MONOLITHIC AND PARTITIONED L-STABLE ROSENBROCK METHODS FOR DYNAMIC SUBSTRUCTURE TESTS

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ABSTRACT

Real-time testing with dynamic substructuring is a hybrid numerical-experimental technique, the challenge of which is to ensure that both numerical and physical substructures interact in real-time, in order to simulate the behaviour of an emulated structure. With this objective in mind, the development and implementation of partitioned real-time compatible Rosenbrock algorithms are presented in this paper. In detail, we shortly introduce monolithic linearly implicit L-stable algorithms with one and two stages, and present an *ad hoc* real-time implementation and application of substructure tests. In view of the analysis of complex emulated structures, we also present novel partitioned algorithms conceived under the broad framework of non-overlapped domain decomposition techniques. Both the stability and accuracy properties of the proposed algorithms are examined through analytical and numerical studies carried out on Single-DoF model problems. Also the parallel interfield case is developed. Finally, a novel test rig conceived to perform both linear and nonlinear substructure tests is introduced.

INTRODUCTION

In recent years real-time hybrid testing techniques, like the Real-time Testing with Dynamic Substructuring (RTDS) and the Hardware-in-the-Loop (HiL) technique, became more and more popular in order to study the performance of components and structures subject to dynamic loads (Saouma and Sivaselvan 2008; Bursi and Wagg 2008). With regard to time-stepping methods, they can be broadly classified in monolithic and partitioned. In a monolithic approach, the method integrates: (i) the Numerical Substructure (NS) only, whilst the Physical Substructure (PS) can be considered as a *black box* (Bursi et al. 2008) or as a *grey box* (Lamarche et al. 2008), with estimates of stiffness and damping of the PS included in the Jacobian matrix -see Fig. 1a-1d-; (ii) both the NS and the PS by means of stiffness estimates (Jung et al. 2007), like in a typical pseudo-dynamic (PsD) test. Conversely, a partitioned approach typically solves both NS and PS through different integrators and takes into account the interface problem, for instance by prediction, substitution and synchronization of Lagrange multipliers (Pegon and Magonette 2002). In detail, partitioned algorithms can be applied to the Euler-Lagrange form of the equations of motion -second-order in time- (Prakash and Hjelmstad 2004, Bonelli et al. 2008) or to the Hamilton form of the equations of motion -first-order in time- (Nakshatrala et al. 2008). In this paper, we consider both monolithic and partitioned approaches based on L-stable real-time compatible Rosenbrock (LSRT) algorithms applied to equations of motion first-order in time.

Most of the aforementioned research works carried out on substructure tests considered structural integrators applied to the equations of motion second-order in time. Nonetheless, it is well known that the motion of the PS in a substructure test, see Fig.1e, is driven by a transfer system –

actuator- and sensors, governed by a control unit. Since the control system is typically described by first-order Differential-Algebraic Equations (DAEs), the utilized integrators have to deal with mixed first- and second-order DAEs. In order to solve this problem, there are mainly three options: (i) to use different integrators for structural and control systems, respectively, –see for instance Wu et al. (2007), that utilizes the Newmark-β method for the emulated structure and a proprietary MTS controller with its own built-in time discretization; (ii) to reformulate the control equations in a second-order form (Brüls and Golinval, 2006), and employ a structural integrator like the Generalized-α (Chung and Hulbert, 1993) for both systems; to use first-order integrators like the LSRT Rosenbrock algorithms, for both structural and control systems. Herein, we adopt the last option owing to the favourable properties of LSRT algorithms employed in control (Vulcan, 2006).

As far as complex emulated structures are concerned, numerical and control requirements impose different time steps for a NS and a PS, respectively. As a result, two main techniques can be identified to tackle this problem: (i) model reduction, that represents an effective way to lower computation burdens related to the integration of a complex NS, but becomes very inaccurate especially for non-linear systems; (ii) multi-time methods that allow to employ different time integrators in distinct subdomains. Moreover, subcycling permits to use different time steps in different subdomains. The last strategy is relatively simple to implement, but it can hinder stability and accuracy properties of the original schemes. Therefore, the paper proposes some novel multi-time method with subcycling strategies, and also investigates relevant stability and accuracy issues.

The remaining part of the paper is organized as follows. Firstly, it introduces LSRT algorithms with one stage (LSRT1) and two stages (LSRT2), as well as the linearly implicit Chang's method (2002), which is nowadays used in real-time substructured tests. Then, some results of real-time tests performed both with Chang's and LSRT monolithic algorithms are presented. Conversely, partitioned algorithms and subcycling strategies based on the progenitor LSRT algorithms that represent the main focus of the paper are considered. These algorithms first solve the interface problem by means of Lagrange multipliers and subsequently advance the solution in all subdomains. Thus, stability and accuracy properties of these algorithms are analysed through numerical experiments on a Single-DoF split-mass system, including subcycling too. Lastly, a novel test rig is introduced, conceived to perform both linear and nonlinear substructure tests on Multiple-DoF systems with different configurations of NSs and PSs.

MONOLITHIC INTEGRATION METHODS FOR REAL-TIME TESTING

Linearly Implicit Real Time Compatible Algorithms

In this subsection, we introduce LSRT compatible algorithms developed and suggested by Bursi et al. (2008) as well as a linearly implicit Newmark-based algorithm, i.e., the Chang's method (Chang 2002). They are linearly implicit because eliminate the need to solve non-linear systems for nonlinear problems. In order to employ LSRT algorithms, the governing system equations of motion $\mathbf{M}\ddot{\mathbf{u}} = \mathbf{r}(\mathbf{u}, \dot{\mathbf{u}}, t)$ can be rewritten into a state-space form

$$\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y}, t), \text{ with } \mathbf{y} = \begin{Bmatrix} \mathbf{u} \\ \dot{\mathbf{u}} \end{Bmatrix}, \; \mathbf{f}(\mathbf{y}, t) = \begin{Bmatrix} \dot{\mathbf{u}} \\ \mathbf{r}(\mathbf{u}, \dot{\mathbf{u}}, t) \end{Bmatrix} \quad (1)$$

where **M** stands for the mass matrix which is assumed to be symmetric positive definite for simplicity and $\mathbf{r}(\mathbf{u},\dot{\mathbf{u}},t)$ for the vector of applied and internal forces. In a FE context, the forces usually split into $\mathbf{r}(\mathbf{u},\dot{\mathbf{u}},t) = -\mathbf{Ku} - \mathbf{C}\dot{\mathbf{u}} + \mathbf{F}_e$ with a stiffness matrix **K**, a damping matrix **C** and a displacement vector **u**. Differentiation with respect to time is expressed by a dot, and thus we set $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$ to define the corresponding velocity and acceleration vectors.

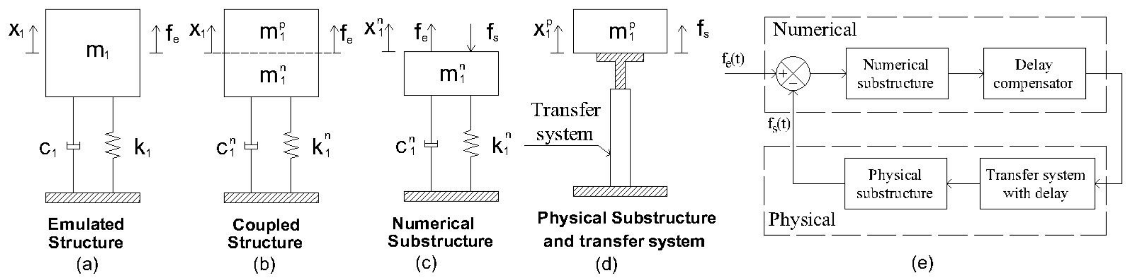


Fig. 1. (a)-(d) Schematic representation of a SDOF split-mass system; (e) block diagram representation including delay.

L-stable real-time one-stage (LSRT1) method. When applied to the differential equation $\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y},t)$ the one-stage real-time method exploits the following expression:

$$\mathbf{k}_1 = [\mathbf{I} - \gamma\Delta t\mathbf{J}]^{-1}\mathbf{f}(\mathbf{y}_k,t_k)\Delta t,\ \ \mathbf{y}_{k+1} = \mathbf{y}_k + b_1\mathbf{k}_1 \tag{2}$$

where $\mathbf{y}_k \approx \mathbf{y}(t)$ at $t = t_k$, $\Delta t = t_{k+1} - t_k$ is the step size and $\mathbf{J} = (\partial\mathbf{f}/\partial\mathbf{y})$ is the Jacobian matrix or an approximation thereof. The method is first order accuracy when $b_1 = 1$. The condition necessary to the method to achieve L-stability is $\gamma = 1$.

L-stable real-time two-stage (LSRT2) method. The two-stage real-time method applied to the differential equation $\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y},t)$ reads:

$$\mathbf{k}_1 = [\mathbf{I} - \gamma\Delta t\mathbf{J}]^{-1}\mathbf{f}(\mathbf{y}_k,t_k)\Delta t,\ \ \mathbf{y}_{k+\alpha_{21}} = \mathbf{y}_k + \alpha_{21}\mathbf{k}_1 \tag{3}$$

$$\mathbf{k}_2 = [\mathbf{I} - \gamma\Delta t\mathbf{J}]^{-1}\left(\mathbf{f}\left(\mathbf{y}_{k+\alpha_{21}},t_{k+\alpha_2}\right) + \mathbf{J}\gamma_{21}\mathbf{k}_1\right)\Delta t,\ \ \mathbf{y}_{k+1} = \mathbf{y}_k + b_1\mathbf{k}_1 + b_2\mathbf{k}_2 \tag{4}$$

For the LSRT2 algorithm we introduce two sets of parameters that satisfy second-order accuracy, L-stability and real-time compatibility: $\gamma = 1 - \sqrt{2}/2$ and $\gamma = 1 + \sqrt{2}/2$, respectively, together with $\alpha_2 = \alpha_{21} = 1/2$, $\gamma_{21} = -\gamma$, $b_1 = 0$ and $b_2 = 1$. The favourable performance of the LSRT2 method with respect to the low and high-frequency components of the response can be observed from Fig. 2, where a comparison with the Generalized-α method (Chung and Hulbert 1993) is illustrated.

A-stable Newmark-based method. For the sake of comparison, we introduce the Newmark-based method proposed by Chang (2002), whose equations read:

$$\begin{aligned} &\mathbf{M}\ddot{\mathbf{u}}_{k+1} + \mathbf{C}\dot{\mathbf{u}}_{k+1} + \mathbf{r}_{k+1} = \mathbf{f}_{e,k+1} \\ &\mathbf{u}_{k+1} = \mathbf{u}_k + \beta_1\Delta t\dot{\mathbf{u}}_k + \beta_2\Delta t^2\ddot{\mathbf{u}}_k \\ &\dot{\mathbf{u}}_{k+1} = \dot{\mathbf{u}}_k + \Delta t\left(\ddot{\mathbf{u}}_k + \ddot{\mathbf{u}}_{k+1}\right)/2 \end{aligned} \tag{5}$$

In detail, the two parameters β_1 and β_2 read

$$\beta_1 = \left[\mathbf{I} + \Delta t \mathbf{M}^{-1}\mathbf{C}/2 + \Delta t^2 \mathbf{M}^{-1}\mathbf{K}_0/4\right]^{-1}\left[\mathbf{I} + \Delta t \mathbf{M}^{-1}\mathbf{C}/2\right]$$
$$\beta_2 = \left[\mathbf{I} + \Delta t \mathbf{M}^{-1}\mathbf{C}/2 + \Delta t^2 \mathbf{M}^{-1}\mathbf{K}_0/4\right]^{-1}/2 \qquad (6)$$

where $\mathbf{K}_0$ represents the initial stiffness matrix. This method is linearly implicit and second order accurate as the LSRT2 algorithm. Nonetheless and differently from the LSRT algorithms, the method is linearly implicit only with respect to nonlinear restoring forces while nonlinear damping forces render the method implicit. Moreover, the method is A-stable but it does not exhibit high-frequency dissipation capabilities as depicted in Fig. 2.

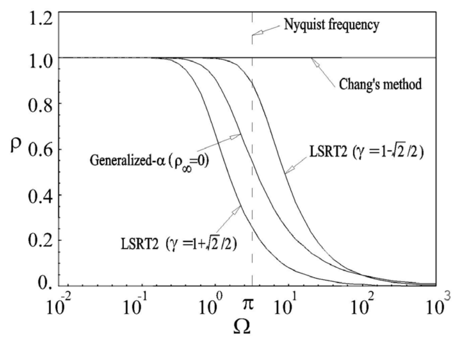


Fig. 2. Spectral radii ρ of linearly implicit algorithms with respect to the Generalized-α method vs. the non-dimensional frequency Ω.

Real-Time Application to a Linear SDOF Split-Mass System

In order to investigate the performance of the aforementioned linearly implicit integrators applied to a Single-DoF system, we consider the split mass system depicted in Fig. 1. System parameters were chosen to be $m_1^n = 5.4\ kg$, $c_1^n = 20\ kgs/m$, $k_1^n = 100\ kg/m$, $m_1^p = 1.8\ kg$ and $c_1^p = 0$. Moreover, a sinusoidal external excitation with f=3 Hz and amplitude A=10 N was considered. No particular care was adopted to minimize load cell errors. The displacements of the Single-DoF system integrated with the Chang's method and the LSRT2 method with $\gamma = 1 + \sqrt{2}/2$ are represented in Fig. 3a and 3b, respectively. The Chang's method leads to instability mainly caused by measurement errors and phase lag. Conversely, the numerical damping introduced by the LSRT2 algorithm allows the substructure test to be carried out.

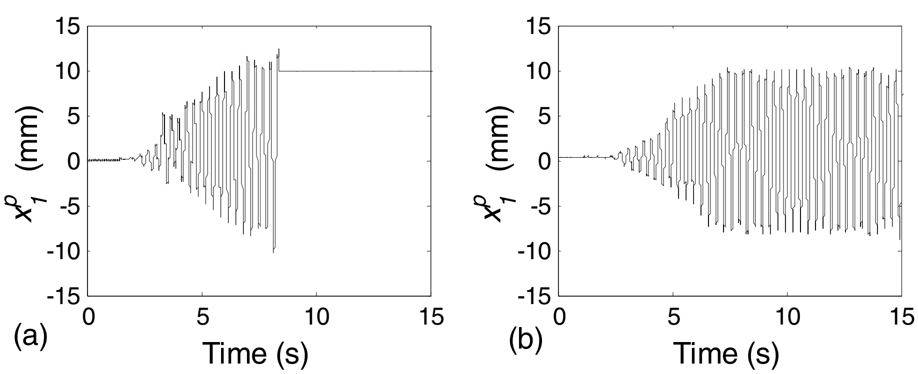


Fig. 3. Experimental results of a Single-DoF split-mass system, excited by a sinusoidal force with A=10 N and f=3 Hz, by using (a) the Chang's method, (b) the LSRT2 method.

PARTITIONED INTEGRATION METHODS FOR REAL-TIME TESTING

With respect to the Newmark's family of integrators, it has been shown by Prakash and Hjelmstad (2004) and Bonelli et al. (2008) that the enforcement of the continuity of velocity along interface enables stable solutions. Because we focus on partitioned integrators applied to the Hamilton form of the equations of motion, we consider the continuity of acceleration. Therefore, the key strategy of the partitioned algorithm is as follows: Lagrange multipliers are computed via an acceleration constraint at interfaces and then each subdomain is advanced in time by using linearly implicit integrators.

The general form of the dynamic equilibrium equations for coupled dynamic systems by using a Lagrange multipliers technique can be written in a compact form as

$$\begin{cases} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}} \\ \ddot{\mathbf{u}} \end{Bmatrix} = \begin{Bmatrix} \dot{\mathbf{u}} \\ \mathbf{f}(\mathbf{u}, \dot{\mathbf{u}}, t) \end{Bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{G}^T \end{bmatrix} \boldsymbol{\lambda} \\ [\mathbf{0}, \mathbf{G}] \begin{Bmatrix} \dot{\mathbf{u}} \\ \ddot{\mathbf{u}} \end{Bmatrix} = \mathbf{0} \end{cases} \tag{7}$$

With the assumption $\mathbf{y} = \{\mathbf{u}^T \quad \dot{\mathbf{u}}^T\}^T$, the system (7) can be transformed into

$$\begin{cases} \mathbf{A}\dot{\mathbf{y}} = \mathbf{F}(\mathbf{y}, t) + \mathbf{C}^T \boldsymbol{\Lambda} \\ \mathbf{C}\dot{\mathbf{y}} = \mathbf{0} \end{cases} \tag{8}$$

Here, we can solve (8) for the vector $\dot{\mathbf{y}}$

$$\dot{\mathbf{y}} = \mathbf{A}^{-1}\mathbf{F}(\mathbf{y}, t) + \mathbf{A}^{-1}\mathbf{C}^T \boldsymbol{\Lambda} \tag{9}$$

and then the Lagrange multiplier vector $\boldsymbol{\Lambda}$, i.e.,

$$\boldsymbol{\Lambda} = -\left[\mathbf{C}\mathbf{A}^{-1}\mathbf{C}^T\right]^{-1} \mathbf{C}\mathbf{A}^{-1}\mathbf{F}(\mathbf{y}, t) \tag{10}$$

Then we can advance all the subdomains from t_k to t_{k+1}. For brevity, we only describe the case that uses the LSRT2 algorithm for two subdomains A and B but we consider subcycling. Also the algorithm with LSRT2 and LSRT1 algorithms was developed but it is not treated here. Let's assume that a subdomain A is integrated with a course time step Δt_A and a subdomain B with a fine time step Δt_B.

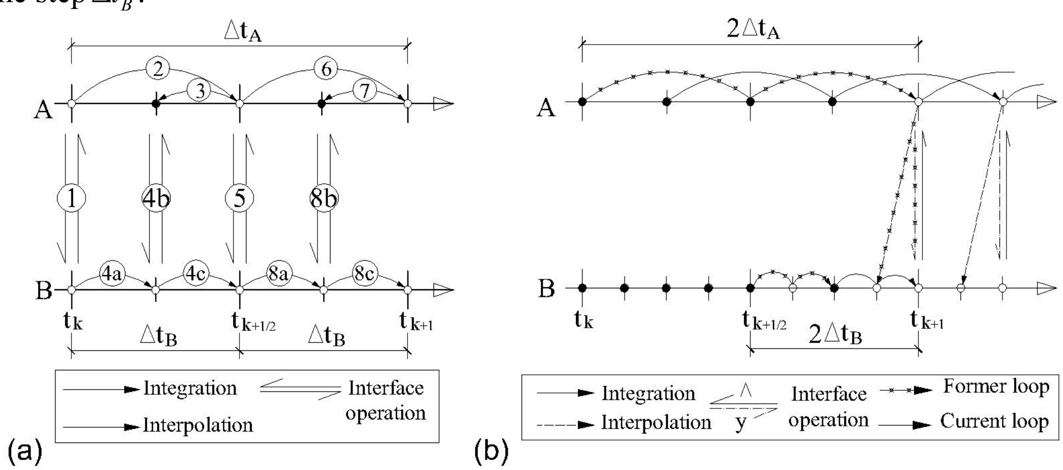


Fig. 4. The multi-time-step partitioned algorithm with *ss*=2: (a) staggered procedure; (b) interfield parallel procedure.

The proposed solution procedure, sketched in Fig. 4a can be summarized in the following pseudo-code:

1. compute the Lagrange multiplier vector $\mathbf{\Lambda}_k$ by solving the interface problem (10);
2. solve for the intermediate point $t_{k+1/2}$ in subdomain A by means of (3) -First stage-;
3. interpolate the internal solutions of A with $\mathbf{y}_{A,k+\frac{j}{2ss}} = (1-\frac{j}{ss})\mathbf{y}_{A,k} + \frac{j}{ss}\mathbf{y}_{A,k+1/2}$;
4. loops on the $ss/2$ substeps of the subdomain B from t_k to $t_{k+1/2}$ with j=1, 2, …, $ss/2$;
   (a) solve for the point $t_{n+j/2/ss}$ in subdomain B by means of (3) -First stage-;
   (b) compute the Lagrange multipliers $\mathbf{\Lambda}_{k+j/2/ss}$ by means of (10);
   (c) solve for the point $t_{n+(j+1)/2/ss}$ in subdomain B by means of (4) -Second stage-;
5. compute the Lagrange multiplier vector $\mathbf{\Lambda}_{k+1/2}$ by means of (10);
6. solve for the point t_{k+1} in subdomain A by means of (4) -Second stage-;
7. interpolate the internal solutions of A with $\mathbf{y}_{A,k+\frac{ss+j}{2ss}} = (1-\frac{j}{ss})\mathbf{y}_{A,k+1/2} + \frac{j}{ss}\mathbf{y}_{A,k+1}$;
8. loops on the $ss/2$ substeps of the subdomain B from $t_{k+1/2}$ to t_{k+1} with j=1, 2, …, ss/2
   (a) solve for the point $t_{n+(ss+j)/2/ss}$ in subdomain B by means of (3) -First stage-;
   (b) compute the Lagrange multiplier vector $\mathbf{\Lambda}_{k+(ss+j)/2/ss}$ by means of (10);
   (c) solve for the point $t_{n+(ss+j+1)/2/ss}$ in subdomain B by means of (4) -Second stage-.

Absolute Stability Properties of the Proposed Multi-Time-Step Partitioned Methods

Herein, the absolute stability of the proposed methods is investigated by means of a spectral approach applied to a Single-DoF split-mass system. When applied to linear problems, the methods can be recast into a recursive form as

$$\mathbf{y}_{k+1} = \mathbf{R}\mathbf{y}_k + \mathbf{L}_k \tag{11}$$

where **R** is the amplification matrix and **L** is the load vector that depends on external forces, respectively. In detail, we consider an SDOF split-mass system, with the assumption

$$b_1 = \frac{m_A}{m_B} = \frac{k_B}{k_A} \tag{12}$$

The absolute values of the eigenvalues $|\lambda_i|$ relevant to the model problem integrated with the staggered partitioned algorithm vs. the numerical frequency $\Omega = \sqrt{k/m}\Delta t_A$ are plotted in Fig. 5. Since the algorithm is real-time compatible, the size of the state vector involved in the recurrence (11) is 4, and therefore, the number of non-zero eigenvalues of the amplification matrix is 4. Among them, only one pair of complex conjugate eigenvalues are principle eigenvalues, whereas the other two are spurious. One of the two spurious eigenvalues is unitary. From the stability analysis, we can conclude that when ss=1, the scheme is stable. This has also been analytically proved by means of linear recurrences. When ss>1, the scheme appears to be stable for all cases examined.

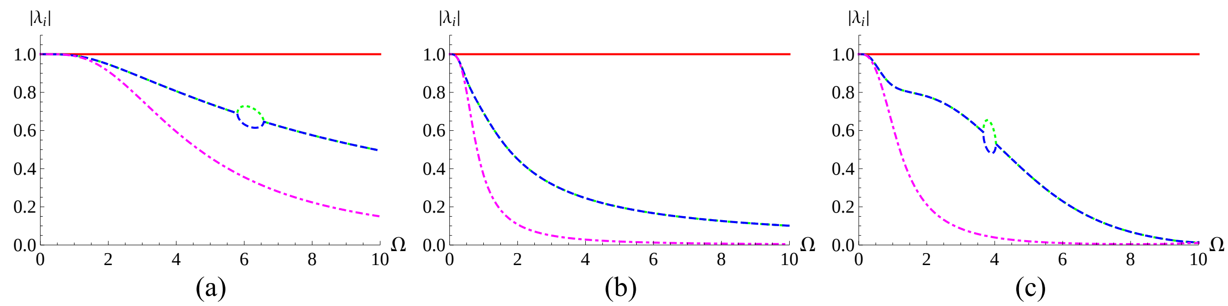


Fig. 5. $\left|\lambda_i\right|$ **of the SDOF problem for the staggered procedure: (a)** b_1**=0.5,** *ss***=1,** $\gamma = 1-\sqrt{2}/2$**;**
(b) b_1**=0.5,** *ss***=1,** $\gamma = 1+\sqrt{2}/2$**; and (c)** b_1**=1,** *ss***=10,** $\gamma = 1+\sqrt{2}/2$**.**

Moreover, both the algorithmic damping $\bar{\xi}$ and the frequency error $\left(\Omega-\bar{\Omega}\right)/\Omega$ determined by the principal eigenvalues are depicted in Fig. 6. We can observe that the partitioned algorithms exhibit favourable dissipation capabilities in the high frequency range as the progenitor algorithms.

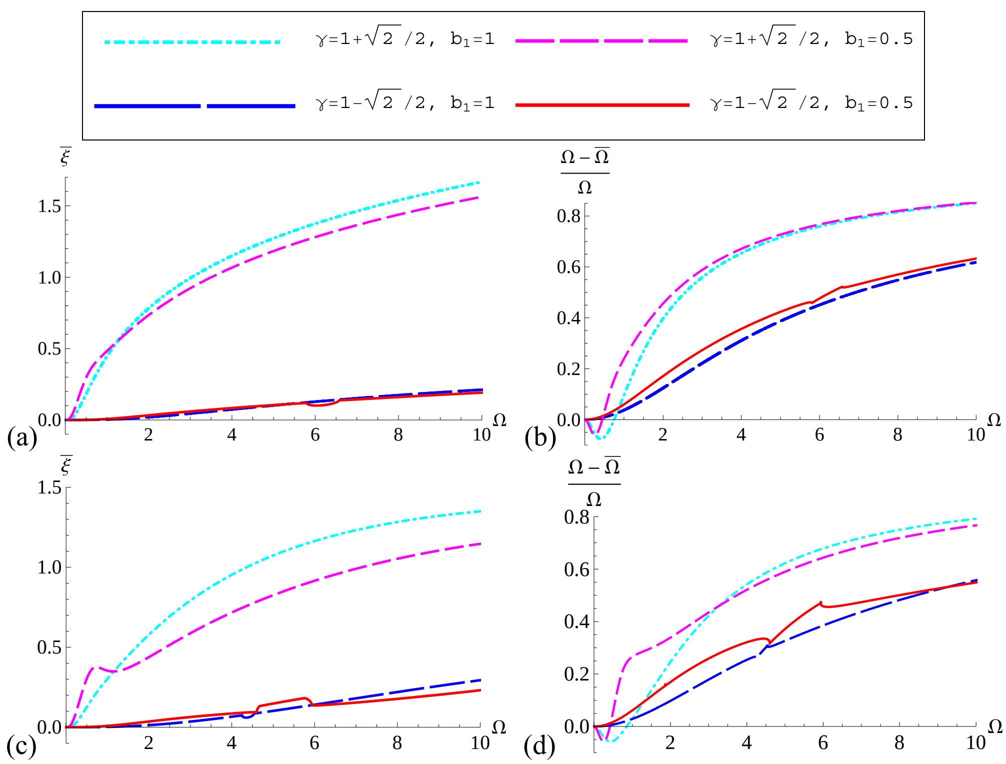
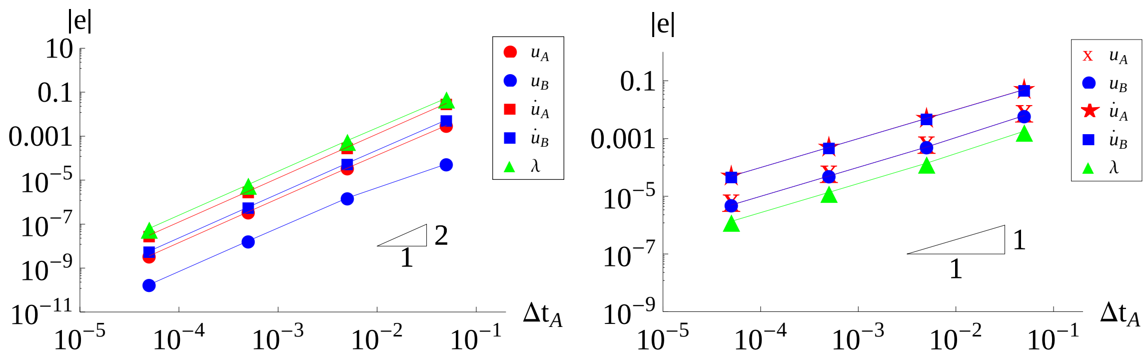


Fig. 6. Algorithmic damping ratio $\bar{\xi}$ **and relative frequency error for the staggered procedure: (a), (b)** *ss***=1; (b), (c)** *ss***=10.**

Convergence Analysis for the Staggered Algorithm

With regard to the convergence of the proposed partitioned algorithm, both the local truncation error and the global error were investigated through purely analytical means. For simplicity, we present herein the global error analysis carried out for the LSRT2 algorithm by means of numerical experiments performed on a Single-DoF split-mass system. Fig. 7(a) shows the global error $|e|$ versus Δt_A for the case with subcycling *ss*=10, which demonstrates that the proposed algorithm preserves second-order convergence even in the subcycling case. In addition, the global error achieved by the partitioned method proposed by Nakshatrala et al. (2008) is presented in Fig. 7(b). One can observe that their method based on the midpoint rule is only first-order accurate.

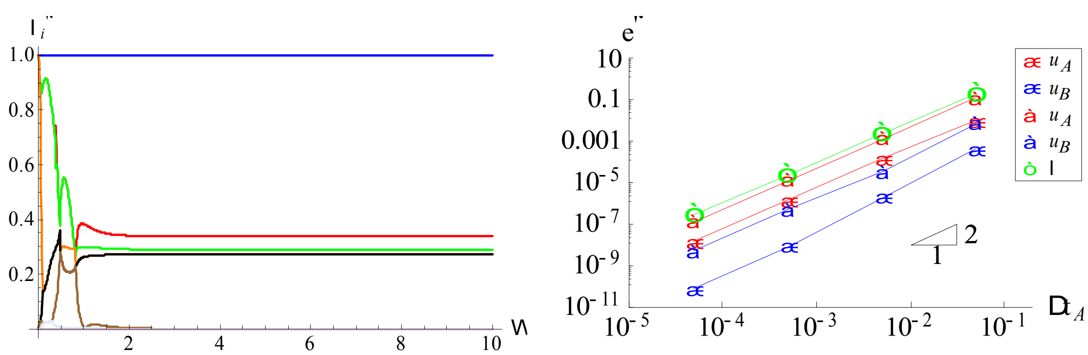


(a) **(b)**

Fig. 7. Global error for the staggered procedure with b_1=0.5 and *ss*=10: (a) the LSRT2 algorithm with $\gamma = 1 - \sqrt{2}/2$, (b) the midpoint rule algorithm.

Extension to a Parallel Partitioned Algorithm

In view of real-time applications, we conceived an interfield parallel algorithm, which is sketched in Fig. 4b. Needless to say that the interfield process ongoing in both subdomains A and B is inherently parallel, i.e., the integrations of the two subdomains are independent in each step. Some convergence properties of the parallel algorithm are presented in Fig. 8. For the case $\gamma = 1 + \sqrt{2}/2$, several numerical experiments show that the parallel algorithm is always stable; while the algorithm with $\gamma = 1 - \sqrt{2}/2$ sometime becomes conditionally stable. In addition, the parallel algorithm preserves second-order accuracy as the progenitor monolithic LSRT2 method.

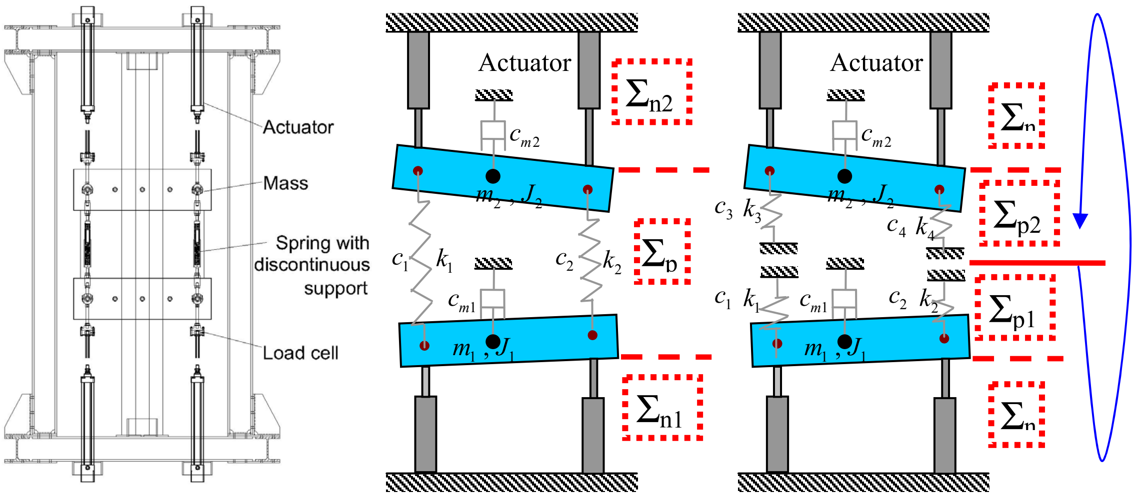


(a) **(b)**

Fig. 8. (a) $|\lambda_i|$, and (b) global error of the SDOF problem for the parallel procedure with $\gamma = 1 + \sqrt{2}/2$, b_1=0.5 and *ss*=10.

A TEST RIG FOR LINEAR AND NON-LINEAR MDOF SYSTEMS

A novel test rig has been designed to perform both linear and nonlinear substructure tests in real time on Multiple-DOFs systems. A sketch of the set-up is shown in Fig. 9a: it is characterized by two masses and four-DOFs. The non-linearities at this stage derive from mass rotations and springs with discontinuous supports. Also non-linear dampers can be introduced. The test rig will also be used to test linear and nonlinear control techniques. To evaluate the performance of the partitioned algorithms for real-time hybrid testing, we conceived two different configurations that are shown in Fig. 9 b and 9c, respectively.



(a) (b) (c)

Fig. 9. Test rig: (a) experimental set-up; (b) and (c) schematic configurations of numerical and physical substructures.

CONCLUSIONS

In this paper, we introduced and applied linearly implicit L-stable Rosenbrock methods with one-stage and two-stages to real-time dynamic substructure tests. The methods are endowed with several favourable characteristics, among which real-time compatibility, explicit evaluation of state variables and user-defined high-frequency dissipation capabilities. In detail, the favourable properties of the monolithic methods were proved through real-time tests on an SDOF split-mass system, in the case of not negligible experimental errors. In view of hybrid testing of complex emulated structures, we developed and illustrated a novel staggered partitioned algorithm based on the progenitor Rosenbrock method that can incorporate subcycling. Through spectral analysis and numerical simulations on an SDOF split-mass system, both stability and accuracy properties were shown. In a greater detail, the partitioned algorithm preserved second-order accuracy as the progenitor monolithic method, and favourable stability properties. Moreover in view of real-time applications, the method was extended to be an interfield parallel procedure. Finally, a novel test rig was conceived and presented to perform both linear and non-linear substructure tests on Multiple-DoF systems. It will allow also an in-depth study of linear and non-linear control strategies of transfer systems.

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