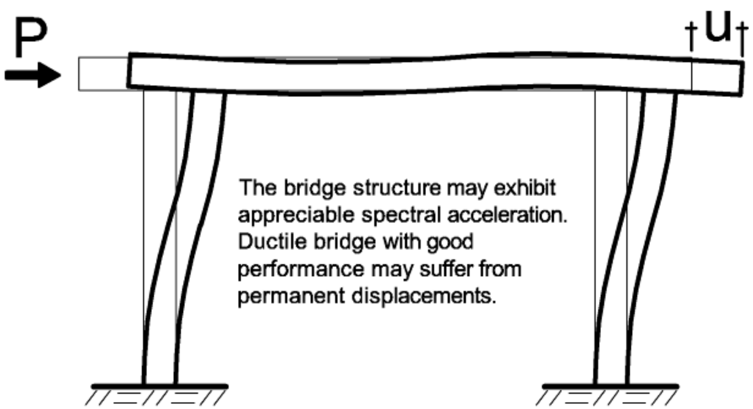
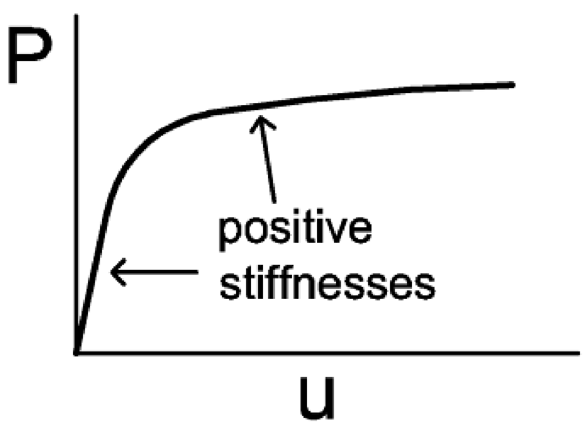
The Dynamics of Rocking Isolation

Nicos Makris
Professor of Structures and Mechanics,
University of Central Florida
email: nicos.makris@ucf.edu

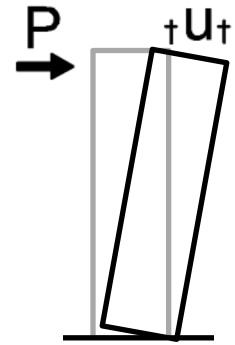
Fundamental Differences between Articulated "Ancient" and Modern Structural Systems

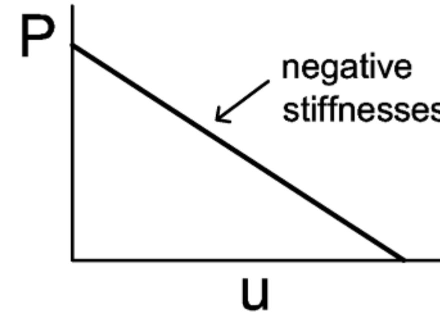
Statically Intermediate Moment-Resisting Frames Ductile behavior



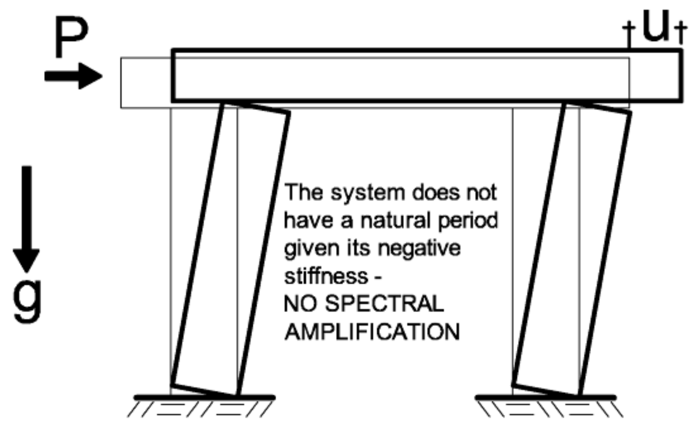


Free-Standing Rocking Structures





One-hinge mechanism



zero ductility

Four-hinge mechanism

Free-standing rocking structures have survived the most severe earthquakes level as dictated by modern building codes: (2% probability in 50 years)

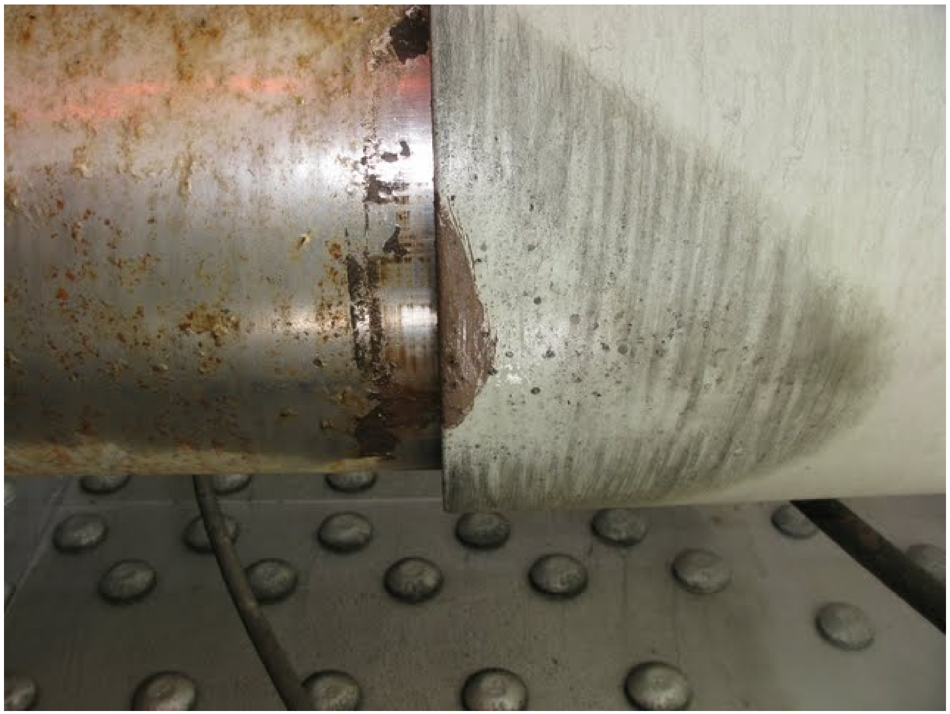
Sustainable Engineering: The design and construction of structures that meet acceptable performance levels at present and in the years to come without compromising the ability of future generations to use them, maintain them and benefit from them.

Articulate Structures emerge as a Triumph for Sustainability (Societal Requirement).

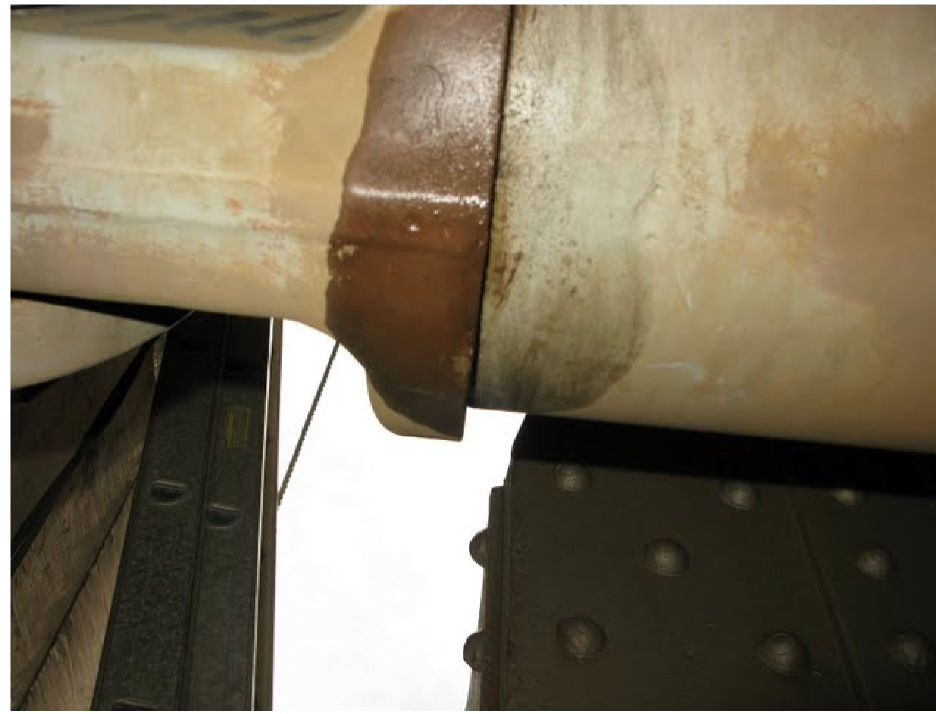
The San Francisco-Oakland Bay Bridge

2007

View of the travelling of fluid dampers
under traffic loads



Evidence of appreciable oil leaking from the fluid dampers in less than five years upon they have been installed

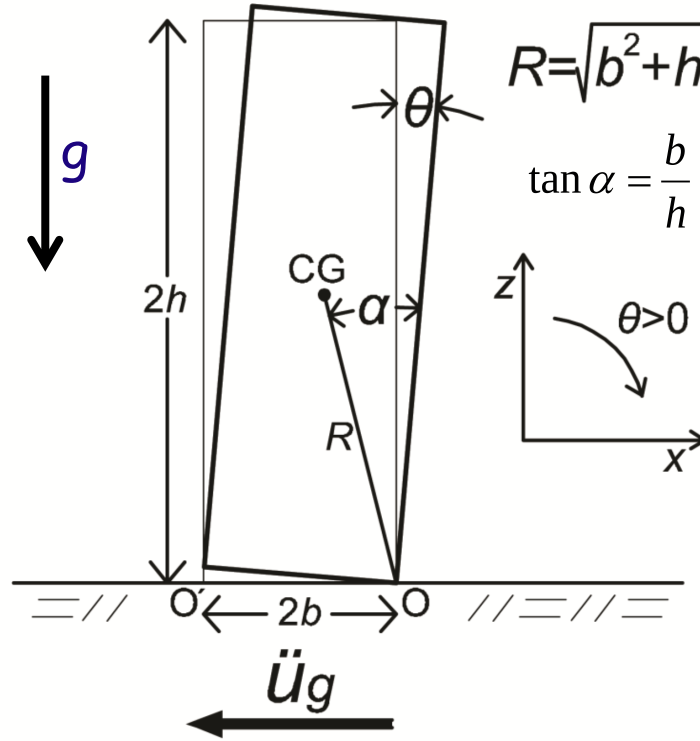


All 94, 450-kip, fluid dampers had to be replaced!

The target of sustainability was not achieved.

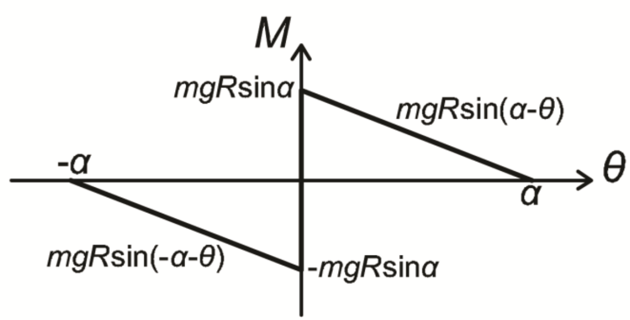
The Free-Standing Rocking Column

510-490 BC



$R=\sqrt{b^2+h^2}$ =size

$\tan\alpha = \dfrac{b}{h}$ =slenderness



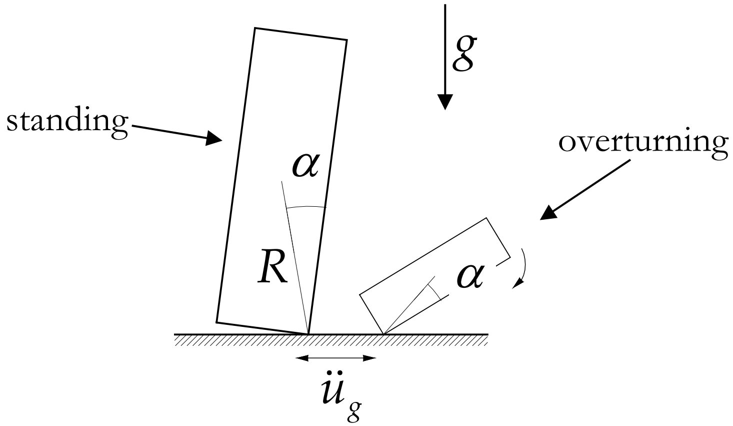
Parameters of the linear oscillator and the free-standing rocking block.

Parameters/ characteristics	Damped oscillator m, c, k	Rocking rigid block b, h, g
Restoring mechanism	Elasticity of the structure	**Gravity**
Restoring force/moment	$F = ku$ ($for\ linear\ springs$)	$M = mgR\sin(\alpha - \theta)$ $R = \sqrt{b^2 + h^2}$
Stiffness at stable equilibrium	Finite	Infinite
Restoring force/moment at stable equilibrium	Zero	Finite: $mgR\sin(\alpha)$
Stiffness away from equilibrium	Positive	**Negative**
Frequency parameter	Undamped natural frequency: $\omega_0 = \frac{2\pi}{T_o} = \sqrt{\frac{k}{m}}$	Frequency parameter: $p = \sqrt{\frac{3g}{4R}}$ ($for\ rectangular\ blocks$)
Damping parameter	Viscous damping ratio: $\xi = \frac{c}{2m\omega_0}$	Slenderness: $\alpha = \tan^{-1}(b/h)$

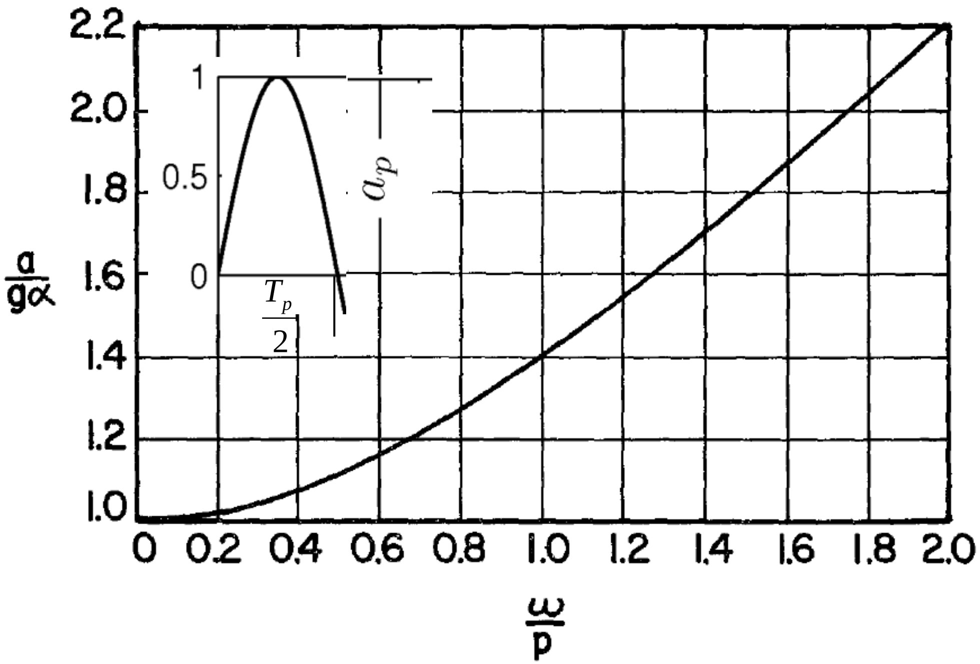
Fundamental size-frequency scale effect
1963 George W. Housner

(a) The larger of two geometrically similar blocks can survive an excitation that will topple the smaller block
(b) Out of two same acceleration amplitude pulses the one with longer duration is more capable to induce overturning

Conclusion reached from studies motivated from the destruction observed after the May 1960 earthquake in Chile.



$$\omega_p = \frac{2\pi}{T_p} \;, \quad p = \sqrt{\frac{3g}{4R}}$$



Small blocks or long-duration pulses

Large blocks or high-frequency pulses

EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS, VOL. 8, 565–587 (1980)

ROCKING RESPONSE OF RIGID BLOCKS TO EARTHQUAKES

CHIK-SING YIM, ANIL K. CHOPRA AND JOSEPH PENZIEN
Department of Civil Engineering, University of California, Berkeley, California, U.S.A.

SUMMARY

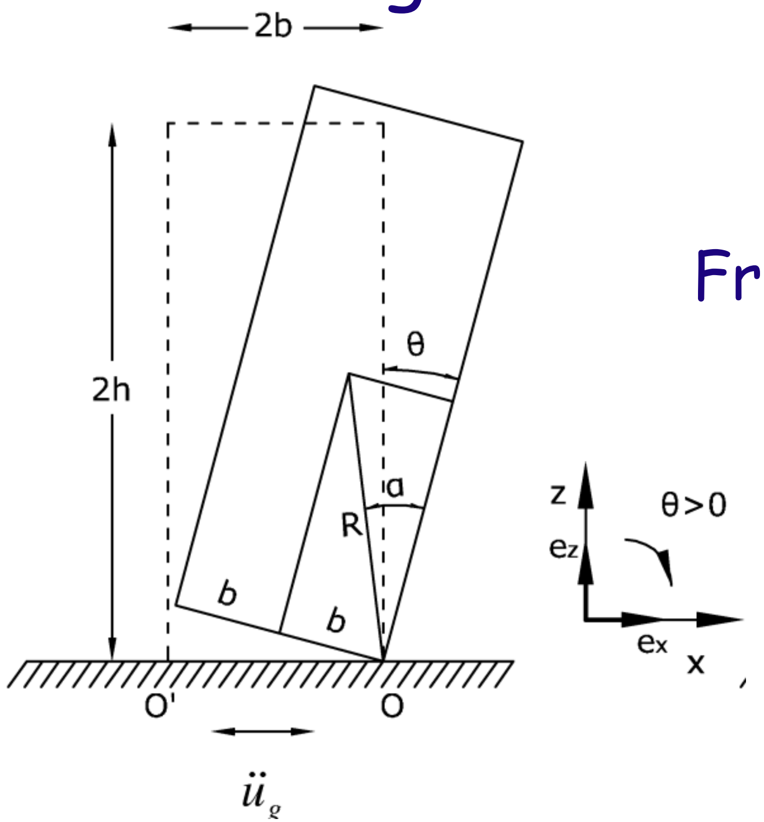
This investigation deals with the rocking response of rigid blocks subjected to earthquake ground motion. A numerical procedure and computer program are developed to solve the non-linear equations of motion governing the rocking motion of rigid blocks on a rigid base subjected to horizontal and vertical ground motion.

The response results presented show that the response of the block is very sensitive to small changes in its size and slenderness ratio and to the details of ground motion. Systematic trends are not apparent: The stability of a block subjected to a particular ground motion does not necessarily increase monotonically with increasing size or decreasing slenderness ratio. Overturning of a block by a ground motion of particular intensity does not imply that the block will necessarily overturn under the action of more intense ground motion.

In contrast, systematic trends are observed when the problem is studied from a probabilistic point of view with the ground motion modelled as a random process. The probability of a block exceeding any response level, as well as the probability that a block overturns, increases with increase in ground motion intensity, increase in slenderness ratio of the block and decrease in its size.

It is concluded that probabilistic estimates of the intensity of ground shaking may be obtained from its observed effects on monuments, minarets, tombstones and other similar objects provided suitable data in sufficient quantity is available, and the estimates are based on probabilistic analyses of the rocking response of rigid blocks, considering their non-linear dynamic behaviour.

Review of the dynamics of the free-standing rocking block



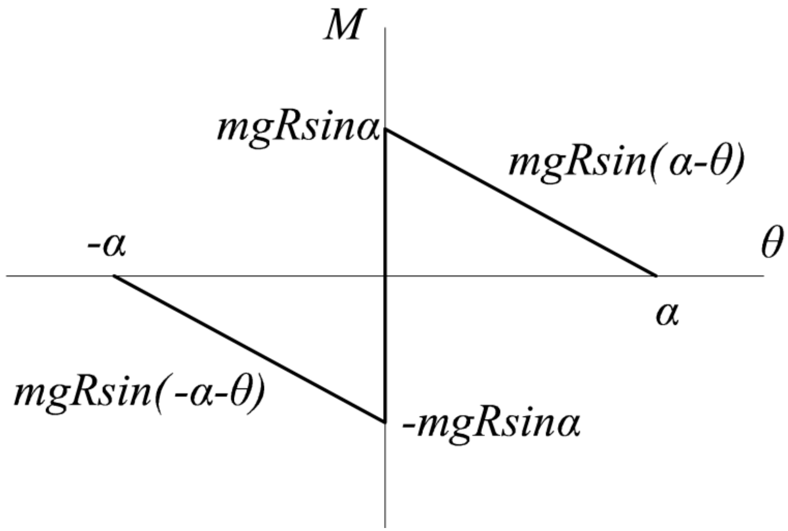
Frequency Parameter

$$p = \sqrt{\frac{3g}{4R}}$$

$$I_0 \ddot{\theta}(t) + mgR\sin[-a-\theta(t)] = -m\ddot{u}_g(t)R\cos[-a-\theta(t)], \quad \theta(t) < 0$$

$$I_0 \ddot{\theta}(t) + mgR\sin[a-\theta(t)] = -m\ddot{u}_g(t)R\cos[a-\theta(t)], \quad \theta(t) > 0$$

$$\boxed{\ddot{\theta}(t) = -p^2\{\sin[a\,\mathrm{sgn}(\theta(t)) - \theta(t)] + \frac{\ddot{u}_g}{g}\cos[a\,\mathrm{sgn}(\theta(t)) - \theta(t)]}$$



$$I_0\dot{\theta}_1 - m\dot{\theta}_1 2bR\sin(a) = I_0\dot{\theta}_2$$

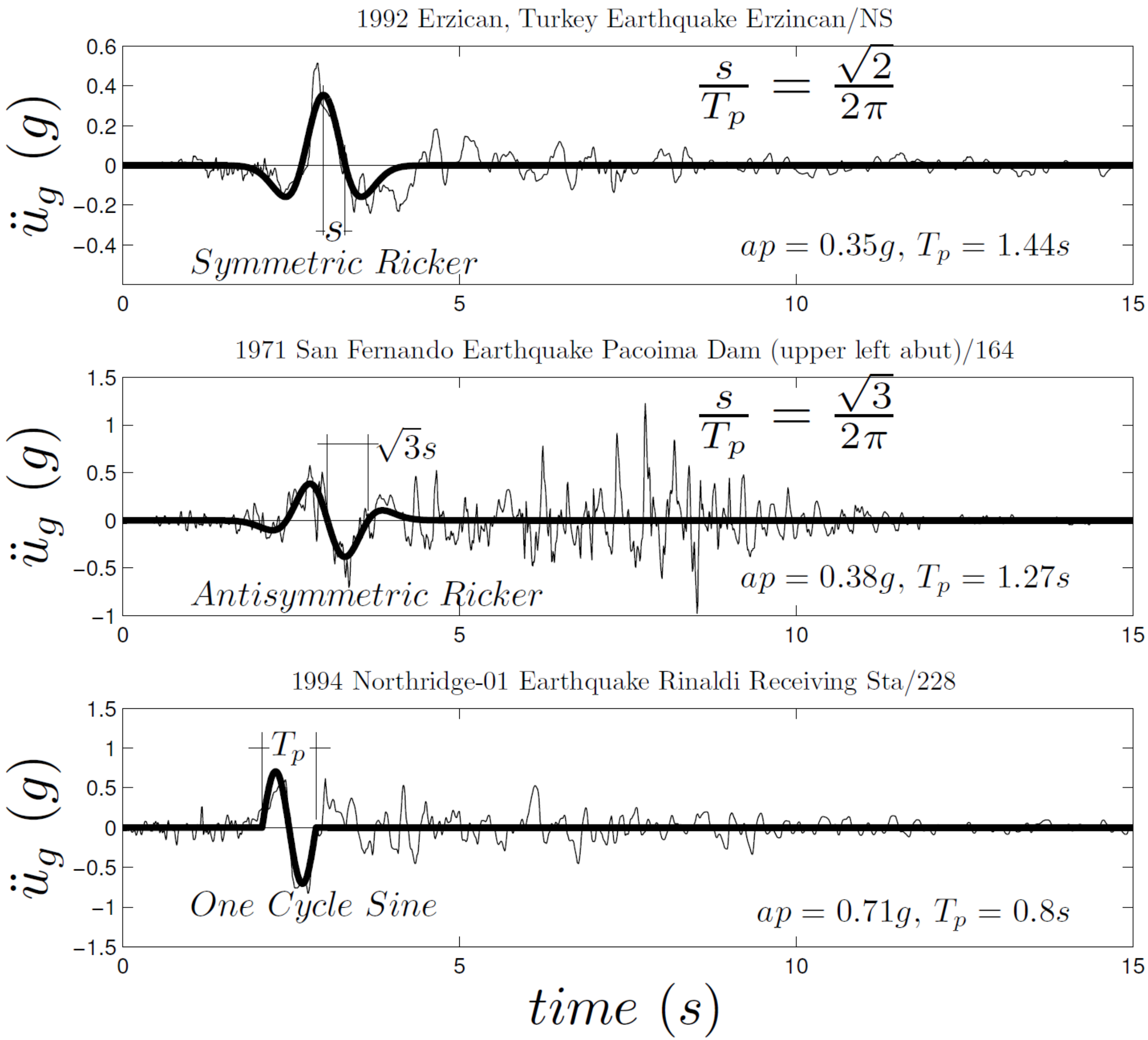
Coefficient of Restitution:

$$r = \frac{\dot{\theta}_2^{\,2}}{\dot{\theta}_1^{\,2}}$$

$$r = [1 - \frac{3}{2}\sin^2 a]^2$$

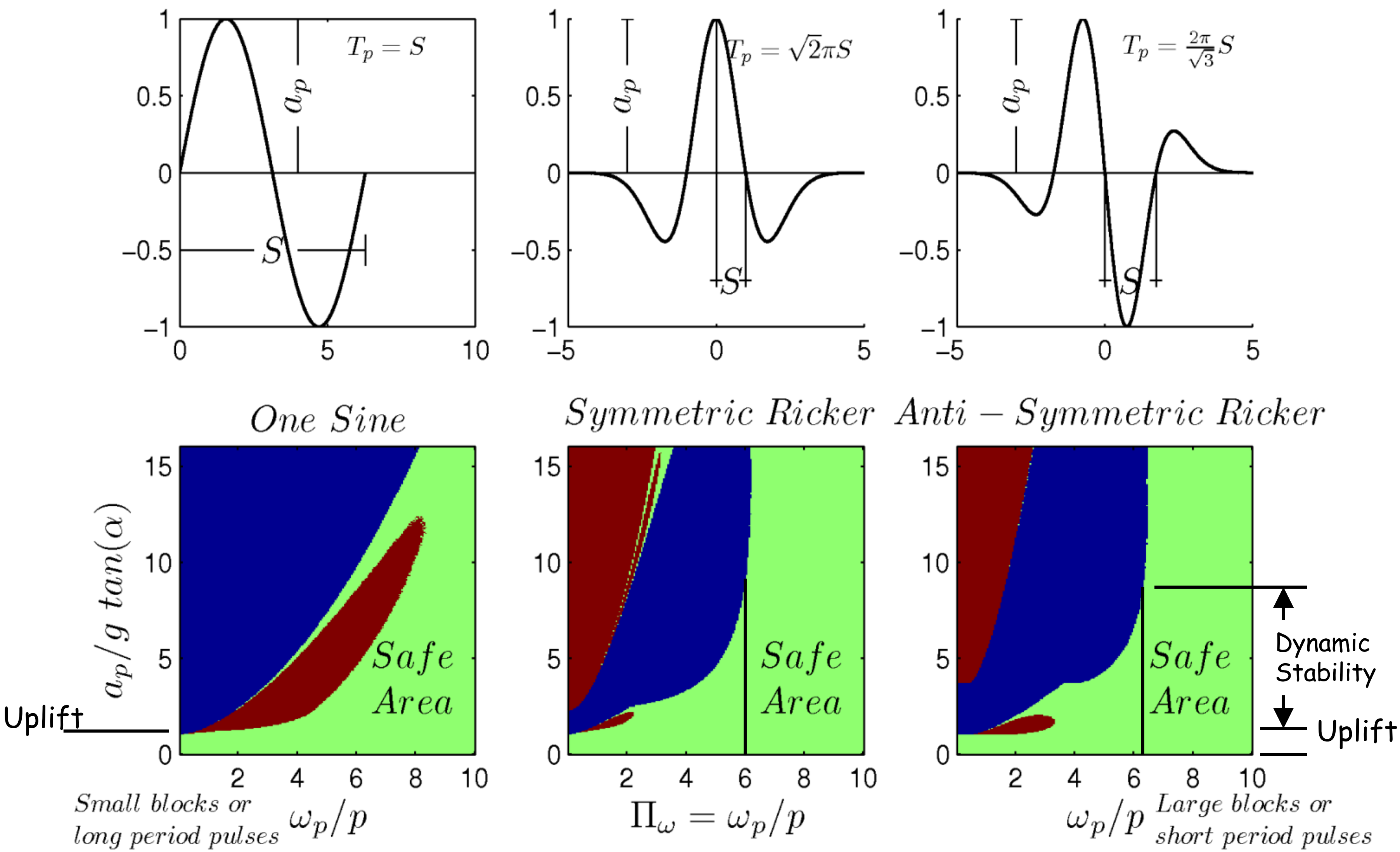
Energy dissipation happens only during impact, while the ductility of the system is zero

Time Scale and Length Scale of Pulse-Like Ground Motions

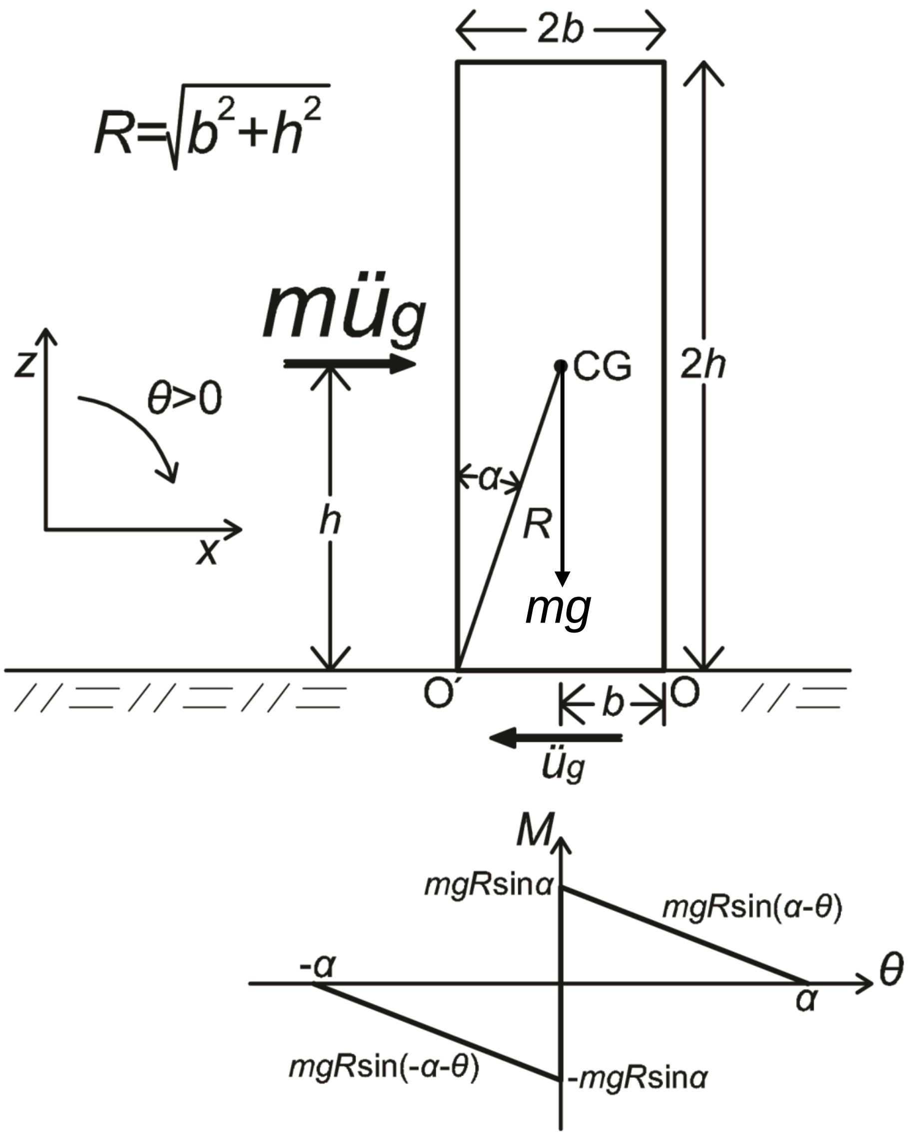


Overturning spectra of a rigid block standing free on a monolithic base

$$a = 14^o$$



A Notable Limitation of the Equivalent Static Lateral Force Analysis



$$\underbrace{m\ddot{u}_g h}_{\text{demand}} = \underbrace{mgb}_{\text{resistance}}$$

or

$$\underbrace{\ddot{u}_g}_{\text{demand}} = g\frac{b}{h} = \underbrace{g\tan\alpha}_{\text{resistance}}$$

The "equivalent static" Lateral Force analysis indicates that the stability of a free-standing column depends solely on the slenderness ($g\tan\alpha$) and is independent to the size $R=\sqrt{b^2+h^2}$

Seismic Resistance of Free-Standing Columns subjected to Dynamic Loads

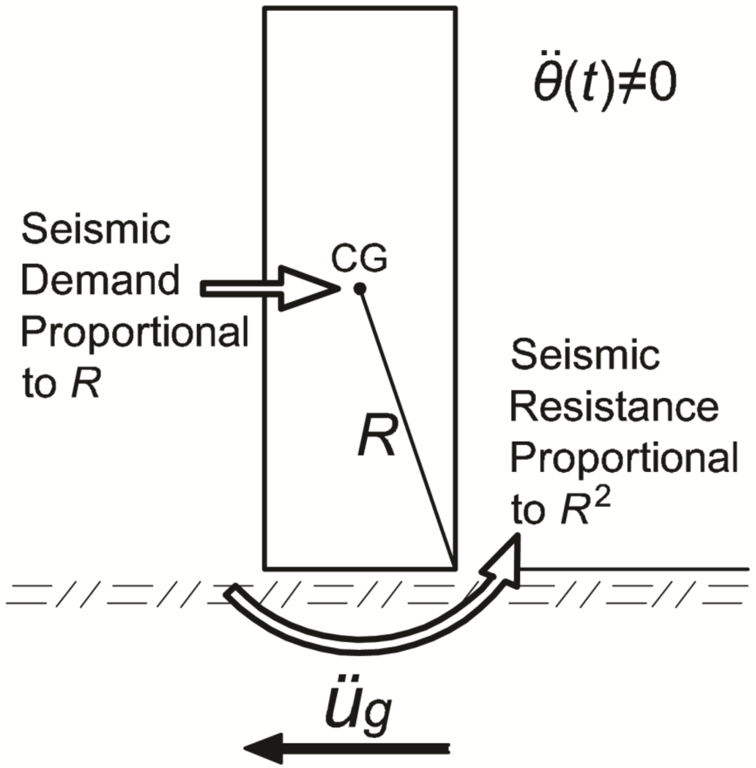
$$\underbrace{-m\ddot{u}_g(t)\overbrace{R\cos[a-\theta(t)]}^{\text{Vertical lever arm}}}_{\text{demand}} = \underbrace{I_o\ddot{\theta}(t) + mg\overbrace{R\sin[a-\theta(t)]}^{\text{Horizontal lever arm}}}_{\text{resistance}}$$

$$\theta > 0$$

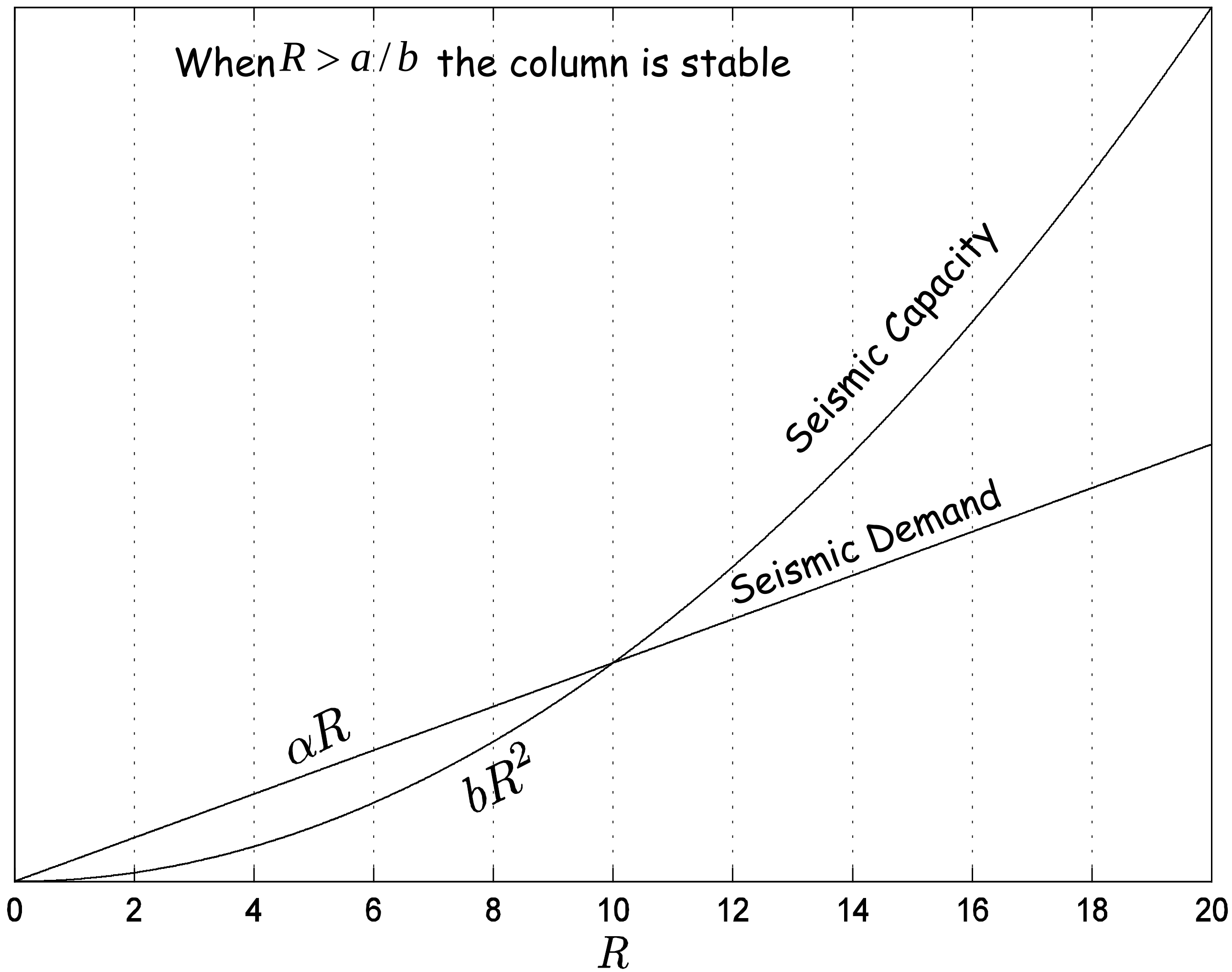
For rectangular blocks, $I_o=(4/3)mR^2$

$$\underbrace{-\ddot{u}_g(t)R\cos[a-\theta(t)]}_{\text{seismic demand}} = \underbrace{\frac{4}{3}R^2\ddot{\theta}(t) + gR\sin[a-\theta(t)]}_{\text{seismic resistance}}$$

$$\theta > 0$$



Simply stated, Housner's size effect uncovered in 1963 is merely a reminder that a quadratic term eventually dominates over a linear term regardless the values of their individual coefficients.


When $R > a/b$ the column is stable
Seismic Capacity
Seismic Demand
αR
bR^2
0
2
4
6
8
10
12
14
16
18
20
R

Basic design concepts and response-controlling quantities associated with:
(a) the **traditional earthquake resistant (capacity) design**;
(b) **seismic isolation**; and
(c) **rocking isolation**.

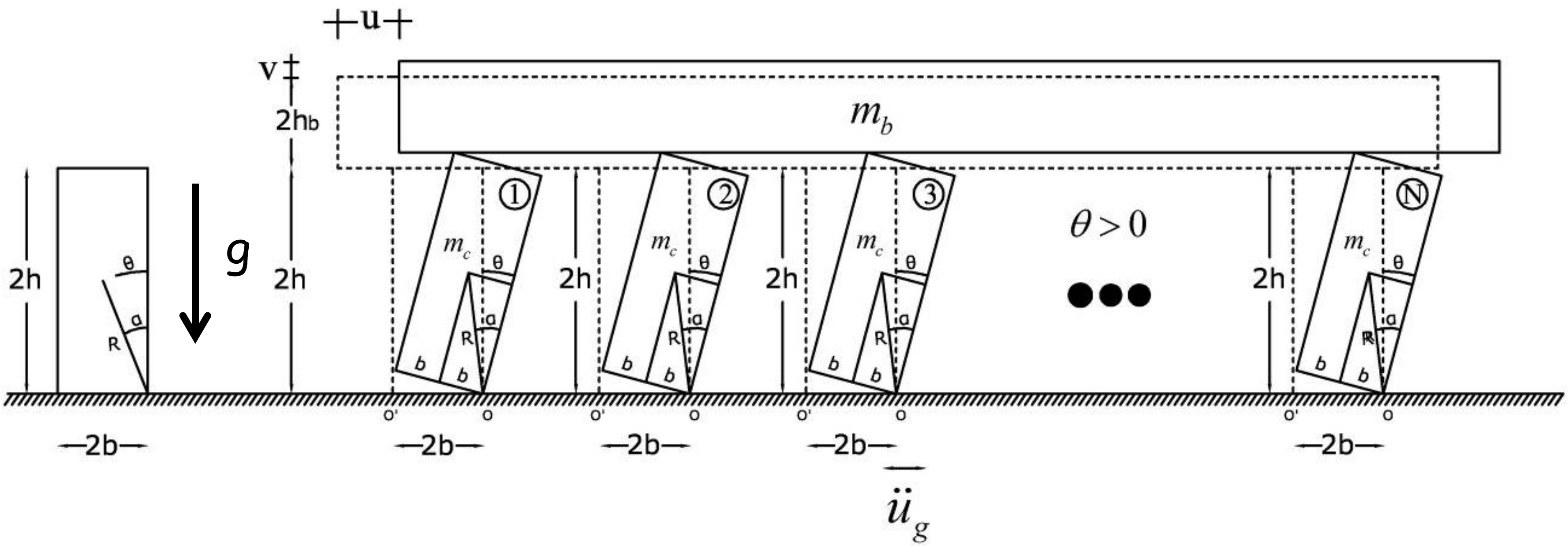
	TRADITIONAL EARTHQUAKE RESISTANCE DESIGN • **Moment Resisting Frames** • **Braced Frames**	**SEISMIC ISOLATION**	**ROCKING ISOLATION**
Strength	Moderate to Appreciable $\ddot{u}_g^y = \frac{Q}{m} =$ 0.10g-0.25g	Low $\ddot{u}_g^y = \frac{Q}{m} =$ 0.03g-0.09g	Low to Moderate $\ddot{u}_g^{up} = g\frac{b}{h} = g \tan a$
Stiffness	Positive and Variable due to Yielding	Positive, Low and Constant	Negative, Constant
Ductility	Appreciable μ=3-6	Very Large/Immaterial* LRB†: μ=10-30 CSB‡: μ=1000-3000	Zero
Damping	Moderate	Moderate to High	Low (only during impact)
Seismic Resistance Originates from:	Appreciable Strength and Ductility	Low Strength and Low Stiffness in association with the capability to accommodate Large Displacements	Low to Moderate Strength and **Appreciable Rotational Inertia**
Equivalent Static Lateral Force Analysis is Applicable?	YES	YES	**NO**
Design Philosophy	Equivalent Static	Equivalent Static	**Dynamic**

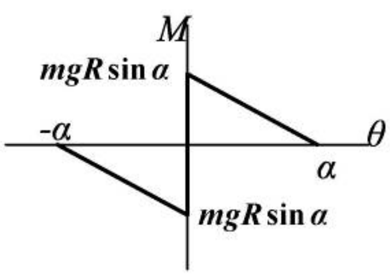
*Makris and Vassiliou (2011)
†LRB=Lead Rubber Bearings
‡CSB=Concave Sliding Bearings

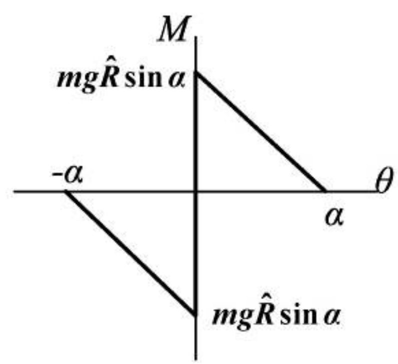
The Dynamics of the Rocking Frame

The Rocking Frame

A one-degree-of-freedom structure



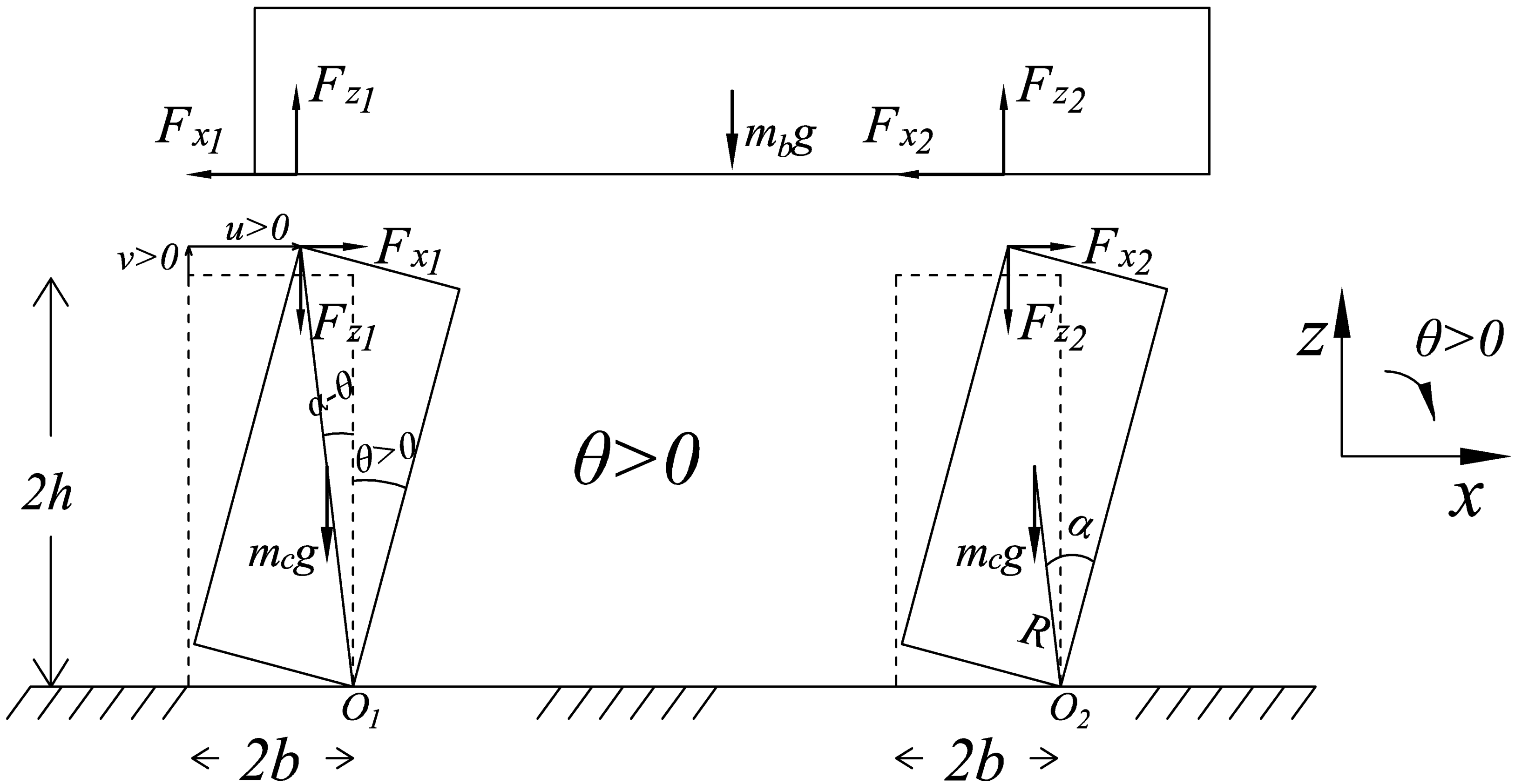




$$\hat{R} = R\left(1 + \frac{\gamma}{1+2\gamma}\right)$$

$$\gamma = \frac{m_b}{N m_c}$$

Direct vs Variational Formulation



Direct Approach: Derivation of the equations of motion by employing Newton's law of dynamic equilibrium. There is a need to calculate the internal forces.

Indirect Approach: The average kinetic energy less the average potential energy is a minimum along the true path from one position to another: Variational formulation – **No need to calculate internal forces.**

Relations of the horizontal and vertical displacements with the angle of rotation

$$u = \mp 2R\left(\sin\alpha - \sin\left(\alpha \pm \theta\right)\right)$$

$$\delta u = \frac{du}{d\theta}\delta\theta$$

$$\dot{u} = 2R\cos\left(\alpha \pm \theta\right)\dot{\theta}$$

$$\ddot{u} = 2R\left(\mp\sin\left(\alpha \pm \theta\right)\left(\dot{\theta}\right)^2 + \cos\left(\alpha \pm \theta\right)\ddot{\theta}\right)$$

$$v = 2R\left(\cos\left(\alpha \pm \theta\right) - \cos\alpha\right)$$

$$\delta v = \frac{dv}{d\theta}\delta\theta$$

$$\dot{v} = \mp 2R\sin\left(\alpha \pm \theta\right)\dot{\theta}$$

$$\ddot{v} = -2R\left(\cos\left(\alpha \pm \theta\right)\left(\dot{\theta}\right)^2 + \sin\left(\alpha \pm \theta\right)\ddot{\theta}\right)$$

Equation of Motion: Variational Formulation

Lagrange's Equation: $$\frac{d}{dt}\left(\frac{dT}{d\dot{\theta}}\right)-\frac{dT}{d\theta}=Q$$

$Q=-\frac{dW}{d\theta}$ Generalized force acting on the system

Kinetic Energy: $$T=N\frac{1}{2}I_o\left(\dot{\theta}\right)^2+\frac{1}{2}m_b\left(\left(\dot{u}\right)^2+\left(\dot{v}\right)^2\right)$$

Variation of the Work: $$\delta W=\left(m_b+\frac{N}{2}m_c\right)\left(\ddot{u}_g\delta u+g\delta v\right)$$

$$\delta W=\frac{dW}{d\theta}\delta\theta,\qquad \frac{dW}{d\theta}=2R\left(m_b+\frac{N}{2}m_c\right)\left(\ddot{u}_g\cos\left(\alpha+\theta\right)-g\sin\left(\alpha+\theta\right)\right)$$

Equation of Motion: $$\left(\frac{\frac{I_o}{2m_cR}+2\gamma R}{\left(\gamma+\frac{1}{2}\right)g}\right)\ddot{\theta}=-\sin\left(a-\theta\right)-\frac{\ddot{u}_g}{g}\cos\left(\alpha-\theta\right)$$

$\theta(t)>0$

Equation of Motion of the Rocking Frame

$$\ddot{\theta}(t) = -\frac{1+2\gamma}{1+3\gamma}\,p^2\left(\sin[a\,\mathrm{sgn}(\theta(t))-\theta(t)] + \frac{\ddot{u}_g}{g}\cos[a\,\mathrm{sgn}(\theta(t))-\theta(t)]\right)$$

Equation of Motion of the Solitary Rocking Column

$$\ddot{\theta}(t) = -p^2\left(\sin[a\,\mathrm{sgn}(\theta(t))-\theta(t)] + \frac{\ddot{u}_g}{g}\cos[a\,\mathrm{sgn}(\theta(t))-\theta(t)]\right)$$

$$\hat{p} = \sqrt{\frac{1+2\gamma}{1+3\gamma}}\,p, \quad \gamma = \frac{m_b}{Nm_c}$$

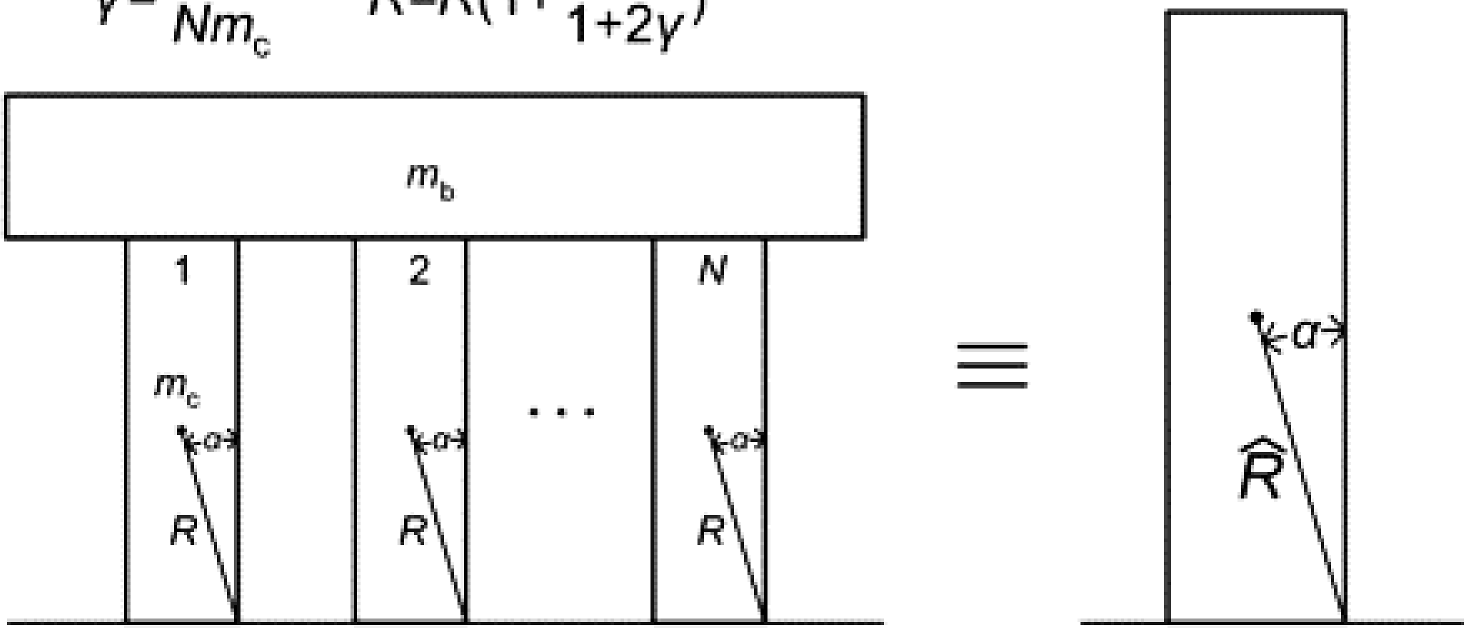
$$\hat{R} = \frac{1+3\gamma}{1+2\gamma}R = \left(1+\frac{\gamma}{1+2\gamma}\right)R$$

Important Finding:

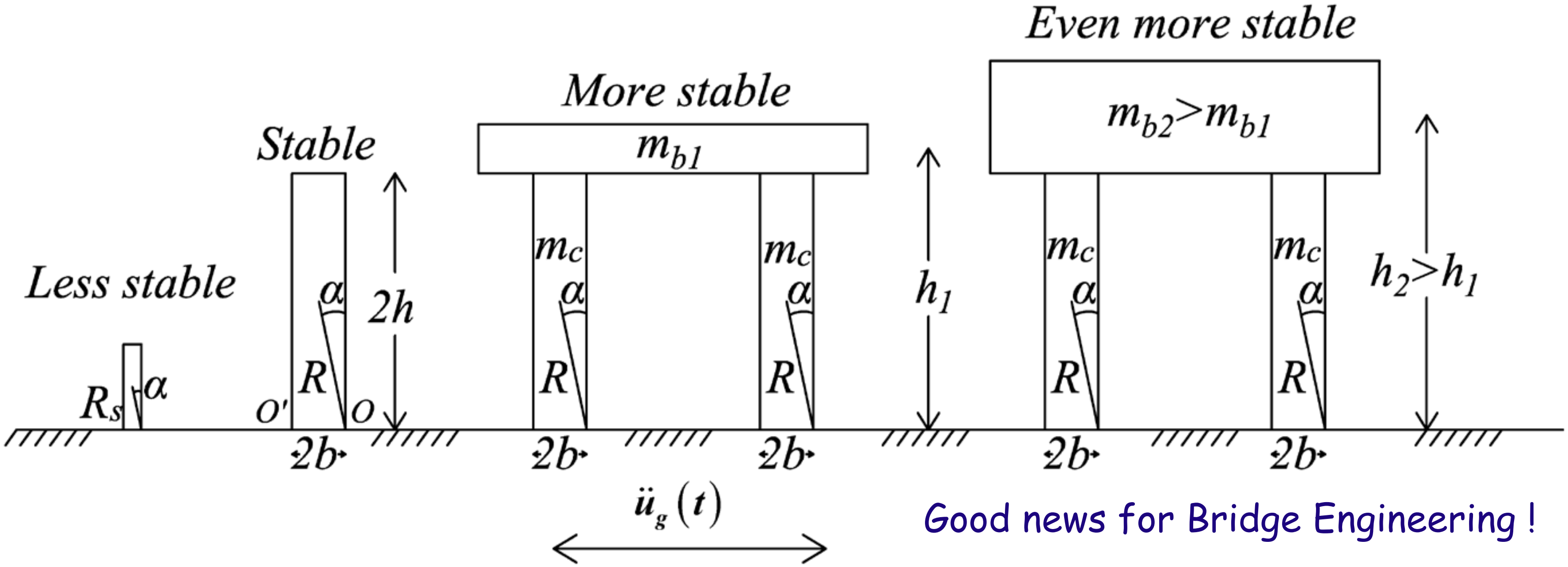
The equation of motion of the rocking frame indicates that the heavier the cap beam is, the more stable is the free-standing rocking frame despite the rise of the center of gravity of the cap beam

Remarkable Finding

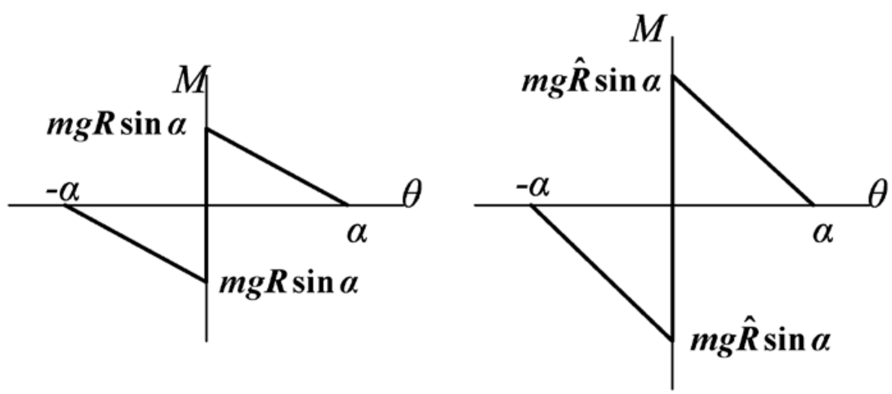
$$\gamma = \frac{m_b}{N m_c} \qquad \widehat{R} = R\left(1 + \frac{\gamma}{1+2\gamma}\right)$$



$$r = \left(\frac{\dot{\theta}_2}{\dot{\theta}_1}\right)^2 = \left(\frac{1 - \frac{3}{2}\sin^2\alpha + 3\gamma\cos 2\alpha}{1 + 3\gamma}\right)^2$$



Good news for Bridge Engineering !



$$\hat{R} = R\left(1+\frac{\gamma}{1+2\gamma}\right)$$

$$\gamma = \frac{m_b}{2m_c}$$

Makris, N. and Vassiliou, M. (2013). Planar rocking response and stability analysis of an array of freestanding columns capped with a freely supported rigid beam, EESD, 42(3): 431-449.

Are Some Top-Heavy Structures More Stable?

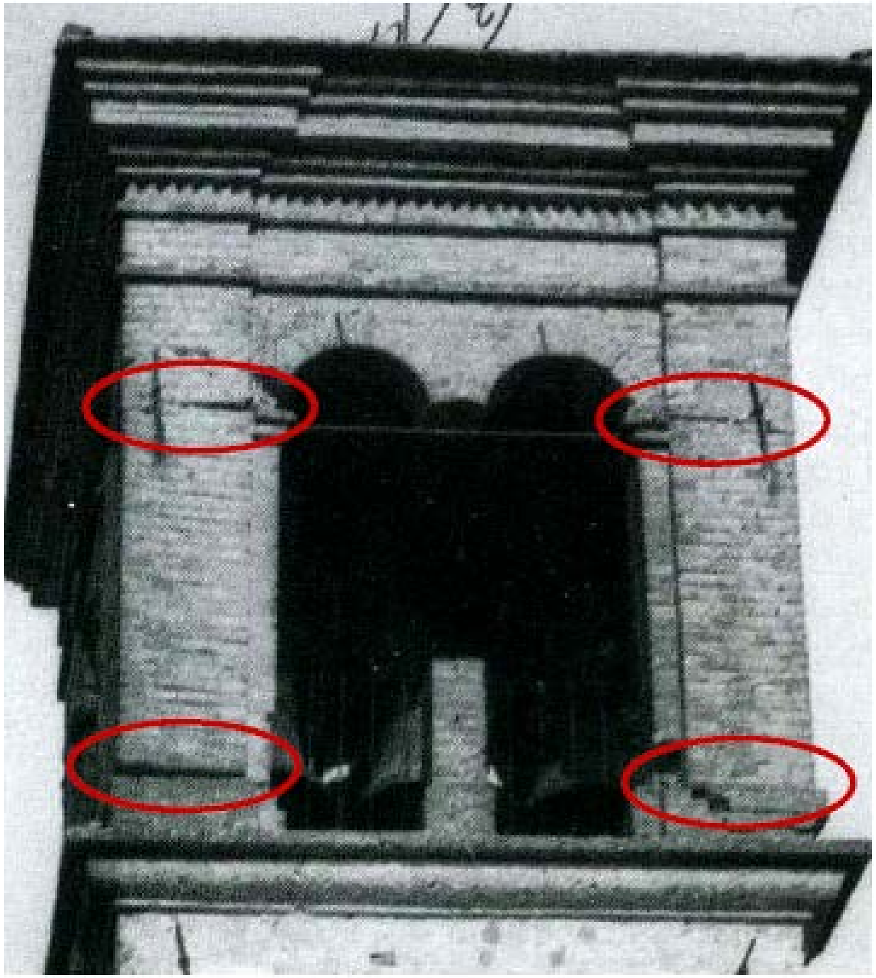
Nicos Makris, M.ASCE[1]; and Michalis F. Vassiliou[2]

Abstract: This technical note investigates the dynamic response and stability of a rocking frame that consists of two identical free-standing slender columns capped with a freely supported rigid beam. Part of the motivation for this study is the emerging seismic design concept of allowing framing systems to uplift and rock along their plane in order to limit bending moments and shear forces— together with the need to stress that the rocking frame is more stable the more heavy is its cap-beam, a finding that may have significant implications in the pre-fabricated bridge technology. In this technical note, a direct approach is followed after taking dynamic force and moment equilibrium of the components of the rocking frame, and the remarkable results obtained in the past with a variational formulation (by the same authors) is confirmed—that the dynamics response of the rocking frame is identical to the rocking response of a solitary, free-standing column with the same slenderness, yet with larger size, which produces a more stable configuration. The motivation for reworking this problem by following a direct approach is to show, in the simplest possible way, that the heavier the freely supported cap beam, the more stable is the rocking frame, regardless of the rise of the center of gravity of the cap beam. The conclusion is that top-heavy rocking frames are more stable that when they are top-light. **DOI: 10.1061/(ASCE)ST.1943-541X.0000933.**

Author keywords: Rocking frame; Seismic isolation; Articulated structures; Prefabricated bridges; Seismic design; Seismic effects.

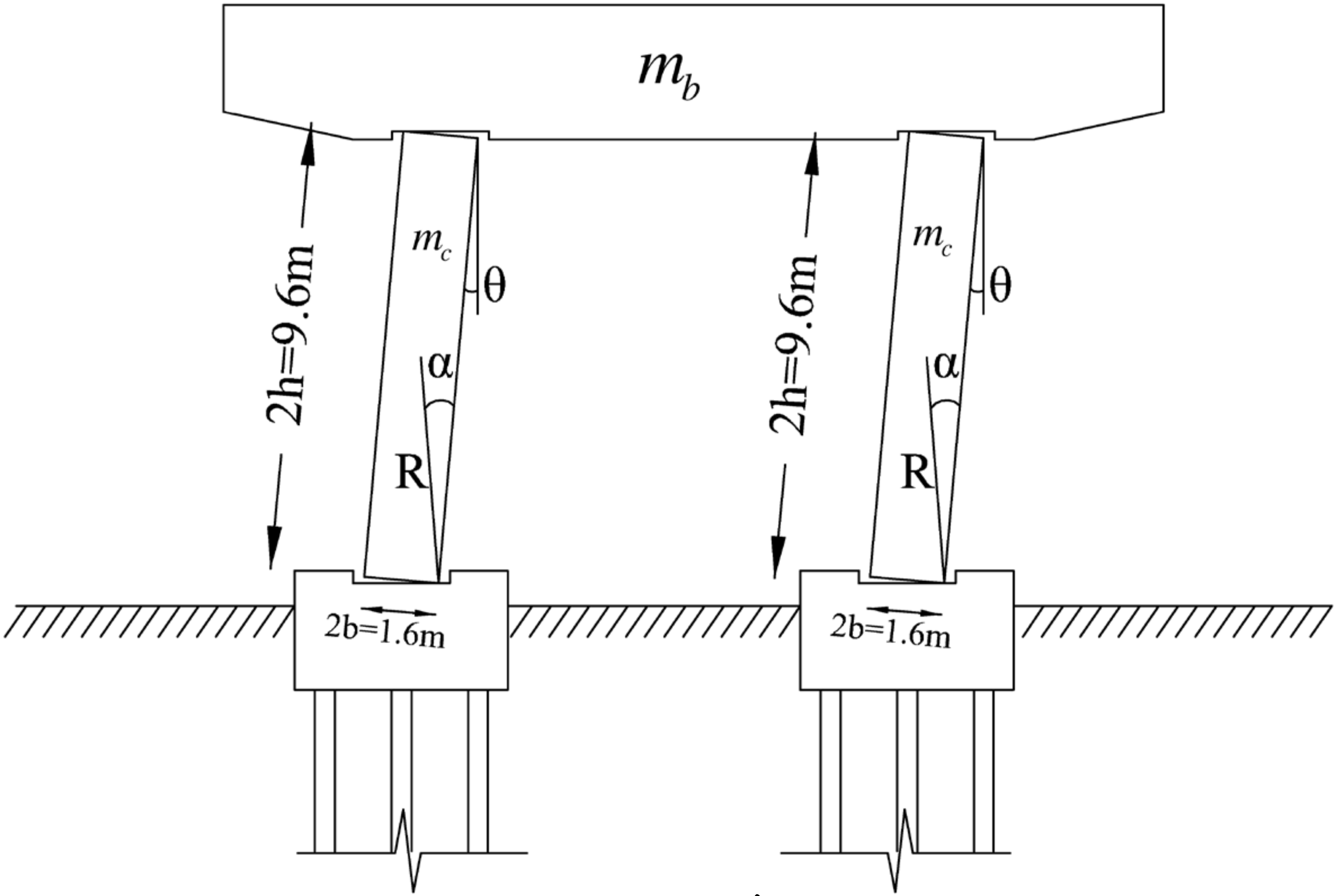
Church of St. Marko in Gaio, Italy

(from S. Lagomarsino, July 2008)



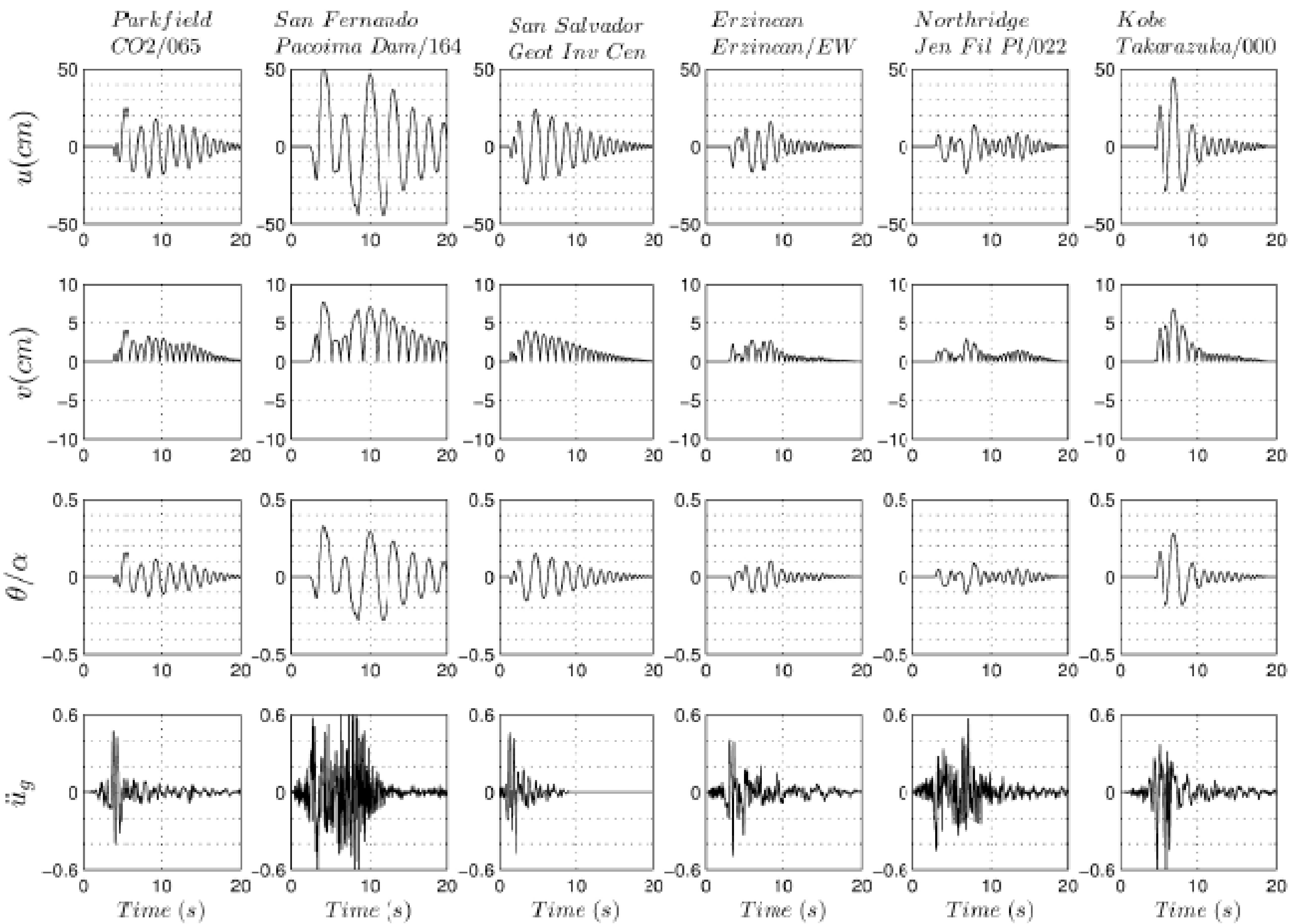
Formation of rocking frame offered dynamic stability which led to collapse prevention

On-Going Research: The Rocking Frame for Bridges



tanα=1/6

Rotation, vertical and horizontal displacement histories of the free standing rocking frame

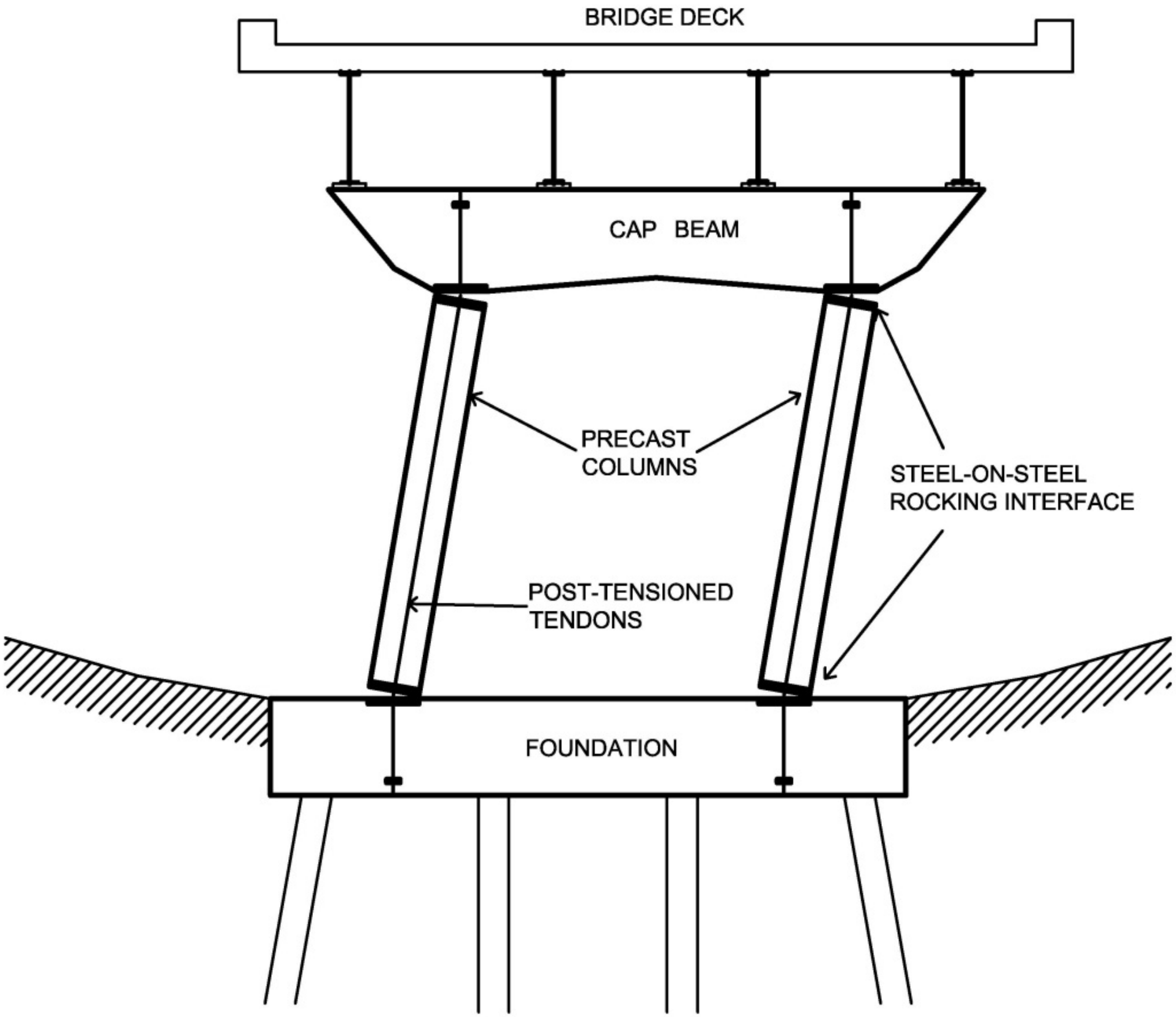


One of the few applications of rocking isolation

South Rangitīkei, New Zealand (78m tall piers)

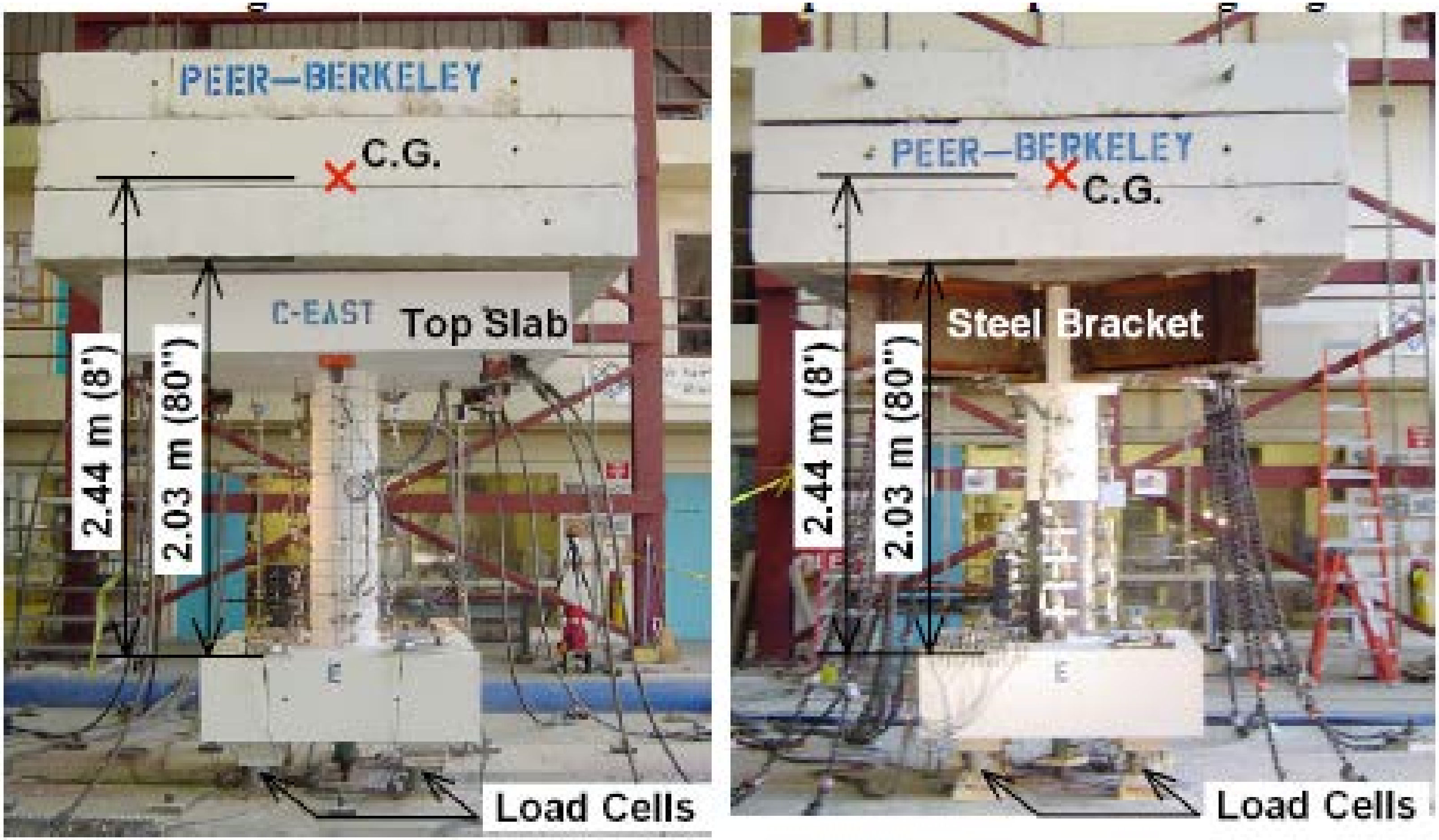
Aim of this work: To develop the theoretical/technical background in an effort to accept and **establish** **rocking isolation** and the associated hinging mechanism **not just as a limit-state mechanism**; but, **as an operational state** (seismic protection mechanism for large, slender structures)

Vertically Restrained Rocking Bridges



Mander, J. B., & Cheng, C. T. (1997). Seismic resistance of bridge piers based on damage avoidance design. *Technical Report NCEER*, *97*. (20 years ago!)

Vertically Restrained Rocking Columns



(a) Specimens tested in 2004 (b) Specimens tested in 2005

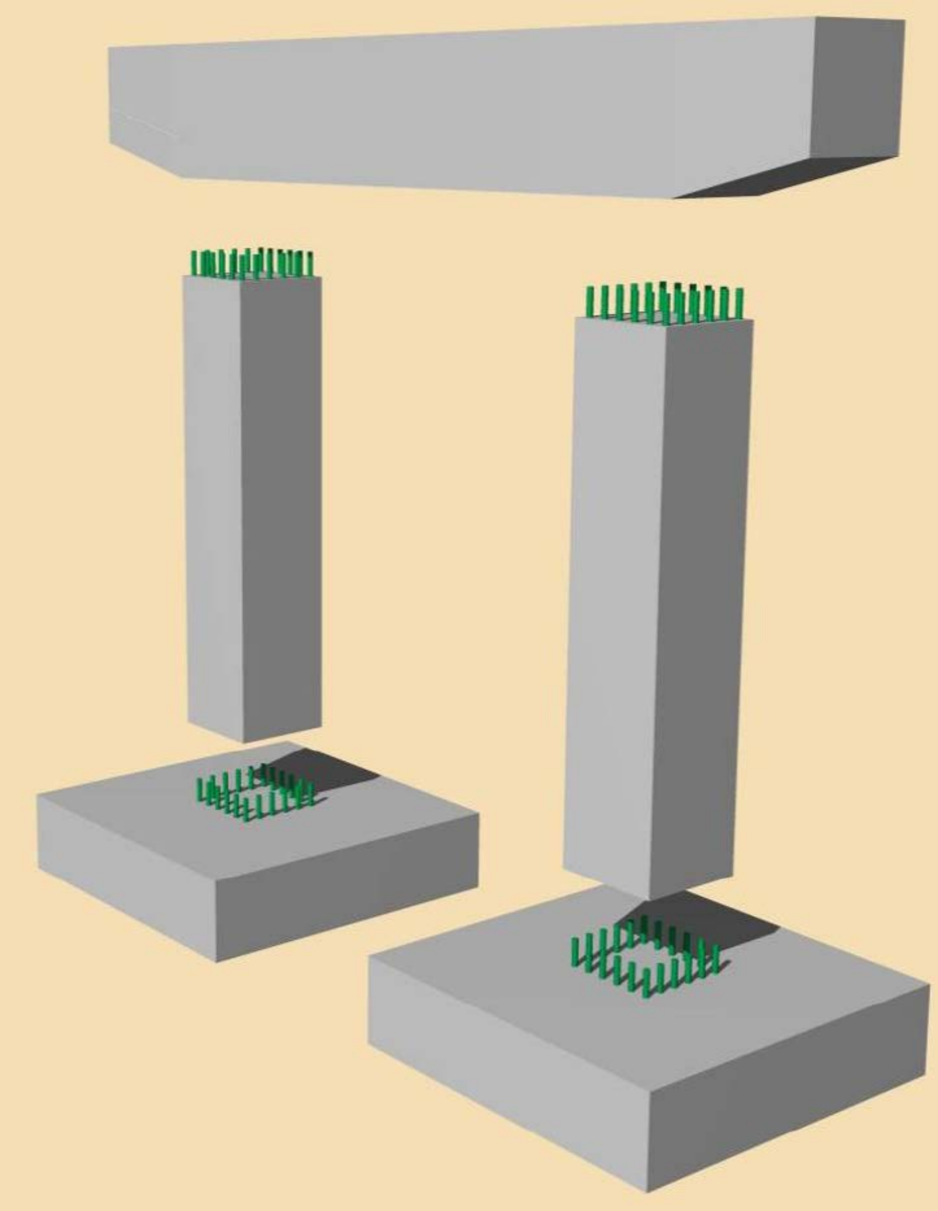
Mahin, S., Sakai, J., & Jeong, H. (2006, September). Use of partially prestressed reinforced concrete columns to reduce post-earthquake residual displacements of bridges. In *Fifth National Seismic Conference on Bridges & Highways, San Francisco, California*.

Accelerated Bridge Construction

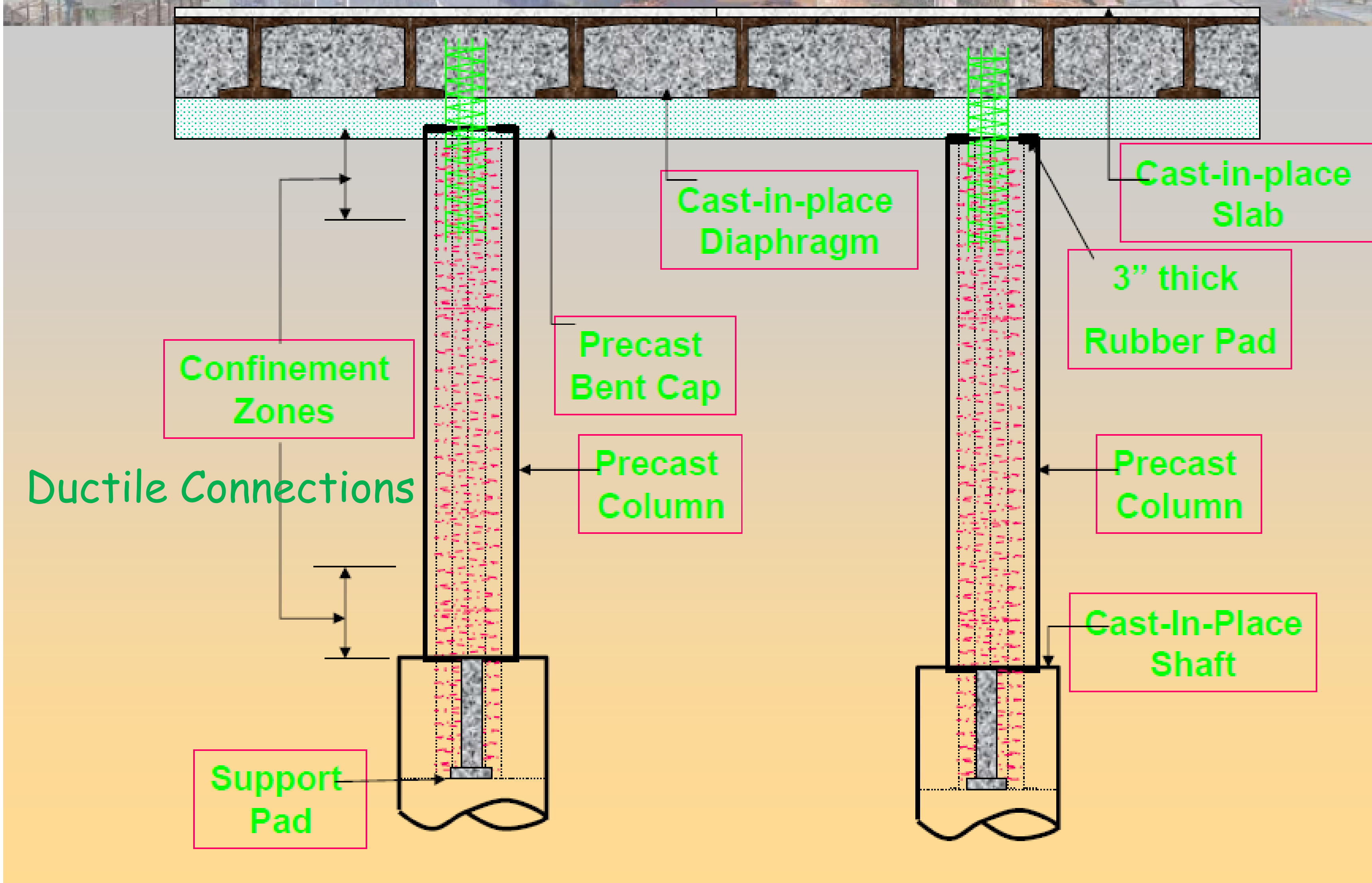
Synthesis and Erection of Prefabricated Bridges

Traditional Prefabricated Concrete Pier System

Growing Accelerated Bridge Construction Technology

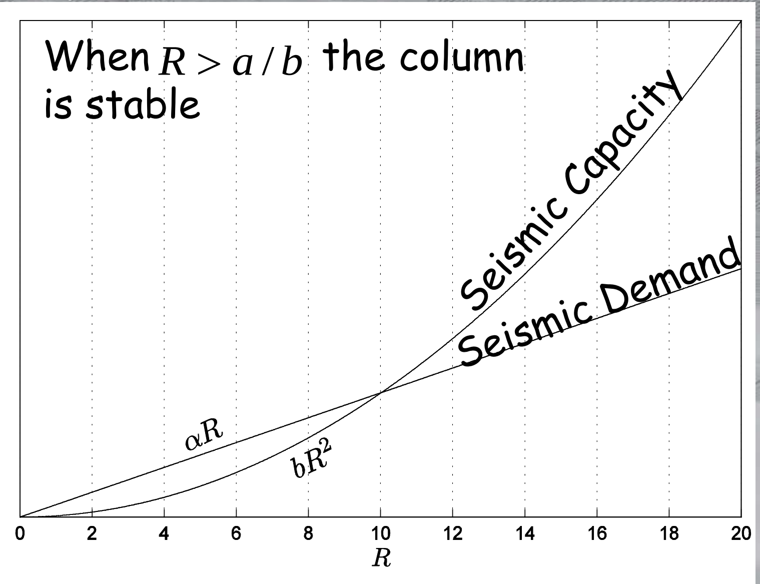


Washington DOT - Precast System



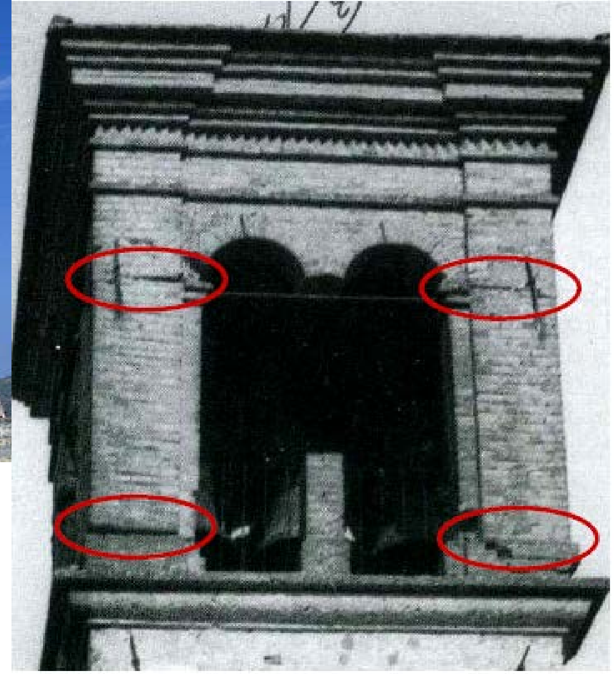
From: TRB Research Proposal Webinar, available on the internet

The Choragic Monument of Thrasyllus at the Foohills of Athens Acroplolis



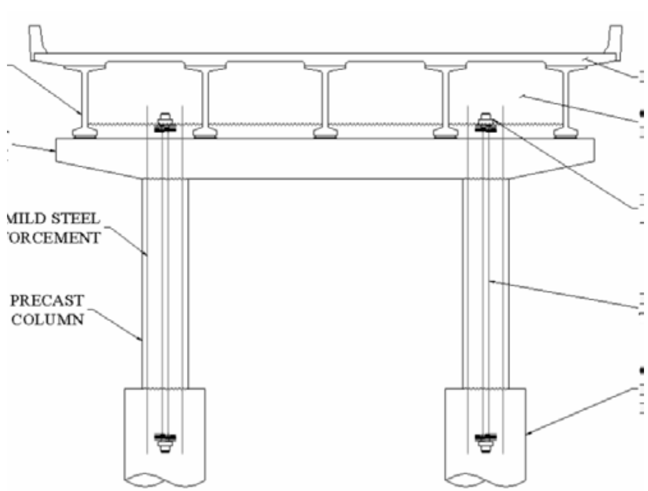
These two slender solitary free-standing columns remain standing for some 2.5 millenias, not because of any ductile connection or any continuation of the steel reinforcement, but because; while most slender, they are tall enough to survive the through-the-centuries intense shaking that has damaged repeatedly the "well-engineered" structures in the city of Athens

1. Free-Standing Rocking Frame

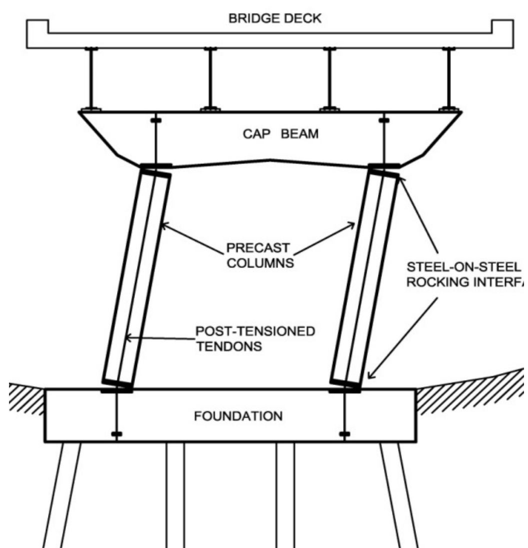


3. Pre-fabricated bridges with ductile connections

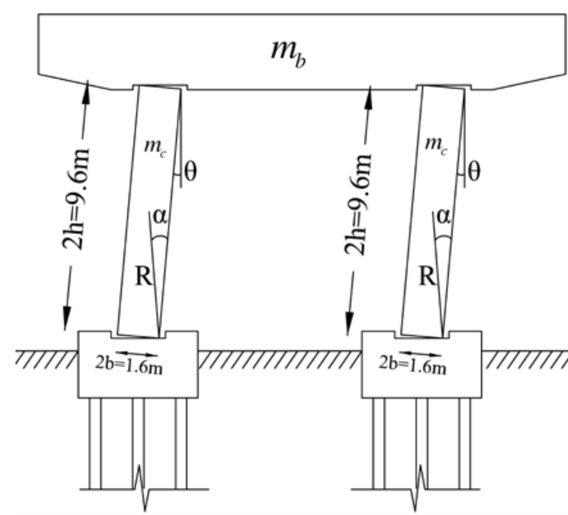
4. The Hybrid Rocking Frame Static Concept



5. Damage Avoidance Design



6. Free-Standing Rocking Frame Dynamic Concept



Dear Anil: *During the last 1/4 century I have enjoyed being either your next-door neighbor on the 7th floor in Davis Hall, or your distant colleague in Greece reviewing manuscripts submitted in EESD. Whatever was our interaction, I was continuously learning from your wisdom and your lucid views.*

Thank you for this life-long mentoring.