A Nonlinear Kinetic Model for Multi-Stage Friction Pendulum Systems

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Introduction

Multi-stage friction pendulum systems (MSFPs) are currently being designed and developed as seismic isolation devices for a wide range of structural and non-structural systems. [1]

Figure: An overview image of an example of a Triple Friction Pendulum (TFP).

While current models have come a long way, no current model for MSFPs utilizes a rigorous setup for the kinematics of the internal sliders; they start directly with scalar equations. Another drawback of current models, is that no one model incorporates the full kinetics of the MSFPs with bi-directional motion; there is either full kinetics for planar motion or bi-directional motion with only kinematics and no kinetics.

Modeling

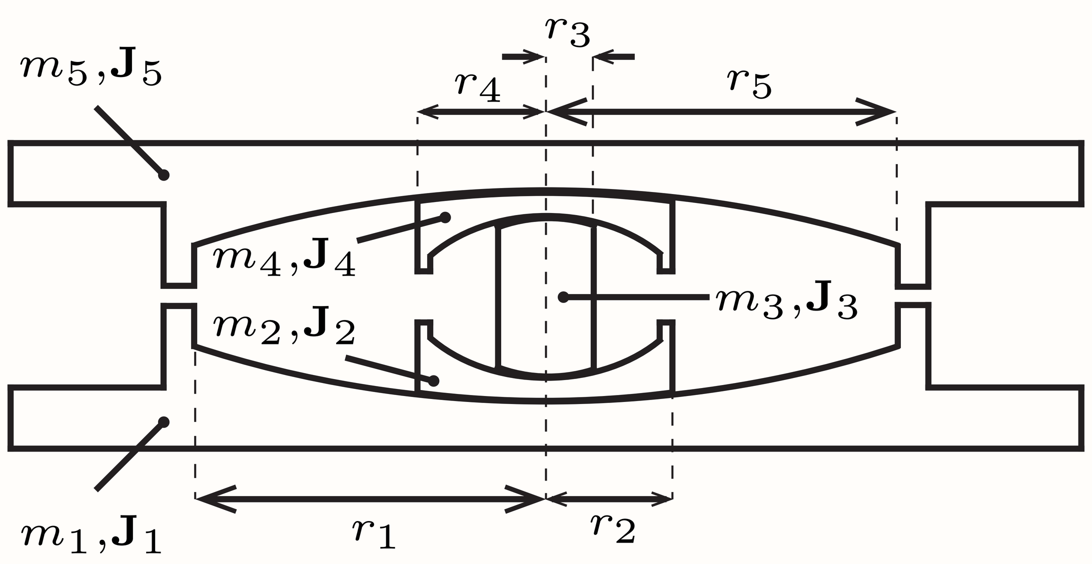

Figure: Diagram of a Triple Friction Pendulum (TFP) model.

Modeling of the TFP is done with a set of 1-2-3 Euler angles to define the kinematics of each bearing with respect to the previous bearing. And the Euler angles are used to define a set of co-rotational basis vectors for each bearing.

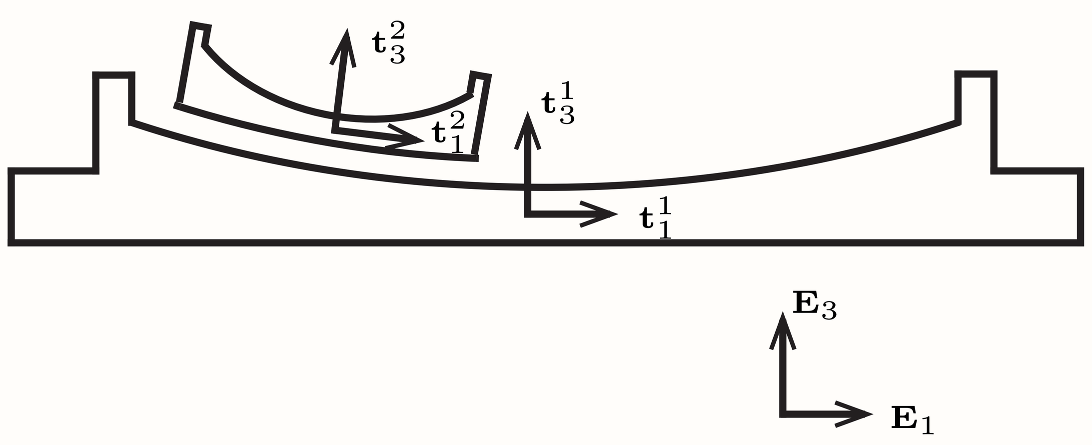

Figure: Locations of the co-rotational basis vectors for the first two bearings.

Normal Forces

New set of 1-2 Euler Angles to define Normal forces

$$\tilde{\mathbf{t}}_i^1 = \tilde{\mathbf{R}}_1 \mathbf{t}_i^1, \quad \tilde{\mathbf{t}}_i^2 = \tilde{\mathbf{R}}_2 \mathbf{t}_i^2, \quad \tilde{\mathbf{t}}_i^3 = \tilde{\mathbf{R}}_3 \mathbf{t}_i^3, \quad \tilde{\mathbf{t}}_i^4 = \tilde{\mathbf{R}}_4 \mathbf{t}_i^4$$

$$\tilde{\mathbf{R}}_1 = \mathbf{R}(\tilde\psi_1, \tilde\theta_1; \mathbf{t}_i^1), \quad \tilde{\mathbf{R}}_2 = \mathbf{R}(\tilde\psi_2, \tilde\theta_2; \mathbf{t}_i^2)$$
$$\tilde{\mathbf{R}}_3 = \mathbf{R}(\tilde\psi_3, \tilde\theta_3; \mathbf{t}_i^3), \quad \tilde{\mathbf{R}}_4 = \mathbf{R}(\tilde\psi_4, \tilde\theta_4; \mathbf{t}_i^4)$$

$$\tilde{\mathbf{r}}_1 = \mathbf{r}_{1c} - R_1 \tilde{\mathbf{t}}_3^1, \ \tilde{\mathbf{r}}_2 = \mathbf{r}_{2c} - R_2 \tilde{\mathbf{t}}_3^2$$
$$\tilde{\mathbf{r}}_3 = \mathbf{r}_{3c} + R_3 \tilde{\mathbf{t}}_3^3, \ \tilde{\mathbf{r}}_4 = \mathbf{r}_{4c} + R_4 \tilde{\mathbf{t}}_3^4$$

$$\mathbf{N}_1 = N_1 \tilde{\mathbf{t}}_3^1, \quad \mathbf{N}_2 = N_2 \tilde{\mathbf{t}}_3^2, \quad \mathbf{N}_3 = N_3 \tilde{\mathbf{t}}_3^3, \quad \mathbf{N}_4 = N_4 \tilde{\mathbf{t}}_3^4$$

Friction Forces

$$\mathbf{F}_{f1} = -\mu_1 N_1 \tilde{\mathbf{f}}_1, \quad \mathbf{F}_{f2} = -\mu_2 N_2 \tilde{\mathbf{f}}_2,$$
$$\mathbf{F}_{f3} = -\mu_3 N_3 \tilde{\mathbf{f}}_3, \quad \mathbf{F}_{f4} = -\mu_4 N_4 \tilde{\mathbf{f}}_4$$

$$\tilde{\mathbf{f}}_1 = Y_1 \tilde{\mathbf{t}}_1^1 + Z_1 \tilde{\mathbf{t}}_2^1, \quad \tilde{\mathbf{f}}_2 = Y_2 \tilde{\mathbf{t}}_1^2 + Z_2 \tilde{\mathbf{t}}_2^2$$
$$\tilde{\mathbf{f}}_3 = Y_3 \tilde{\mathbf{t}}_1^3 + Z_3 \tilde{\mathbf{t}}_2^3, \quad \tilde{\mathbf{f}}_4 = Y_4 \tilde{\mathbf{t}}_1^4 + Z_4 \tilde{\mathbf{t}}_2^4$$

Use a modified Bouc-Wen model for biaxial hysteresis [2]

$$\dot Y_1 = \frac{R_1}{R_0}\Big((1 - a_1 Y_1^2)\tilde u_1 - b_1 Y_1 Z_1 \tilde v_1\Big), \ a_1 = \begin{cases} 1, & Y_1 \tilde u_1 > 0 \\ 0, & Y_1 \tilde u_1 \le 0 \end{cases}$$

$$\dot Z_1 = \frac{R_1}{R_0}\Big((1 - b_1 Z_1^2)\tilde v_1 - a_1 Y_1 Z_1 \tilde u_1\Big), \ b_1 = \begin{cases} 1, & Z_1 \tilde v_1 > 0 \\ 0, & Z_1 \tilde v_1 \le 0 \end{cases}$$

Contact Forces

$$s_1 = R_1 \cos^{-1}(\mathbf{t}_3^1 \cdot \mathbf{t}_3^2), \quad s_2 = R_2 \cos^{-1}(\mathbf{t}_3^2 \cdot \mathbf{t}_3^3)$$
$$s_3 = R_3 \cos^{-1}(\mathbf{t}_3^3 \cdot \mathbf{t}_3^4), \quad s_4 = R_4 \cos^{-1}(\mathbf{t}_3^4 \cdot \mathbf{t}_3^5)$$

$$g_1 = s_{c1} - s_1, \ g_2 = s_{c2} - s_2, \ g_3 = s_{c3} - s_3, \ g_4 = s_{c4} - s_4$$

$$\gamma_1 = \dot g_1, \quad \gamma_2 = \dot g_2, \quad \gamma_3 = \dot g_3, \quad \gamma_4 = \dot g_4$$

$$F_{c1} = \begin{cases} 0 & , g_1 > 0 \\ k_{c1} g_1 + c_{c1}\gamma_1 & , g_1 \le 0 \end{cases}$$

$$\mathbf{F}_{c1} = F_{c1}\bar{\mathbf{f}}_1, \quad \mathbf{F}_{c2} = F_{c2}\bar{\mathbf{f}}_2, \quad \mathbf{F}_{c3} = F_{c3}\bar{\mathbf{f}}_3, \quad \mathbf{F}_{c4} = F_{c4}\bar{\mathbf{f}}_4$$

Result and discussions

Uni-directional Motions

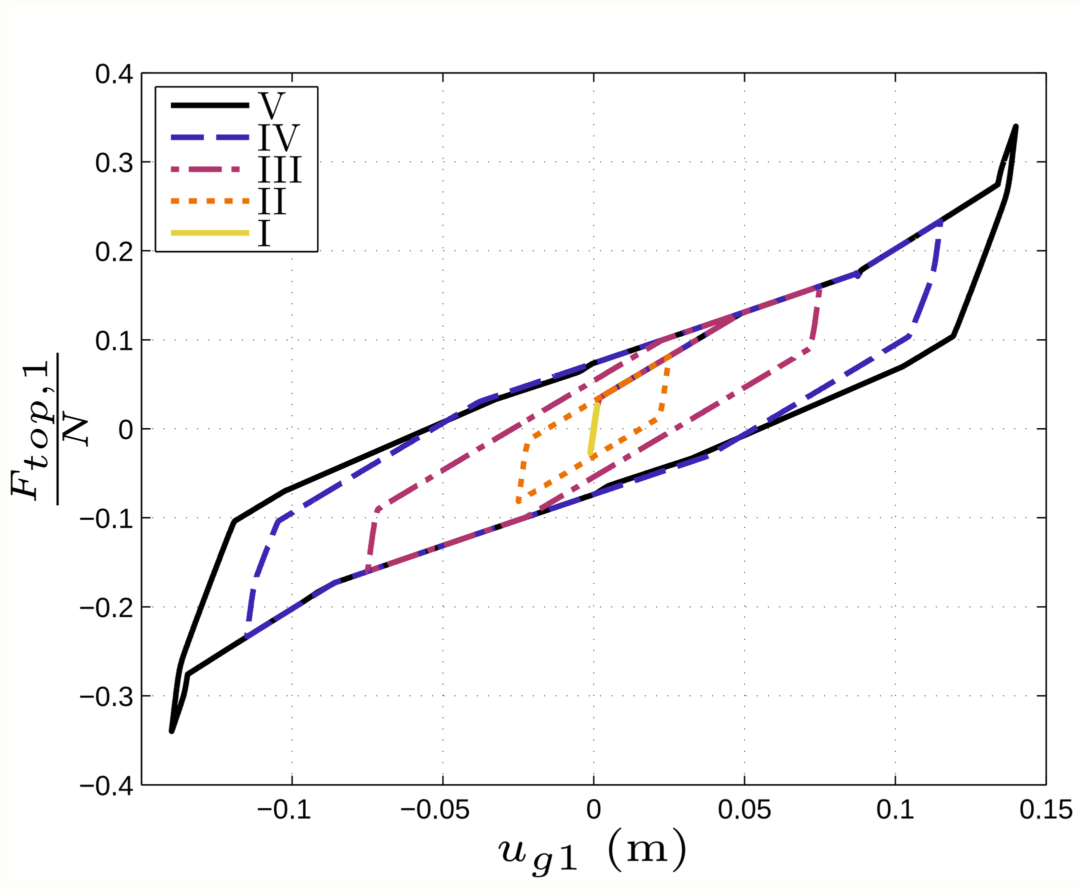

Figure: Hysteresis loop for uni-directional motions for ground motions in the five stages of motion.

	Analytical†	Experimental†	Kinetic Model
u^* (mm)	0.1	2	1.9
u^{**} (mm)	38.4	42	49
u_{dr1} (mm)	92.1	90	87
u_{dr4} (mm)	130.4	130	134
$\frac{F_{dr1}}{N}$	0.161	0.173	0.175
$\frac{F_{dr4}}{N}$	0.240	0.272	0.275

† Analytical and Experimental values come from Regime V Data from [3]

Bi-directional Motions

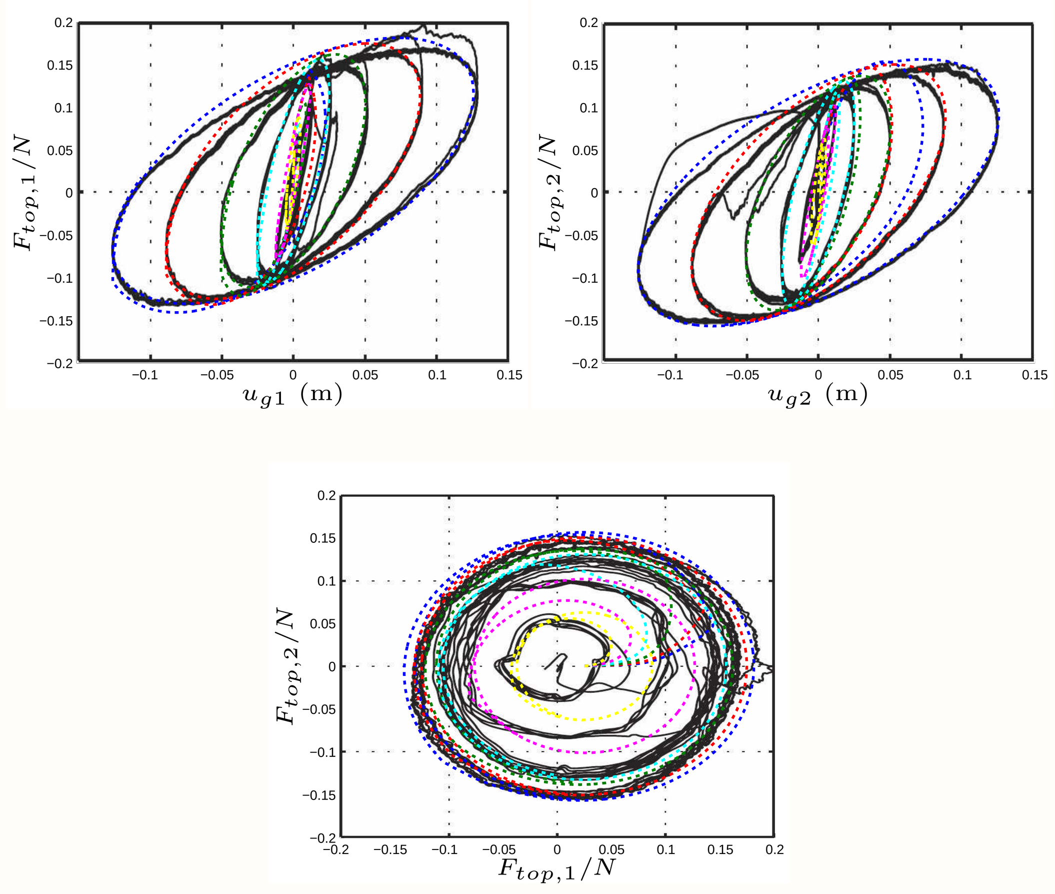

Figure: Hysteresis and force curves for a circular ground motion with experimental results. [4]

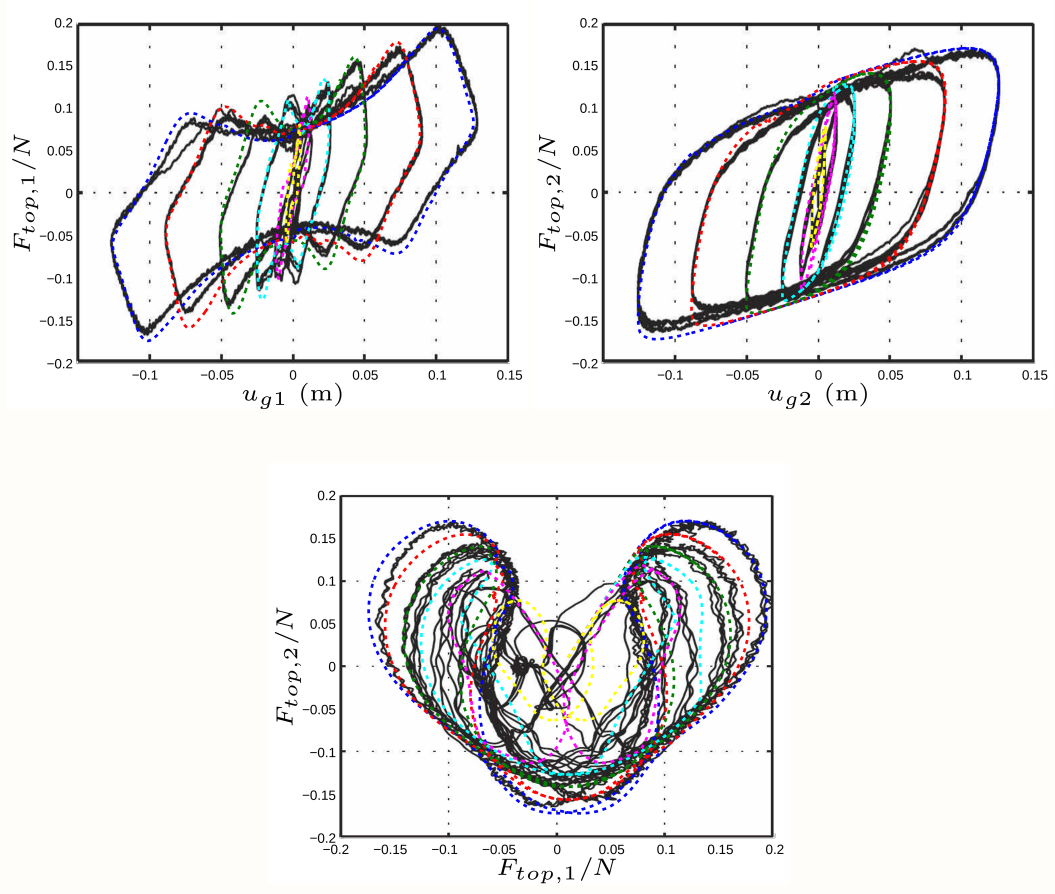

Figure: Hysteresis and force curves for a figure-eight ground motion with experimental results. [4]

Summary and conclusions

The model presented here can work for both uni-directional and bi-directional ground motions with no linearization assumption. For that reason, in the case of uni-directional ground motions, it was shown that the nonlinear model can more accurately predict the experimental values than previous analytical models. The only assumption that the nonlinear kinetic model makes is that the bearings are axisymmetric. Thus, this model can be used to analyze the simplest, as well as the more complicated ground motions that one would like to test. The nonlinear kinetic model has the capability to be connected numerically to models of different superstructures, such as frames, trusses, or any type of finite element model. This allows one to model an entire system, including the TFP, in one complete simulation, while accounting for the non-linear and inertial nature of the TFP.

References

[1] A. S. Mokha et al., *Journal of Structural Engineering* 122(3)(1996)298–308.

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