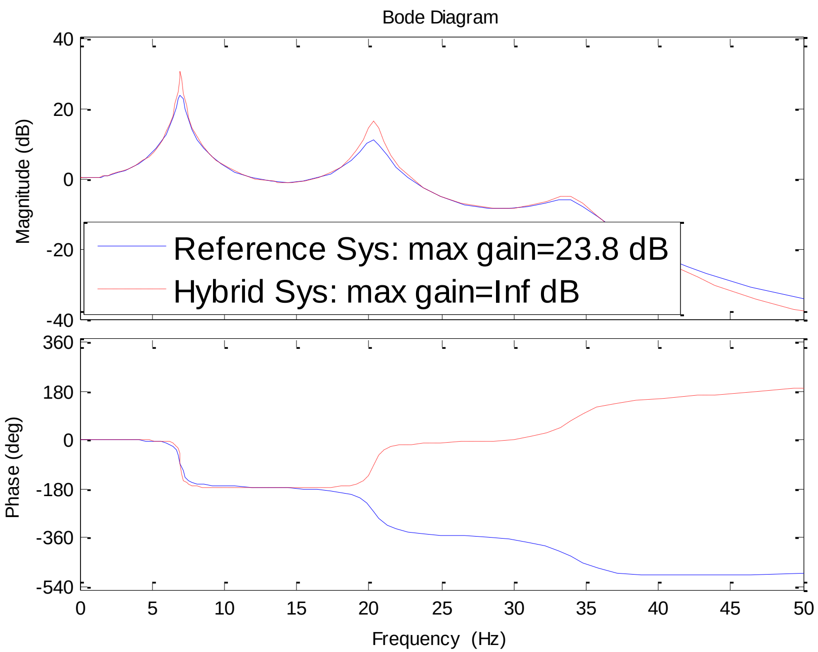
Experimental Substructure

Demo Example 1, Design Case A

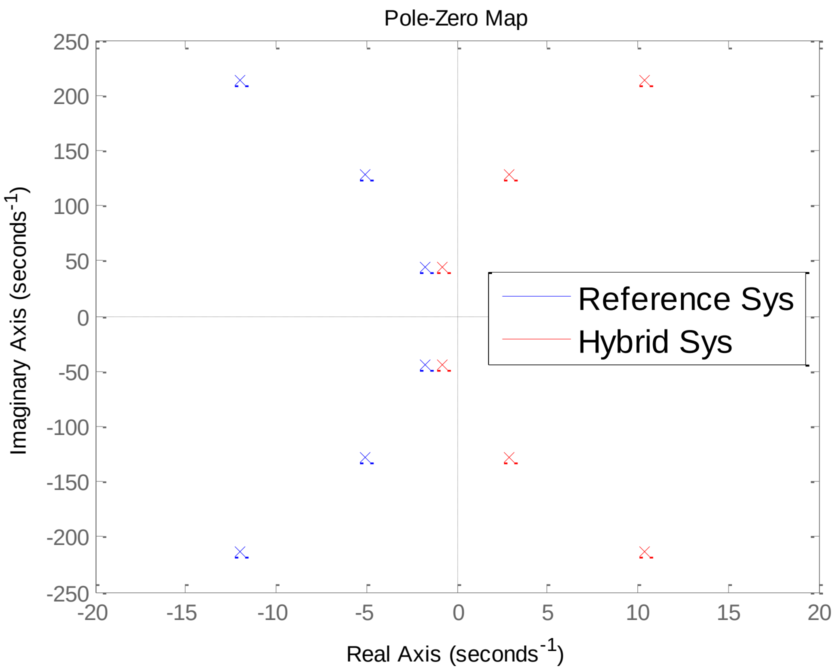
Seismic mass 7.3 metric tons per floor, dynamic modes 7.0Hz, 20.3Hz, and 34.1Hz

4% Rayleigh damping @ mode 1 & 2

Assume 1ms delay @ all modes in the hybrid implementation



Frequency Response Function (FRF)



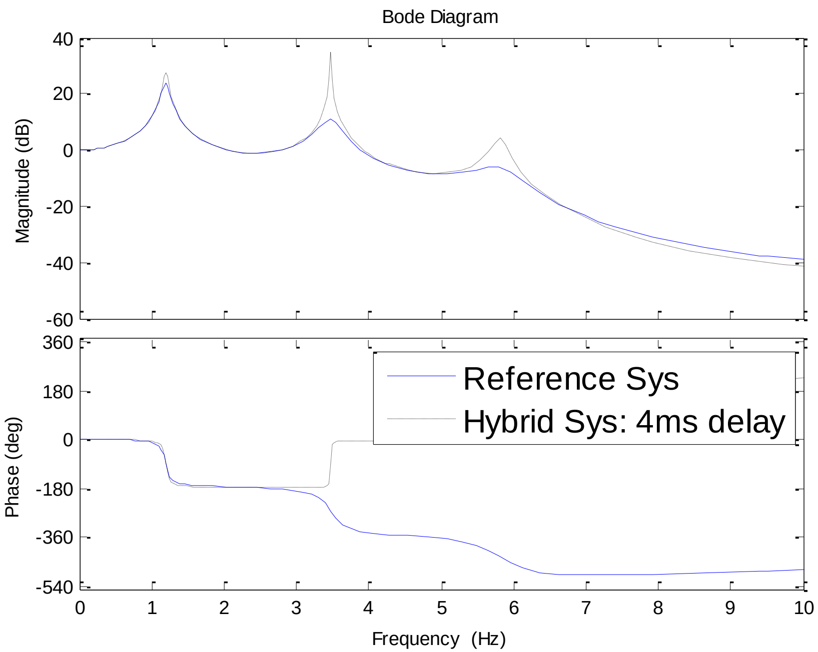
System Poles

Demo Example 1, Design Case B

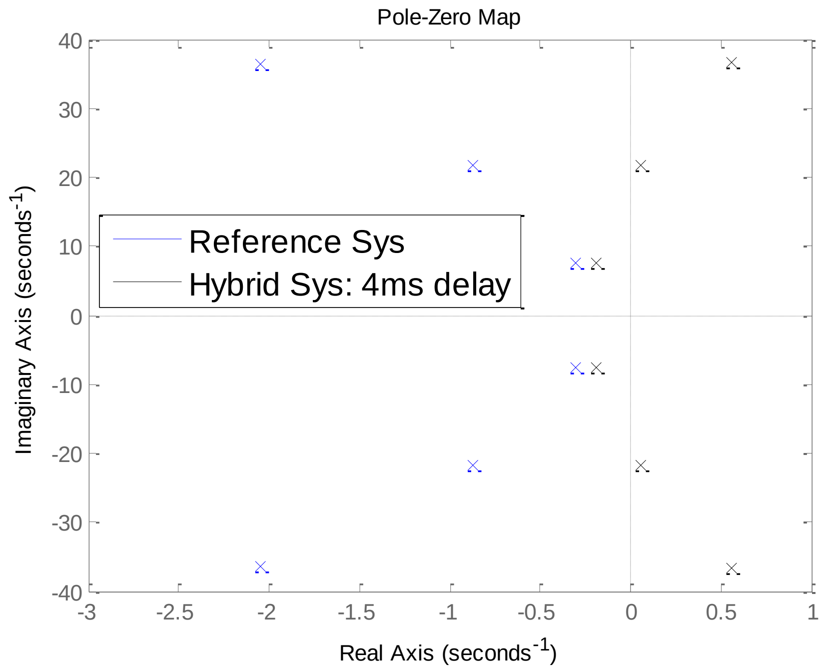
Seismic mass 250 metric tons per floor, dynamic modes 1.2Hz, 3.5Hz, and 5.8Hz

4% Rayleigh damping @ mode 1 & 2

Assume various delay values in the hybrid implementation

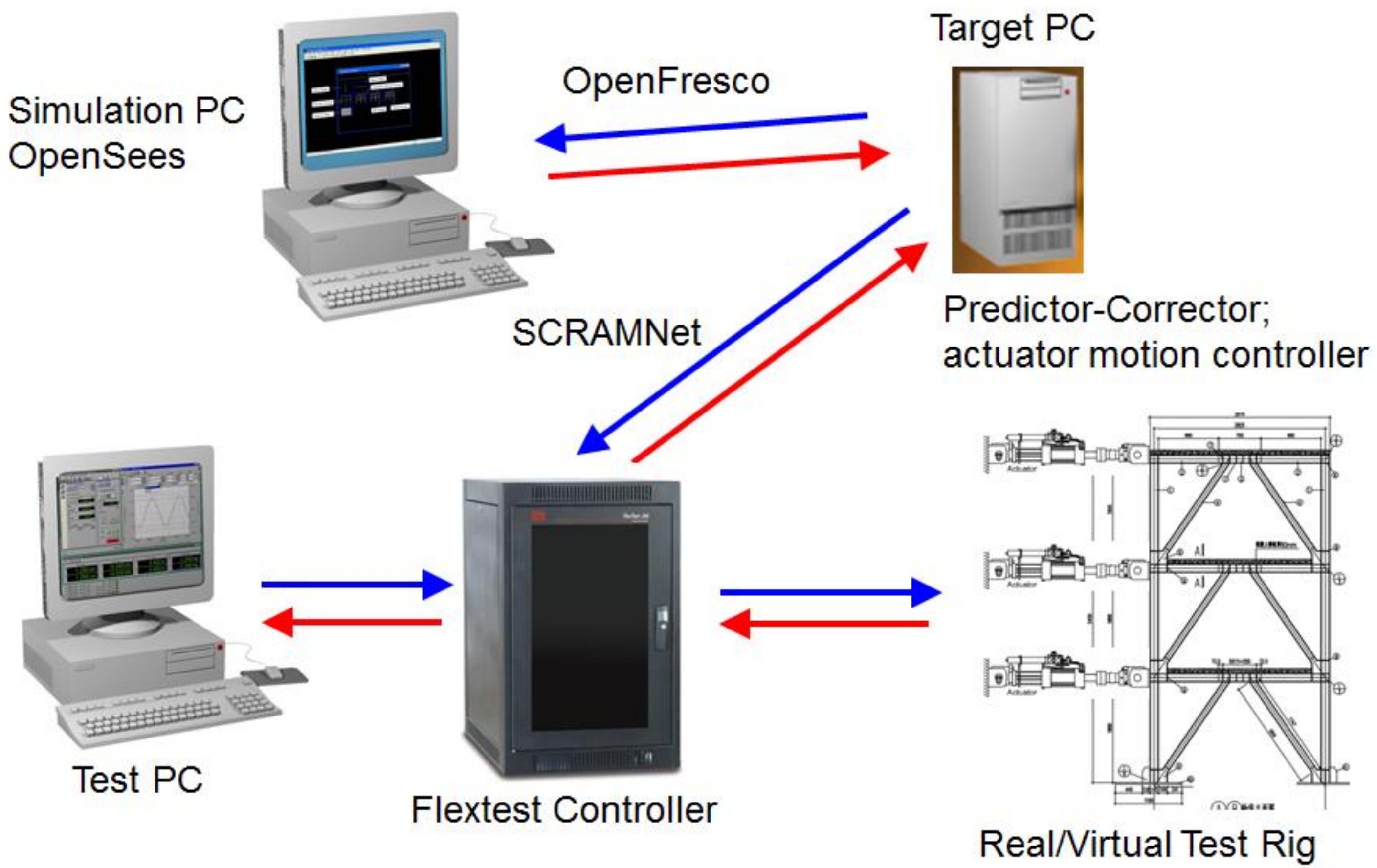


Frequency Response Function (FRF)

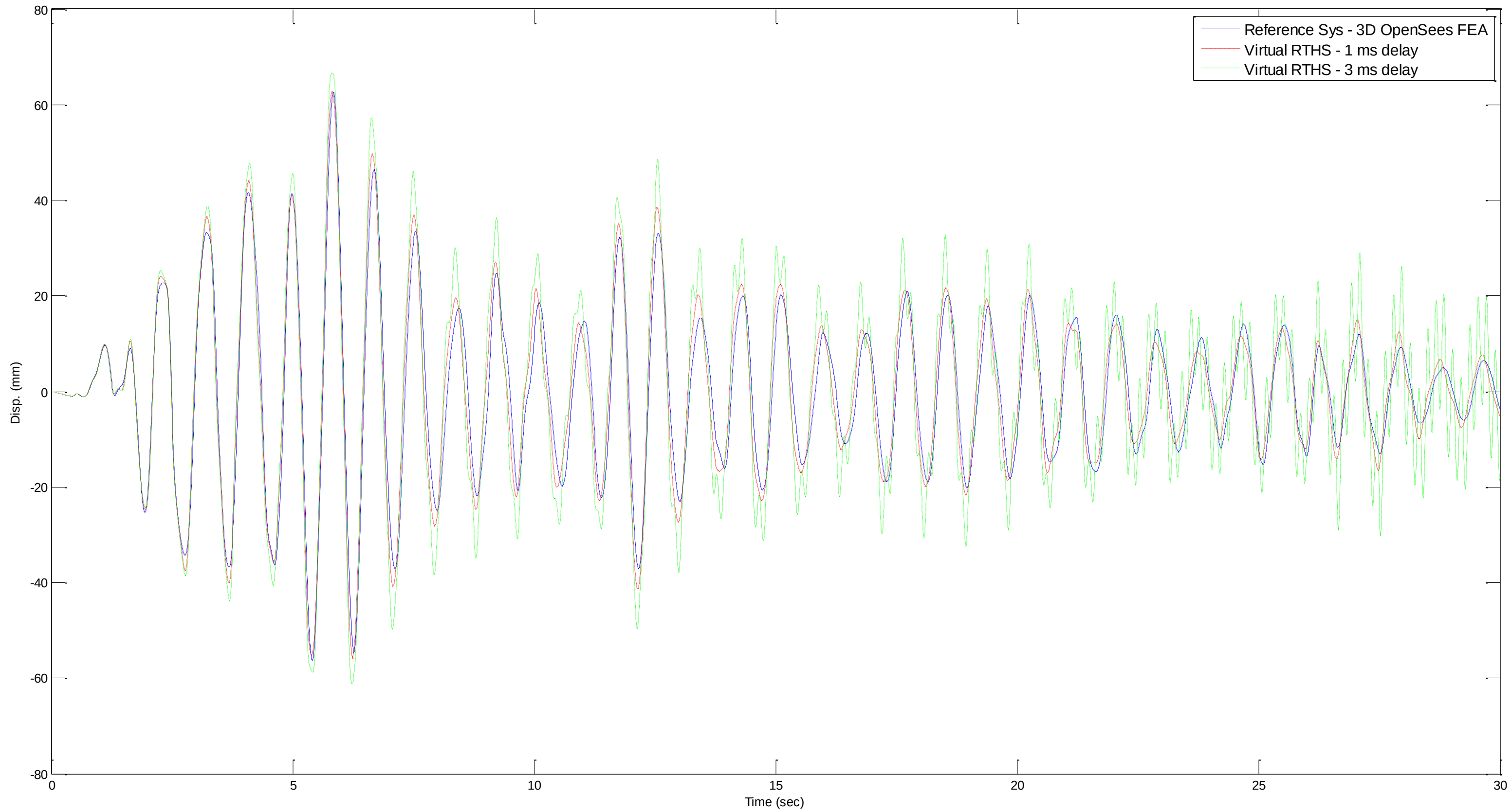


System Poles

Virtual Testing Validation

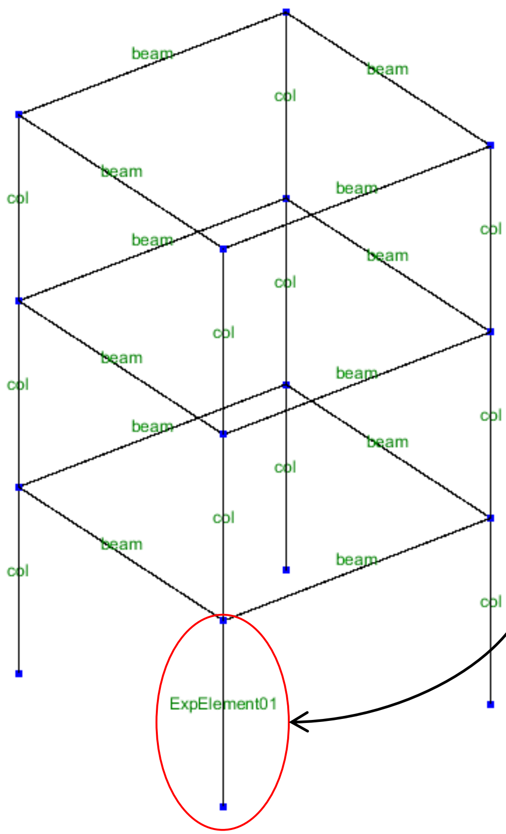


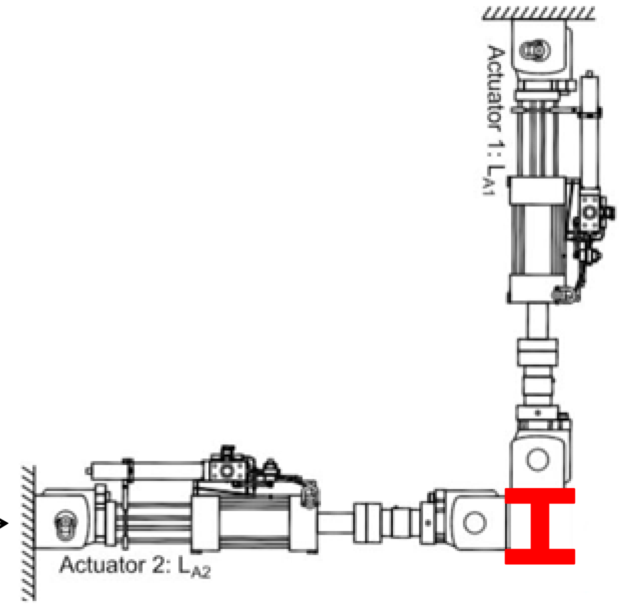
Example 1, Design Case B Virtual Hybrid Simulation Results



Demo Example – 2

$$K_{ref} = \begin{bmatrix} 32.6 & -19.7 & 5.0 \\ -19.7 & 25.9 & -11.7 \\ 5.0 & -11.7 & 7.5 \end{bmatrix} \frac{kN}{mm}$$





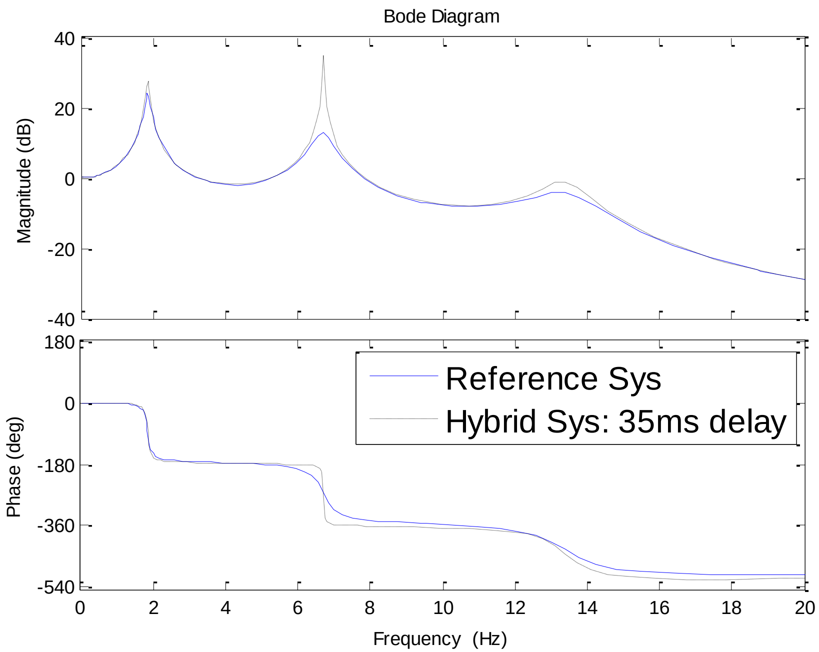
In hybrid simulation, bi-axial single column testing, all other structural components simulated in FEA

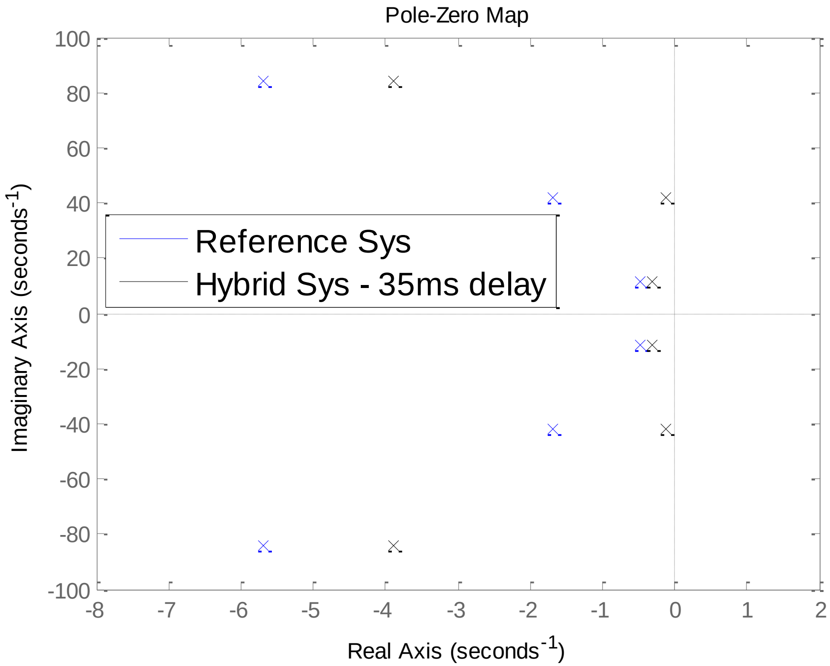
Hybrid System Analysis

Seismic mass 7.3 metric tons per floor, global modes are 1.8Hz, 6.7Hz, and 13.4Hz

4% Rayleigh damping @ mode 1 & 2

Assume various delay values in the hybrid implementation

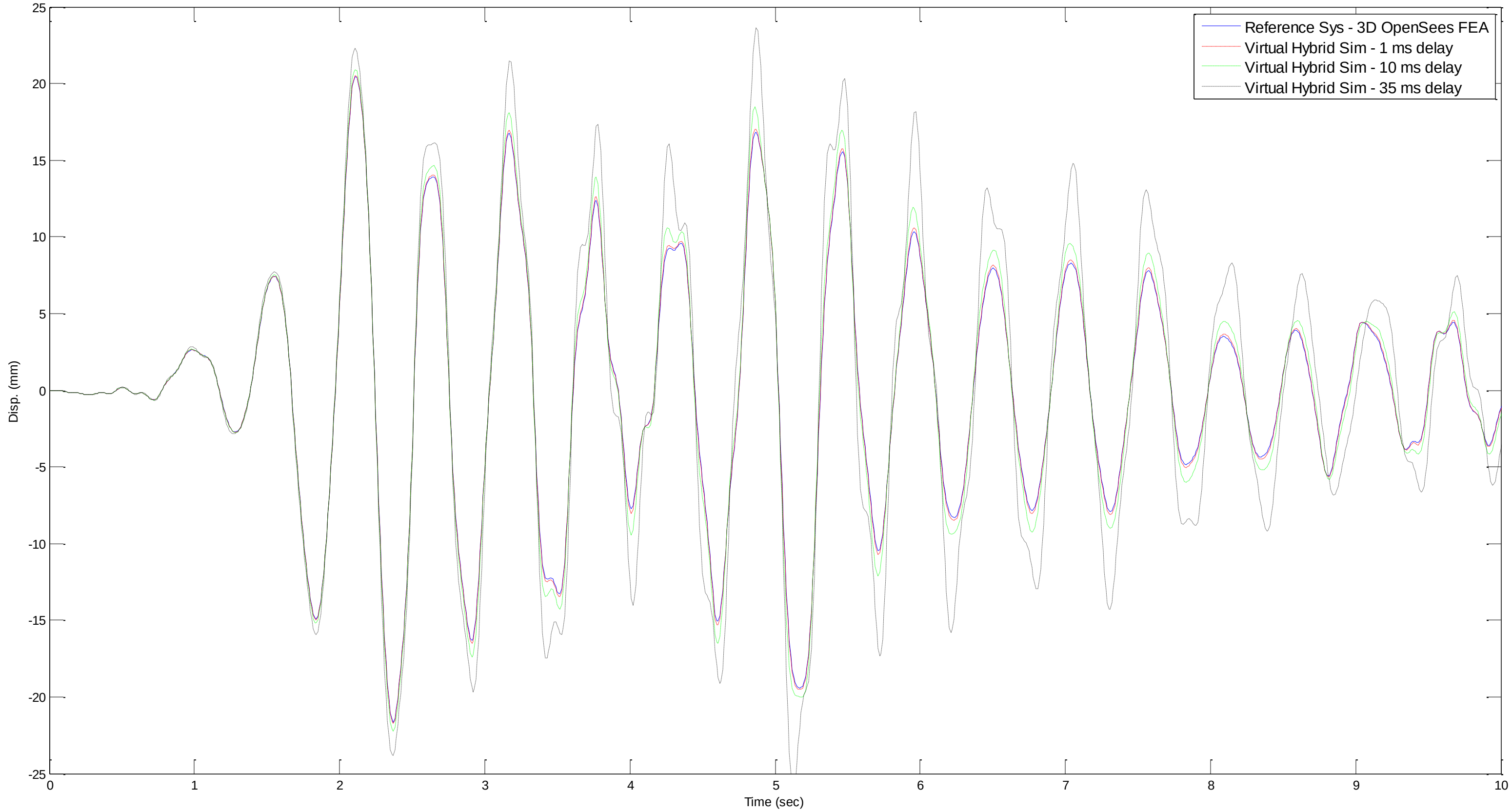




Frequency Response Function (FRF)

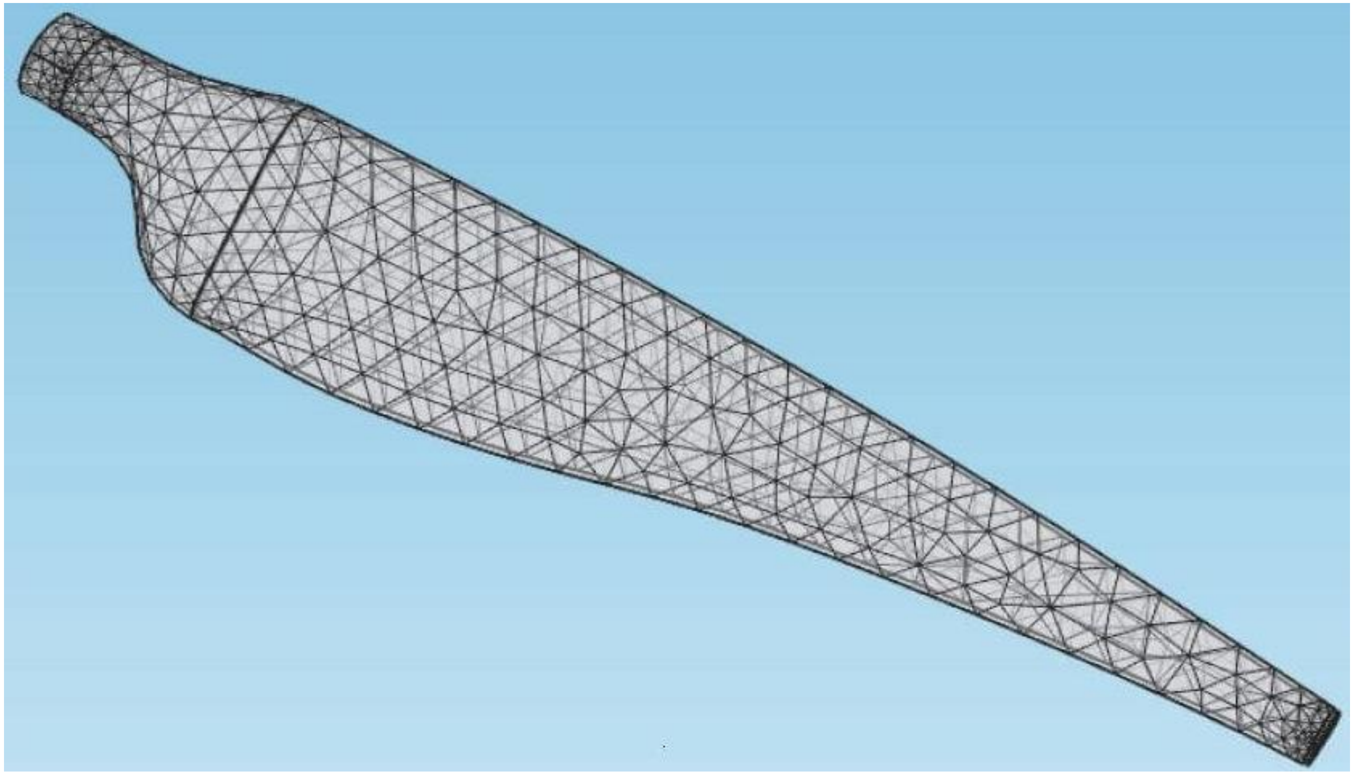
System Poles

Virtual Testing Results



Real-time Hybrid Simulation with Reduced Order Models

In many cases, FEA models are too complicated to run in real-time. However, the specimens are rate dependent. Therefore, real-time hybrid simulation is a must. The solution is Reduced Order Model (ROM).



ROM for Structural Dynamics

$$M_p\ddot{X}_p + C_p\dot{X}_p + K_pX_p = K_p\Gamma x_n + C_p\Gamma\dot{x}_n.$$

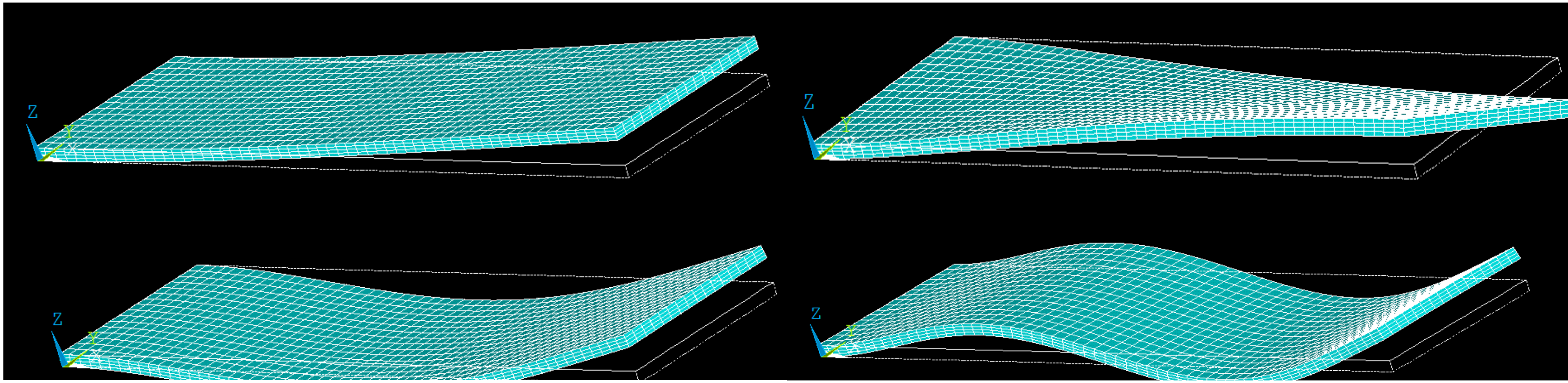
$$\begin{bmatrix} \dot{X}_p \\ \ddot{X}_p \end{bmatrix} = \begin{pmatrix} 0_{p\times p} & I_{p\times p} \\ -M_p^{-1}K_p & -M_p^{-1}C_p \end{pmatrix} \begin{bmatrix} X_p \\ \dot{X}_p \end{bmatrix} + \begin{pmatrix} 0_{p\times 1} & 0_{p\times 1} \\ -M_p^{-1}K_p\Gamma & -M_p^{-1}C_p\Gamma \end{pmatrix} \begin{bmatrix} x_n \\ \dot{x}_n \end{bmatrix}$$

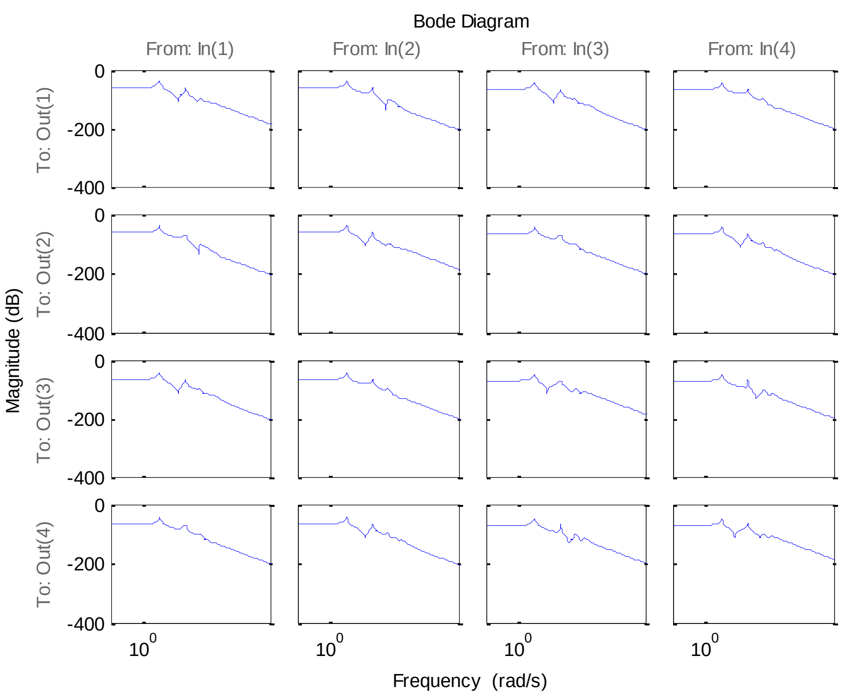
$$F_p = \begin{pmatrix} k_{n+1} & 0_{1\times p-1} & c_{n+1} & 0_{1\times p-1} \end{pmatrix} \begin{bmatrix} X_p \\ \dot{X}_p \end{bmatrix} + \begin{pmatrix} -k_{n+1} & -c_{n+1} \end{pmatrix} \begin{bmatrix} x_n \\ \dot{x}_n \end{bmatrix}$$

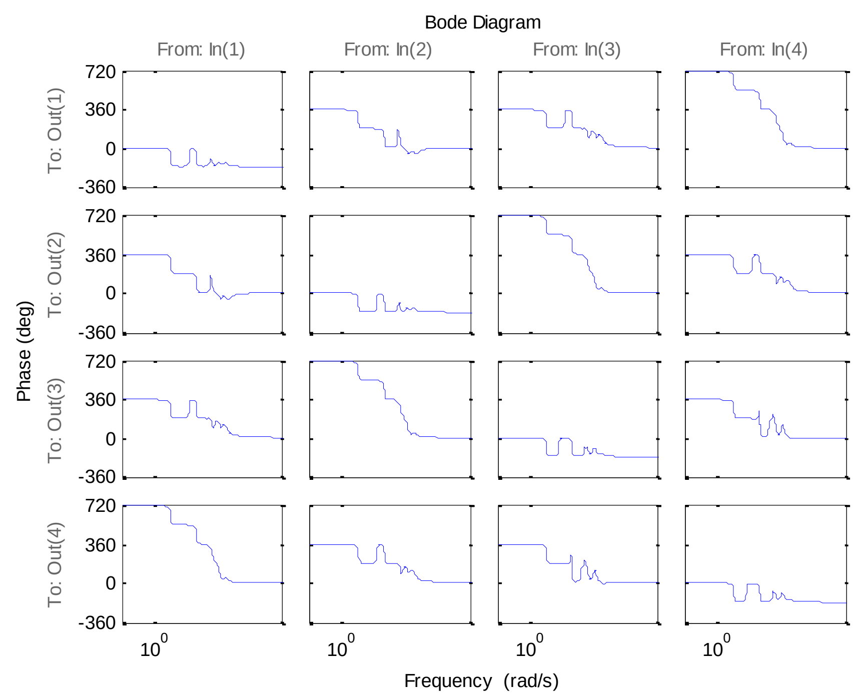
- Convert higher order dynamic model into 1st order state space model
- State space model can easily be integrated with other dynamical components, for system analysis and control design purposes
- Low computational cost to facilitate real-time execution

ROM for FEA Models (ANSYS)

FEA → modal superposition → state space ROM

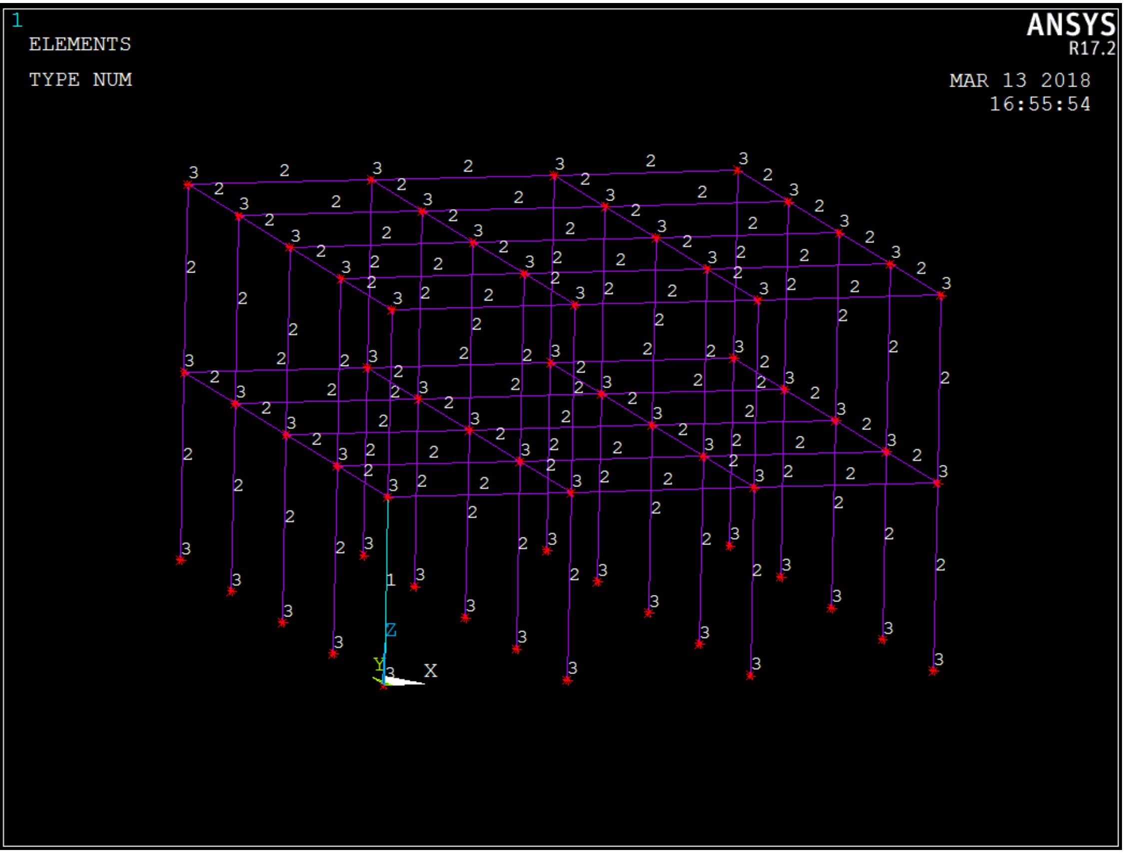


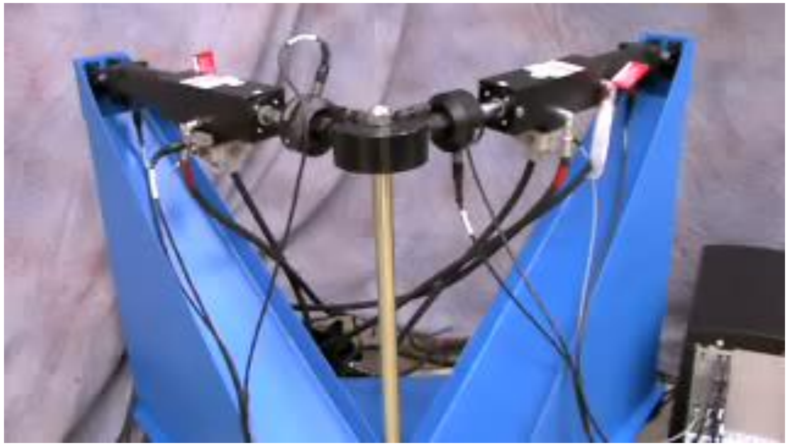


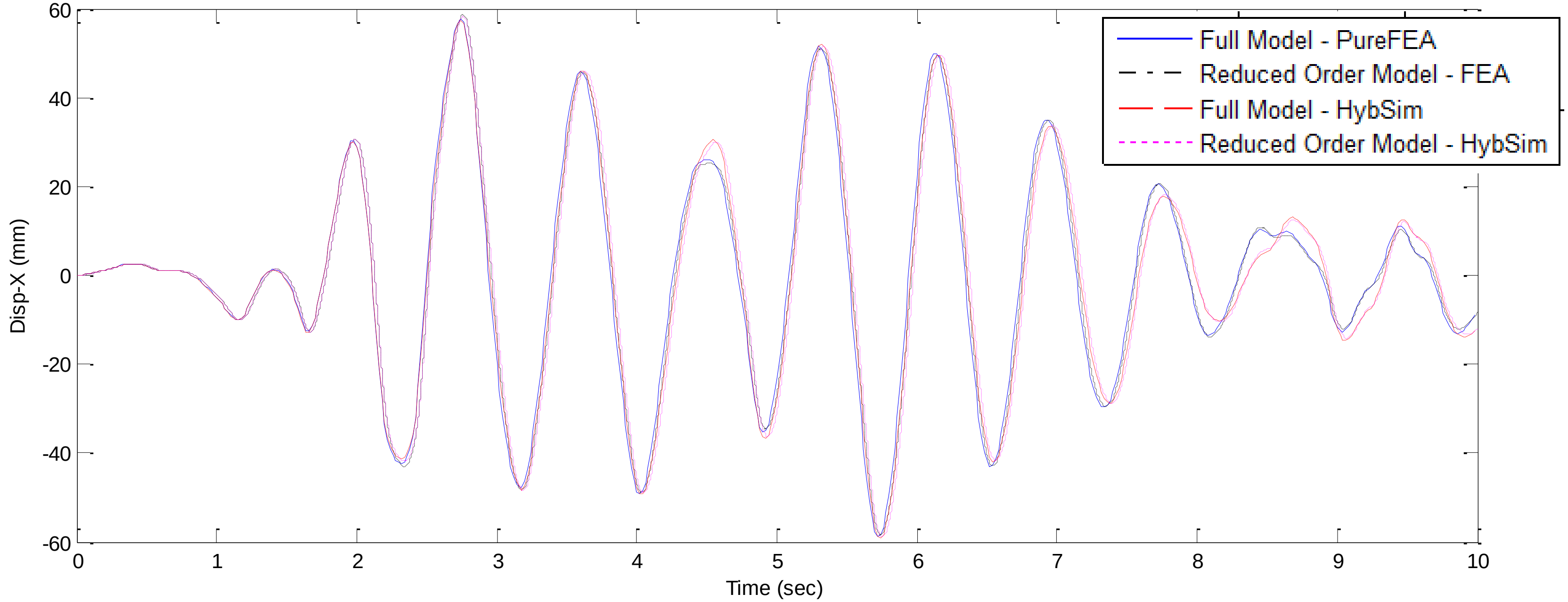


ROM for FEA Models (ANSYS)

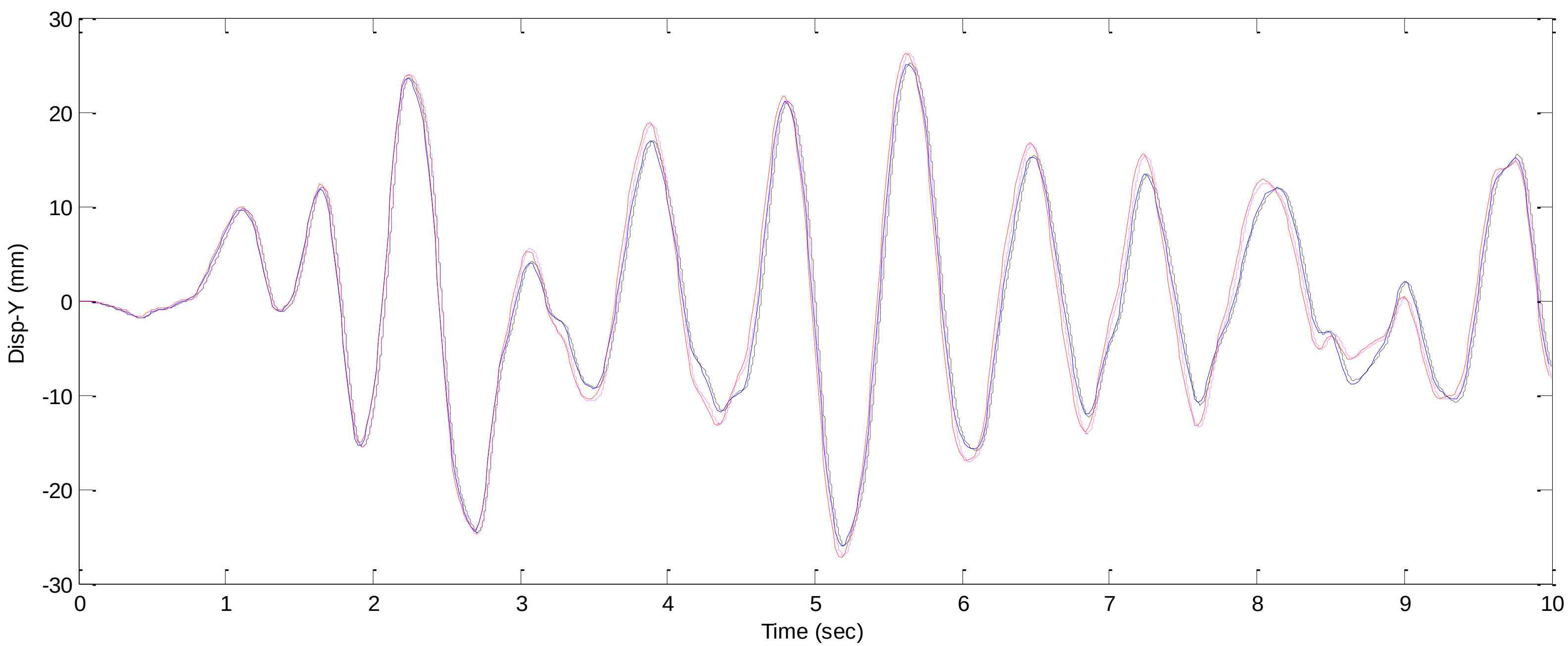
- State space ROM constructed offline from ANSYS
- Hard real-time hybrid simulation, 1024 Hz simulation rate







60
40
20
0
-20
-40
-60
Disp-X (mm)
0
1
2
3
4
5
6
7
8
9
10
Time (sec)
Full Model - PureFEA
Reduced Order Model - FEA
Full Model - HybSim
Reduced Order Model - HybSim



30
20
10
0
-10
-20
-30
Disp-Y (mm)
0
1
2
3
4
5
6
7
8
9
10
Time (sec)

Summary

- Dynamical analysis approach is needed to gain system level understanding about real-time hybrid simulation.
- The hybrid system negative damping not only depends on the actuator control error, but also on the substructure partition.
- The hybrid system EOM can be used with virtual testing procedure to predict the real testing stability limit/performance.
- ROM is an effective technique for real-time hybrid simulation